All India Aakash Test Series for JEE (Advanced)-2020

TEST - 2A (Paper-1) - Code-C

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Test Date : 24/11/2019

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(C)	21.	(B)	41.	(B)
2.	(B)	22.	(A)	42.	(C)
3.	(B)	23.	(B)	43.	(B)
4.	(D)	24.	(B)	44.	(D)
5.	(D)	25.	(B)	45.	(A)
6.	(C)	26.	(D)	46.	(B)
7.	(A, B, D)	27.	(B, D)	47.	(B, C, D)
8.	(B)	28.	(A, B, D)	48.	(A, B)
9.	(B, D)	29.	(A, B, C)	49.	(A, B, C)
10.	(B, D)	30.	(C)	50.	(C, D)
11.	(B, C, D)	31.	(A)	51.	(A, D)
12.	(A)	32.	(C)	52.	(A)
13.	(C)	33.	(C)	53.	(D)
14.	(C)	34.	(B)	54.	(B)
15.	(D)	35.	(C)	55.	(C)
16.	A $\rightarrow$ (P, S)	36.	A $\rightarrow$ (Q, S, T)	56.	A $\rightarrow$ (Q, R, S)
	B $\rightarrow$ (Q, R)		B $\rightarrow$ (P, R, S)		B $\rightarrow$ (Q)
	C $\rightarrow$ (P, S)		C $\rightarrow$ (P, R, S, T)		C $\rightarrow$ (R, S)
	D $\rightarrow$ (P, R)		D $\rightarrow$ (Q, R, S)		D $\rightarrow$ (P, T)
17.	A $\rightarrow$ (P, T)	37.	A $\rightarrow$ (Q, R, T)	57.	A $\rightarrow$ (S)
	B $\rightarrow$ (Q, R)		B $\rightarrow$ (P)		B $\rightarrow$ (Q, R, S, T)
	C $\rightarrow$ (R, S, T)		C $\rightarrow$ (R, S, T)		C $\rightarrow$ (R)
	D $\rightarrow$ (Q, R)		D $\rightarrow$ (T)		D $\rightarrow$ (P, Q, R, S, T)
18.	(16)	38.	(25)	58.	(36)
19.	(50)	39.	(04)	59.	(16)
20.	(29)	40.	(12)	60.	(45)

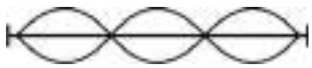
HINTS & SOLUTIONS

PART - I (PHYSICS)

1. Answer (C)

Hint: Second overtone contains 3 loops

Solution:



$$\frac{\lambda}{2}=\frac{2}{3}\Rightarrow\lambda=\frac{4}{3}\text{m}$$

Amp. = 2*A* sin *kx* = A_{max} sin(kx)

$$\therefore A=(A_{\max})\sin\left(\frac{2\pi\times3}{4}\times\frac{1}{6}\right)$$

$$\Rightarrow A=(2)\times\left(\frac{1}{\sqrt{2}}\right)\text{mm}$$

$$=\sqrt{2}\text{ mm}$$

2. Answer (B)

Hint : At maximum temperature $\frac{dT}{dV}=0$

Solution :

$$\left[P_0+(1-\alpha)V^2\right]V=nRT$$

$$\Rightarrow T=\frac{P_0V+(1-\alpha)V^3}{nR}$$

$$\therefore\frac{dT}{dV}=0\text{ at }V^2=\frac{P_0}{3(\alpha-1)}$$

$$\therefore P=P_0+(1-\alpha)\times\frac{P_0}{3(\alpha-1)}$$

$$\Rightarrow P=\frac{2P_0}{3}$$

3. Answer (B)

Hint : $Q = Q_0e^{-t/\tau}$ during discharging

Solution :

$$Q_{.0}=CV_0, C_2\left(\frac{C}{K}\right)$$

$$\therefore V_2=\frac{CV_0}{\left(\frac{C}{K}\right)}=KV_0$$

$$\tau=R\times C_2=\frac{RC}{K}$$

$$\therefore V=V_2e^{-t/\tau}$$

$$\Rightarrow\frac{V_0}{2}=KV_0\times e^{-\frac{t}{\tau}}$$

$$\Rightarrow\frac{1}{2K}=e^{-t/\tau}$$

$$\Rightarrow\ln(2K)=\frac{t}{\tau}$$

$$\Rightarrow t=\tau\ln(2K)$$

$$t=\frac{RC}{K}\ln(2K)$$

4. Answer (D)

Hint: Use reverse symmetry concept

Solution:

Using KVL and KCL, we get

$$R_{eq}=\frac{2R_1R_2+R_2R_3+R_3R_1}{(R_1+R_2+2R_3)}$$

$$=\frac{2\times(2\times3)+(3\times1)+(1\times2)}{(2+3+1\times2)}$$

$$=\frac{12+3+2}{7}=\frac{17}{7}\Omega$$

5. Answer (D)

Hint: Heat current remains constant

Solution :

$$\frac{\frac{(T_1-T)}{L}}{k2\pi a\times\left(\frac{a+b}{2}\right)}=\frac{\frac{T_1-T_2}{L}}{k\times\pi a\times b}$$

$$\Rightarrow T=\frac{T_1a+T_2b}{(a+b)}$$

6. Answer (C)

Hint : $E_{\text{axis}}=\frac{qx}{4\pi\varepsilon_0(R^2+x^2)^{3/2}}$

Solution:

$$E=\frac{q}{4\pi\varepsilon_0d^2}-\frac{q\times d}{4\pi\varepsilon_0(d^2+R^2)^{3/2}}$$

$$=\frac{3qR^2}{8\pi\varepsilon_0d^4}$$

7. Answer (A, B, D)

Hint: Use KVL and KCL.

Solution :

$$R_{eq}=1+\frac{20}{9}=\frac{29}{9}\ \Omega$$

$$\therefore I_0=\frac{58}{(29/9)}=18\text{ A}$$

$$\therefore I_{(2\Omega)} = \frac{18}{2} = 9\text{ A}$$

$$I_{(3\Omega)} = \frac{6}{6+3} \times 9 = 6\text{ A}$$

$$I_{(5\Omega)} = \frac{4}{9} \times (18) = 8\text{ A}$$

$$I_{(4\Omega)} = \frac{5}{9} \times (18) = 10\text{ A}$$

$$\therefore V_{(4\Omega)} = 4 \times 10 = 40\text{ V}$$

$$P_{(5\Omega)} = 8^2 \times 5 = 320\text{ W}$$

8. Answer (B)

Hint : Use Gauss's law

Solution :

σ on outer surface becomes uniform. Potential at outside points is only due to charge on outer surface of shell.

$\therefore V_A = V_B$

9. Answer (B, D)

Hint : Apparent wavelength changes when source moves.

Solution :

$$f' = \frac{(340-10)}{(340-20)} \times (200) = 206\text{ Hz}$$

$$\lambda' = \lambda_0 = V_s \times T = \frac{340}{200} - 20 \times \frac{1}{200} = 1.6\text{ m}$$

10. Answer (B, D)

Hint : $V_{rms}^2 = \frac{\int u^2 dN}{N}$

Solution :

$$N = \text{Area} = \frac{1}{2} \times 10 \times 10 = 50$$

$$\frac{dN}{du} = u + 10$$

$$\therefore V_{rms}^2 = \frac{\int u^2 \times (10-u)du}{N} = \frac{\int_0^{10} (10u^2 - u^3)du}{50}$$

$$V_{rms}^2 = \frac{1000 \times (4-3)}{12 \times 50} = \frac{2500}{3 \times 50}$$

$$\Rightarrow V_{rms} = \sqrt{\frac{50}{3}}\text{ m/s}$$

11. Answer (B, C, D)

Hint : Use KVL and KCL.

Solution :

Q_{total} = 180 – 70 = 110 μC

$$q_A = \frac{2}{2+6+3} \times (110) = 20\ \mu\text{C}$$

$$q_B = \frac{6}{2+3+6} \times (110) = 60\ \mu\text{C}$$

$$q_C = \frac{3}{2+6+3} \times (110) = 30\ \mu\text{C}$$

$$\Delta q_S = (20+30) - (-70) = 120\ \mu\text{C}$$

12. Answer (A)

Hint: Flux is proportional to charge

Solution :

$$\frac{2\pi(1-\cos\alpha)}{4\pi} \times \left(\frac{q_1}{\varepsilon_0}\right)$$

$$= \frac{2\pi(1-\cos\beta)}{4\pi} \times \left(\frac{q_2}{\varepsilon_0}\right)$$

$$\Rightarrow \frac{q_1}{q_2} = \frac{1-\cos\beta}{1-\cos\alpha} = \frac{1-0}{1-\frac{1}{2}} = 2$$

13. Answer (C)

Hint : Flux is proportional to charge

Solution:

$q_1 = 3q_2$

$\Rightarrow$ one third of total flux of q_1 will terminate at q_2

$$\therefore \frac{4\pi}{3} = 2\pi(1-\cos\alpha_{max})$$

$$\Rightarrow \cos(\alpha_{max}) = 1 - \frac{2}{3} = \frac{1}{3}$$

$$\Rightarrow \tan(\alpha_{max}) = 2\sqrt{2} \quad \Rightarrow \alpha_{max} = \tan^{-1}(2\sqrt{2})$$

14. Answer (C)

Hint : $A \to B$ Isochoric $\quad C \to D$ Isochoric

$B \to C$ Isothermal $\quad D \to A$ Isothermal

15. Answer (D)

Hint : W isothermal $= nRT_0 \ln\left(\frac{V_2}{V_1}\right)$

Solution of Q.Nos. 14 and 15

$$W_{BC} = 2P_0V_0 \ln\left(\frac{V_C}{V_B}\right) = -P_0V_0 \ln(2)$$

$$\Rightarrow V_C = \frac{V_0}{\sqrt{2}}$$

$$\therefore P_C = \frac{2P_0V_0}{V_C} = 2\sqrt{2}P_0$$

$$\therefore W_{DA} = \left(\sqrt{2}P_0\right)\left(\frac{V_0}{\sqrt{2}}\right)\ln\left(\sqrt{2}\right) = \frac{P_0V_0}{2}\ln(2)$$

$$\therefore W_{ABCDA} = 0 + -P_0V_0\ln(2) + 0 + \frac{1}{2}P_0V_0\ln(2)$$

$$= -\frac{P_0V_0}{2}\ln(2)$$

16. Answer A(P, S); B(Q, R); C(P, S); D(P, R)

Hint : Capacitance increases due to slab.

Solution :

Total capacitance increases, so charge on *A* increases as well as voltage increases.

$\therefore$ Voltage on *B* decreases. So, charge on it decreases

$\therefore$ Charge on *C* and *D* increases

17. Answer A(P, T); B(Q, R); C(R, S, T); D(Q, R)

Hint : Use Gauss's law.

Solution :

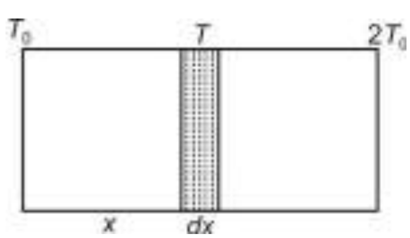
Electric field is uniform in spherical cavity in sphere and in cylindrical cavity in cylinder.

18. Answer (16)

Hint : Use $PV = nRT$

Solution :

$$T = T_0 + \frac{T_0}{L}x$$



$$\therefore \int dn = \int_{x=0}^{L} \frac{P \times Adx}{R\left(T_0 + \frac{T_0}{L}x\right)}$$

$$\Rightarrow n = \frac{PAL}{RT_0}\ln(2)$$

$$\Rightarrow n = \frac{PAL}{4RT_0}\ln(16)$$

$\therefore$ 16

19. Answer (50)

Hint : Voltmeter are not ideal

Solution :

Let resistance of each voltmeter be R_0

$\therefore$ $Ri' = 20\, R_0\, (I - i)$ ....(i)

and $2Ri' = 30$ ....(ii)

$$\Rightarrow i' = \frac{3}{4}i, \qquad \therefore\ i_{(V_2)} = I - i' = I - \frac{3i}{4}$$

$$\therefore 2Ri' = 30 = R_0(I - i) = R_0\left(I - \frac{3i}{4}\right)$$

$$\Rightarrow i = 400\ \mu\text{A}$$

$$\therefore R = \frac{20}{400 \times 10^{-6}} = 50 \times 10^{3}\,\Omega$$

$= 50\ \text{k}\Omega$

20. Answer (29)

Hint: *BC* is isothermal

Solution :

$$(3P_0) \times V_C = P_0 \times V_0$$

$$\Rightarrow V_C = \frac{V_0}{3}$$

$\because$ *CA* is a adiabatic.

$$\therefore\ (3P_0) \times \left(\frac{V_0}{3}\right)^{\gamma} = \left(\frac{P_0}{2}\right)(V_0)^{\gamma}$$

$$\Rightarrow\ \gamma = \frac{\ln 6}{\ln 3} = \frac{\ln 2 + \ln 3}{\ln 3} = \frac{18}{11}$$

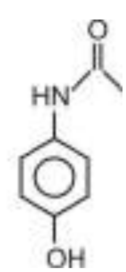
$\therefore$ $p + q = 18 + 11 = 29$

PART - II (CHEMISTRY)

21. Answer (B)

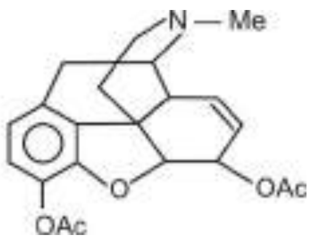
Hint :

Paracetamol

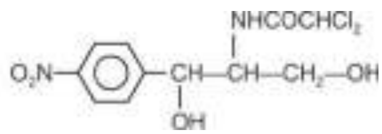


Solution :

Heroin



Chloramphenicol



22. Answer (A)

Hint : A paired with T (A = T)

Solution :

G paired with C (G ≡ C)

23. Answer (B)

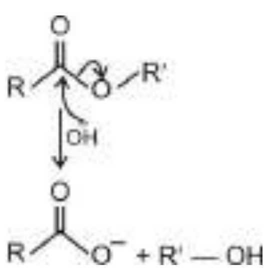
Hint : In DMF S_N2 mechanism is favoured during nucleophilic substitution reaction.

Solution :

Electron withdrawing group increases the tendency of S_N2.

24. Answer (B)

Hint : Hydrolysis of ester under alkaline condition occurs as



Solution :

Greater the extent of electron withdrawing strength of R, greater will be the rate of reaction

25. Answer (B)

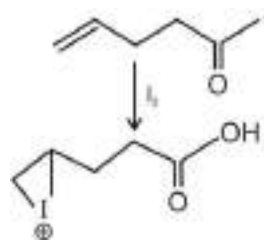
Hint : Compound which are planar, has (4n + 2) πe^- are aromatic

Solution :

Boron has vacant 2*p* orbital hence planar (*sp*^{2}) and has $6\pi e^-$

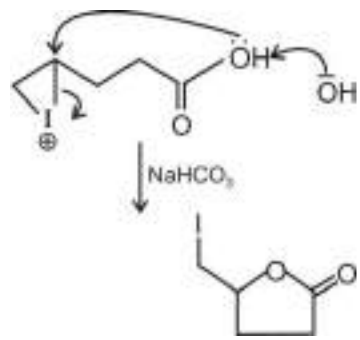
26. Answer (D)

Hint :



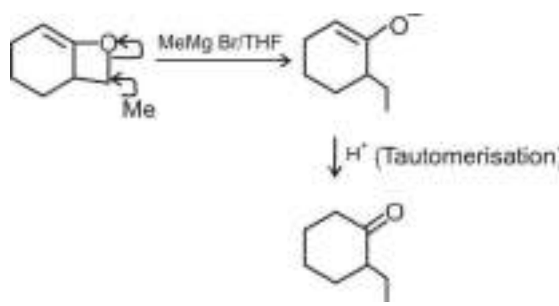
intermediate

Solution :

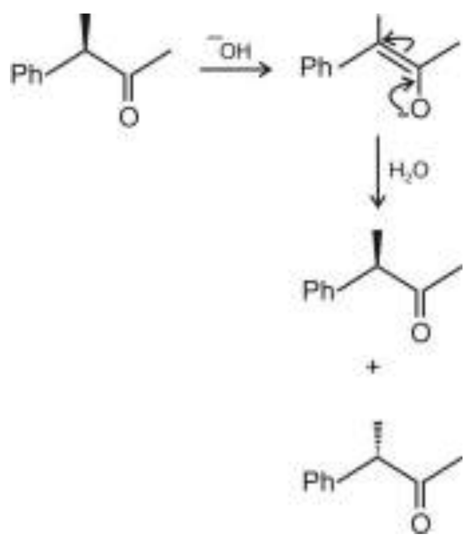


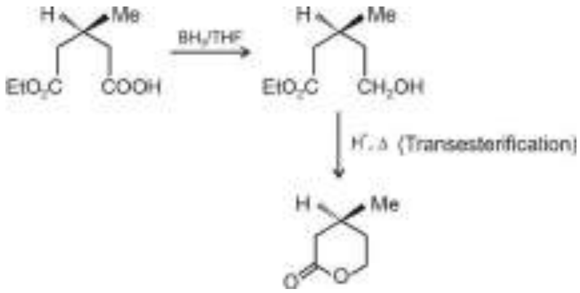
27. Answer (B, D)

Hint :



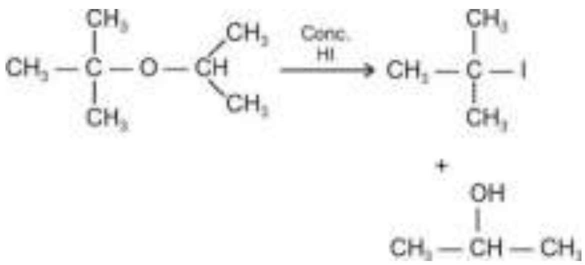
Solution :





28. Answer (A, B, D)

Hint :



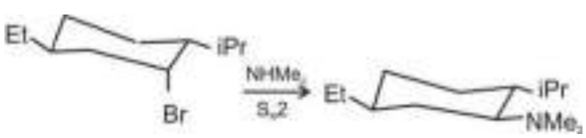
Solution :

Formed alcohol is 2°

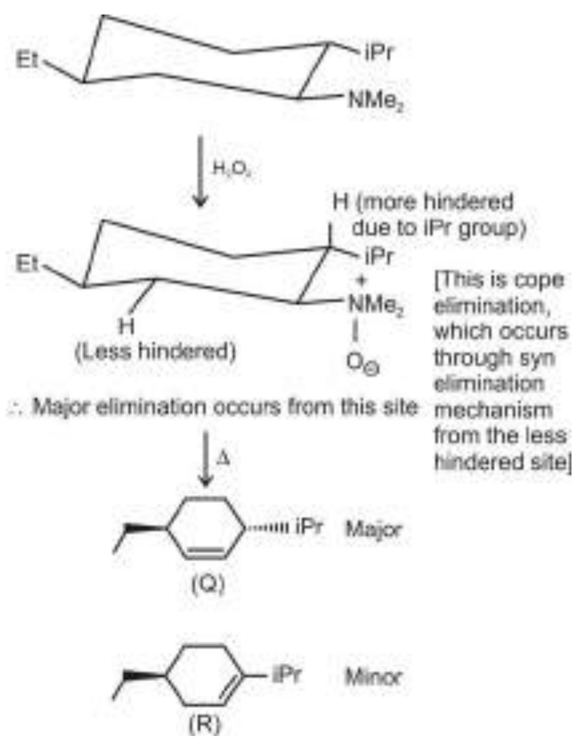
$$CH_3-\underset{\displaystyle CH_3}{\underset{|}{CH}}-OH \xrightarrow{Br_2/NaOH} CH_3-COO^- + CHBr_3 + NaBr$$

29. Answer (A, B, C)

Hint :



Solution :



30. Answer (C)

Hint : Hoffmann bromamide reaction.

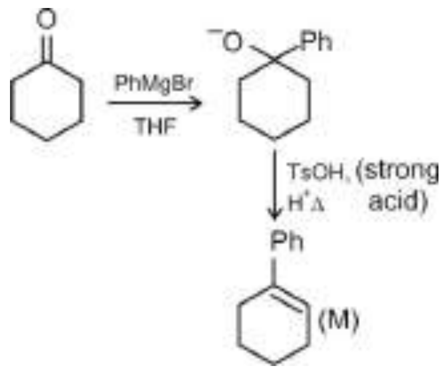
Solution :

$$Ph-CH_2-\overset{\displaystyle O}{\overset{\|}{C}}-NH_2 \xrightarrow[NaOH]{Br_2} Ph-CH_2-NH_2$$

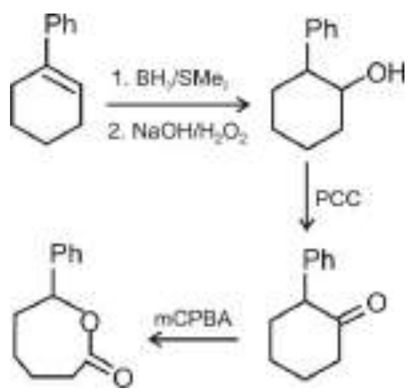
T

31. Answer (A)

Hint :



Solution :



32. Answer (C)

Hint : $SO_2 + O_3 \longrightarrow SO_3 + O_2$

O_3 is consumed by SO_2 only

Solution :

$$NO + \dot{O} \longrightarrow NO_2$$

So more of O_3 is consumed

33. Answer (C)

Hint : CCl_2F_2 is Freon-12

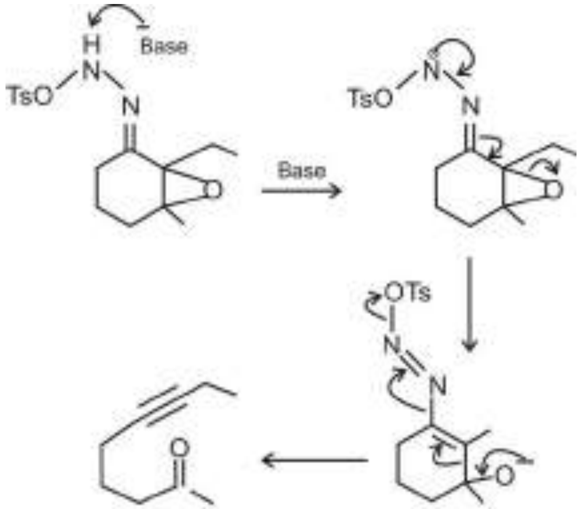
Solution :

Freons initiate radical chain reactions.

34. Answer (B)

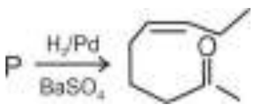
35. Answer (C)

Hint and Solution for Q. No. 34 and 35



(P)

Solution :



36. Answer A(Q, S, T); B(P, R, S); C(P, R, S, T); D(Q, R, S)

Hint :

Reducing sugars	Non-reducing Sugars
Maltose	
Lactose	Cellulose
Glucose	Sucrose
Fructose	

Solution :

Sucrose $\longrightarrow$ α-glucose + β-fructose

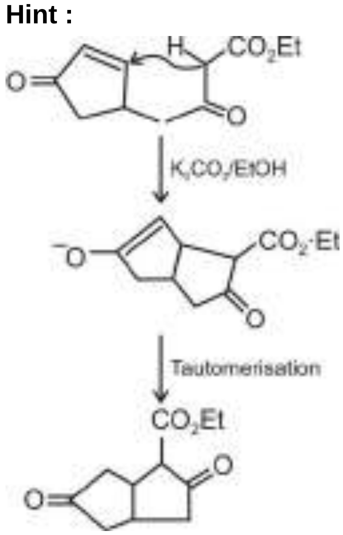
Maltose $\longrightarrow$ 2 α-glucose

Lactose $\longrightarrow$ β-galactose + β-glucose

Cellulose $\longrightarrow$ β-glucose

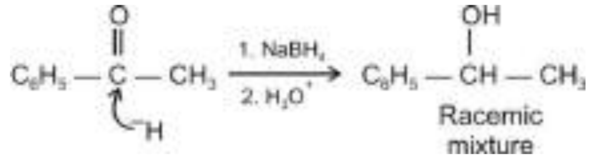
37. Answer A(Q, R, T); B(P); C(R, S, T); D(T)

Hint :



Solution :

In aldol condensation H_2O elimination through E1cB mechanism.



38. Answer (25)

Hint : Since, the sample has [α] to be +4.25 it means (+) alanine is present in excess.

Solution :

Optical purity = $\dfrac{4.25}{8.5}\times 100 = 50\%$. This means that 50% of the sample is pure (+) alanine and the other 50% is racemic. In which equal amount (i.e. 25% each) of (+) and (–) alanine is present.

39. Answer (04)

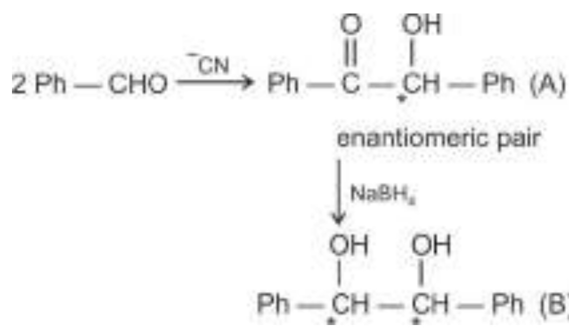
Hint : Since, six 1° H's contribute to the 42% yield of 1-chloro propane, we can say that one 1° H leads to 7% (42/6) of this product. Similarly each 2° hydrogen contributes 28% (56/2) yield to the 2-chloro propane product.

Solution :

So the relative rate of the reaction of each 2°H compared to 1° H is $\dfrac{28}{7} = 4$

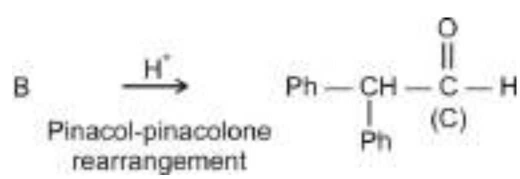
40. Answer (12)

Hint :



Solution :

Total 3 isomers of (B) are formed



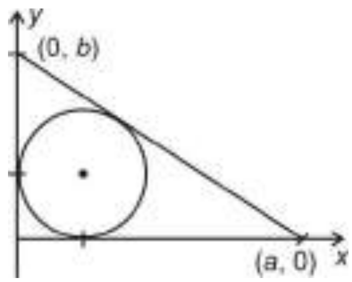
Degree of unsaturation of (C) is 9

PART - III (MATHEMATICS)

41. Answer (B)

Hints : Circumcentre is mid point of hypotenuse.

Solution :



Clearly $a > 2$, $b > 2$

$\Rightarrow \quad \frac{1}{a} < \frac{1}{2}, \ \frac{1}{b} < 2$

$\Rightarrow \quad \frac{1}{a} + \frac{1}{b} < 1$

Also, $rS = \Delta$

$\Rightarrow \quad 1\left(\frac{a+b+\sqrt{a^2+b^2}}{2}\right) = \frac{1}{2}ab$

$\Rightarrow \quad \frac{1}{a} + \frac{1}{b} + \sqrt{\frac{a^2+b^2}{a^2b^2}} = 1$

42. Answer (C)

Hint : First find point of intersection of lines.

Solution :

The vertices of the triangle are

$O(0, 0),\ A\left(\frac{1}{\ell+m}, \frac{1}{\ell+m}\right), B\left(\frac{1}{\ell-m}, \frac{-1}{\ell-m}\right)$

Let circumcenter is (h, k)

$\therefore \quad h = \frac{\ell}{\ell^2-m^2},\ k = \frac{-m}{\ell^2-m^2}$

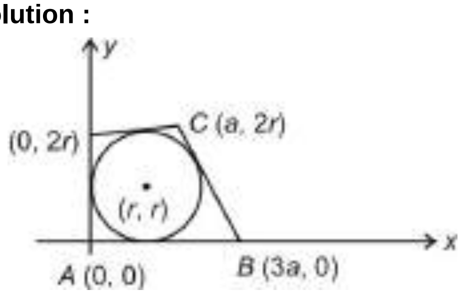
$\Rightarrow \quad h^2 + k^2 = \frac{1}{\left(\ell^2-m^2\right)^2}$ and $h^2 - k^2 = \frac{1}{\ell^2-m^2}$

$\Rightarrow$ Required locus $x^2+y^2 = \left(x^2-y^2\right)^2$

43. Answer (B)

Hint : Use condition for tangency.

Solution :



Area of trapezium $= \frac{1}{2}(a+3a)(2r) = 4$

$\Rightarrow \quad ar = 1$

Equation of BC is $y = -r^2\left(x - \frac{3}{r}\right)$

$\Rightarrow \quad y + r^2x - 3r = 0$

As BC is a tangent

$\Rightarrow \quad \frac{\left|r + r^3 - 3r\right|}{\sqrt{1+r^4}} = r$

$\Rightarrow \quad r = \frac{\sqrt{3}}{2}$

44. Answer (D)

Hint : Find chord of contact equation.

Solution :

Equation of tangent at (1, 2) to C_1 is

$x + 2y - 5 = 0$ …(1)

Let point T is (h, k)

$\therefore$ Equation of C.O.C. w.r.t. C_2 is

$xh + yk - 9 = 0$ …(2)

$\Rightarrow \quad \frac{h}{1} = \frac{k}{2} = \frac{9}{5}$

$\Rightarrow \quad h = \frac{9}{5},\ k = \frac{18}{5}$

45. Answer (A)

Hint : Think of quadratic equation to solve.

Solution :

Let equation of circle is

$(x - r)^2 + y^2 = r^2$ …(1)

$\Rightarrow \quad (at^2 - r)^2 + 4a^2t^2 \geq r^2$

$\Rightarrow \quad a^2t^4 + r^2 - 2rat^2 + 4a^2t^2 \geq r^2$

$\Rightarrow \quad a^2t^4 - 2art^2 + 4a^2t^2 \geq 0$

$\Rightarrow \quad at^2 - 2r + 4a \geq 0$

$\Rightarrow \quad r \leq \frac{a}{2}(t^2+4) \leq 2a$

$\therefore$ Maximum value of $r = 2a$

46. Answer (B)

Hint : Tangency condition.

Solution :

Let the line is $y = mx + 5$

$\because m > 0$ and is least $\therefore$ the line should touch the ellipse

$\Rightarrow \quad 25 = 16m^2 + 9$

$\Rightarrow \quad 16m^2 = 16$

$\Rightarrow \quad m = \pm 1 \qquad \Rightarrow m = 1$

47. Answer (B, C, D)

Hint : A.M $\geq$ G.M

Solution :

$\therefore uv < 0 \Rightarrow u + \frac{1}{u} \geq 2, \quad v + \frac{1}{v} \leq -2$

or $\quad u + \frac{1}{u} \leq -2 \quad$ or $\quad v + \frac{1}{v} \geq 2$

$$\Rightarrow \quad \sec^{-1}\left(u + \frac{1}{u}\right) \in \left[\frac{\pi}{3}, \frac{\pi}{2}\right)$$

$$\sec^{-1}\left(v + \frac{1}{v}\right) \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right]$$

$$\therefore \quad t \in \left(\frac{5\pi}{6}, \frac{7\pi}{6}\right)$$

48. Answer (A, B)

Hint : Conversion into trigonometric function values.

Solution :

$$\because \tan\alpha = \frac{36}{77}, \qquad \tan\beta = \frac{3}{4}, \qquad \tan\gamma = \frac{8}{15}$$

$$\tan(\alpha + \beta + \gamma) = \frac{\Sigma(\tan\alpha) - \pi(\tan\alpha)}{1 - \Sigma\tan\alpha\tan\beta} = \infty$$

$$\Rightarrow \quad \alpha + \beta + \gamma = \frac{\pi}{2}$$

$\therefore$ Option (A) and (B) are correct.

49. Answer (A, B, C)

Hint : Concept of orthogonality of two curves.

Solution :

Due to orthogonal intersection of ellipse and hyperbola

$a^2 + b^2 = 16$

$\Rightarrow \quad a^2e^2 = 16$

$\Rightarrow \quad a^2 = 4 \qquad \Rightarrow \quad b^2 = 12$

$\therefore$ No director circle of hyperbola is possible.

50. Answer (C, D)

Hint : Property of normal.

Solution :

$\because$ Normal intersects the parabola $y^2 = 4ax$ again

$\therefore x_1 x_2 = 4a^2$ and $y_1 y_2 = 8a^2$

$\because a = 2 \qquad \Rightarrow x_1 x_2 = 16$ and $y_1 y_2 = 32$

51. Answer (A, D)

Hint : Form family of circles.

Solution :

Circle with points $\left(2t_1, \frac{2}{t_1}\right)$ and $\left(2t_2, \frac{2}{t_2}\right)$ as diameter is

$$(x - 2t_1)(x - 2t_2) + \left(y - \frac{2}{t_1}\right)\left(y - \frac{2}{t_2}\right) = 0$$

Also $t_1t_2 = -1$

Hence the equation of circle is $(x^2 + y^2 - 8) - 2(t_1 + t_2)(x - y) = 0$

The point of intersection of $x^2 + y^2 = 8$ and $x - y = 0$ are (2, 2) and (–2, –2)

52. Answer (A)
53. Answer (D)

Hint for Q. No. 52 and 53

Hint : Family of circles.

Solution for Q. No. 52 and 53

Let Σ is $x^2 + y^2 - 9x - 12y + 53 + \lambda(2x + 3y - 27) = 0$

Given circle $x^2 + y^2 - 4x - 6y - 3 = 0$

$\therefore$ Equation of common chord

$-5x + 6y + 56 + \lambda(2x + 3y - 27) = 0$

$\therefore$ Chord passes through the point of intersection of $5x + 6y - 56 = 0$ and $2x + 3y - 27 = 0$

i.e. $\left(2, \frac{23}{3}\right)$

$\because$ Σ intersects $x^2 + y^2 = 29$ orthogonally.

$53 - 27\lambda - 29 = 0$

$$\lambda = \frac{24}{27} = \frac{8}{9}$$

$\therefore$ Circle is

$$x^2 + y^2 + \left(\frac{16}{9} - 9\right)x + \left(\frac{29}{9} - 12\right)y + 29 = 0$$

$\therefore$ Center is $\left(\frac{65}{18}, \frac{14}{3}\right)$

54. Answer (B)
55. Answer (C)

Hint for Q. No. 54 and 55

Mathematical induction approach.

Solution for Q. No. 54 and 55

Put $n = 2$

$\Rightarrow \quad f(1) + 2f(2) = 6f(2)$

$\Rightarrow \quad 4f(2) = f(1)$

$\Rightarrow \quad f(2) = \frac{1}{8}$

Similarly $f(3) = \frac{1}{12}$, $f(4) = \frac{1}{16}$ … and so on

$\therefore \quad f(n) = \frac{1}{4n} \qquad \therefore \quad f(1010) = \frac{1}{4040}$

56. Answer A(Q, R, S); B(Q); C(R, S); D(P, T)

Hint : Equality hold conditions for I.T.F.

Solution :

(A) $(\sin^{-1}x)^2 = (\sin^{-1}y)^2 = \frac{\pi^2}{4}$

$\Rightarrow \quad x = \pm 1$ and $y = \pm 1$

$\therefore \quad x^3 + y^3 = -2, 0, 2$

(B) $(\cos^{-1}x)^2 = (\cos^{-1}y)^2 = \pi^2$

$\Rightarrow$ $x = y = -1$

$\therefore$ $x^5 + y^5 = -2$

(C) $(\sin^{-1}x)^2 = \dfrac{\pi^2}{4}$ and $(\cos^{-1}y)^2 = \pi^2$

$\Rightarrow$ $\sin^{-1}x = \pm\dfrac{\pi}{2}$ and $\cos^{-1}y = \pi$

$\Rightarrow$ $x = \pm 1$ and $y = -1$

(D) $|\sin^{-1}x - \sin^{-1}y| = \pi$

$\Rightarrow$ either $\sin^{-1}x = -\dfrac{\pi}{2}$ and $\sin^{-1}y = \dfrac{\pi}{2}$

or $\sin^{-1}x = \dfrac{\pi}{2}$ and $\sin^{-1}y = -\dfrac{\pi}{2}$

$x = -1$ and $y = 1$ or $x = 1$ and $y = -1$

$\therefore$ $x^y = (-1)^1$ or $(1)^{-1}$

$= -1$ or 1

57. Answer A(S); B(Q, R, S, T); C(R); D(P, Q, R, S, T)

Hint : Eccentricity formula for conic.

Solution :

(A) $\because \sqrt{c^2 + d^2} = a;\ \sqrt{a^2 - b^2} = c$

$\Rightarrow$ $c^2 + d^2 = a^2$ and $a^2 - b^2 = c^2$

$\Rightarrow$ $d = b$ $\Rightarrow$ $\dfrac{d}{b} = 1$

(B) Now $e_1 = 1 - \dfrac{b^2}{a^2}$ $e_2 = 1 + \dfrac{d^2}{c^2}$

$\Rightarrow$ $e_1^2 + e_2^2 = 2 + b^2\left(\dfrac{a^2 - c^2}{a^2c^2}\right)$

$e_1 + e_2 = e_1^2 + \dfrac{1}{e_1^2} > 2$

(C) $2\tan^{-1}\left(\dfrac{d}{c}\right) = \dfrac{2\pi}{3}$ $\Rightarrow$ $d = \sqrt{3}c$ $\Rightarrow$ $d^2 = 3c^2$

$\Rightarrow$ $a^2 = 4c^2$ $\Rightarrow$ $a = 2c$

$\therefore$ $4e_1 = 4\sqrt{1 - \dfrac{b^2}{a^2}}$

$= 4\sqrt{1 - \dfrac{3c^2}{4c^2}} = 2$

(D) $b^2 = a^2\left(1 - e_1^2\right)$

$\Rightarrow$ $a^2 = 2b^2$ $\Rightarrow$ $c^2 = b^2$

For P.O.I. $\dfrac{h^2}{b^2} - \dfrac{k^2}{b^2} = \dfrac{h^2}{a^2} + \dfrac{k^2}{b^2}$

$\Rightarrow$ $h^2\left(\dfrac{a^2 - b^2}{a^2b^2}\right) = \dfrac{2k^2}{b^2}$

$\Rightarrow$ $\dfrac{h^2}{k^2} = \dfrac{2a^2}{a^2e_1^2} = 4$

58. Answer (36)

Hint : Point of intersection of two normals.

Solution :

Let $P(t_1)$ and $Q(t_2)$ are points

$\therefore$ $t_2 = 2t_1$

$\because$ P.O.I of normals

$R\left(2a + a\left(t_1^2 + t_1t_2 + t_2^2\right), -t_1t_2\left(t_1 + t_2\right)\right)$

$R\left(2 + t_1^2 + t_1t_2 + t_2^2,\ -t_1t_2\left(t_1 + t_2\right)\right)$

$\therefore$ $x = 2 + 7t_1^2,$ $y = -6t_1^3$

$\left(\dfrac{x-2}{7}\right)^3 = t_1^6 = \left(\dfrac{-y}{6}\right)^2 = \dfrac{y^2}{36}$

$\therefore$ Locus is $y^2 = \dfrac{36}{343}(x-2)^3$

$\therefore$ $k = 36$

59. Answer (16)

Hint : Monotonicity of function.

Solution :

$\because$ $x \in [-1, 1]$

Also $f(x)$ is an increasing function in domain

$\therefore$ $p = f(-1)$ and $q = f(1)$

$\Rightarrow$ $p = -\dfrac{\pi}{2} - \dfrac{\pi}{2} + (-2) = -\pi - 2$

and $q = \dfrac{\pi}{2} + \dfrac{\pi}{2} + 6 = \pi + 6$

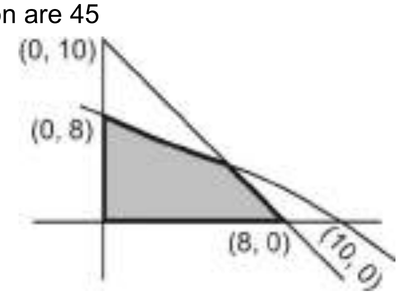
$\therefore$ $p + q = 4$ $\Rightarrow$ $(p + q)^2 = 16$

60. Answer (45)

Hint : Linear inequalities of two variables.

Solution :

Total number of integral co-ordinates in shaded region are 45



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All India Aakash Test Series for JEE (Advanced)-2020

TEST - 2A (Paper-1) - Code-D

Test Date : 24/11/2019

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(C)	21.	(D)	41.	(B)
2.	(D)	22.	(B)	42.	(A)
3.	(D)	23.	(B)	43.	(D)
4.	(B)	24.	(B)	44.	(B)
5.	(B)	25.	(A)	45.	(C)
6.	(C)	26.	(B)	46.	(B)
7.	(B, C, D)	27.	(A)	47.	(A, D)
8.	(B, D)	28.	(C)	48.	(C, D)
9.	(B, D)	29.	(A, B, C)	49.	(A, B, C)
10.	(B)	30.	(A, B, D)	50.	(A, B)
11.	(A, B, D)	31.	(B, D)	51.	(B, C, D)
12.	(A)	32.	(C)	52.	(A)
13.	(C)	33.	(C)	53.	(D)
14.	(C)	34.	(B)	54.	(B)
15.	(D)	35.	(C)	55.	(C)
16.	A → (P, T)	36.	A → (Q, R, T)	56.	A → (S)
	B → (Q, R)		B → (P)		B → (Q, R, S, T)
	C → (R, S, T)		C → (R, S, T)		C → (R)
	D → (Q, R)		D → (T)		D → (P, Q, R, S, T)
17.	A → (P, S)	37.	A → (Q, S, T)	57.	A → (Q, R, S)
	B → (Q, R)		B → (P, R, S)		B → (Q)
	C → (P, S)		C → (P, R, S, T)		C → (R, S)
	D → (P, R)		D → (Q, R, S)		D → (P, T)
18.	(29)	38.	(12)	58.	(45)
19.	(50)	39.	(04)	59.	(16)
20.	(16)	40.	(25)	60.	(36)

HINTS & SOLUTIONS

PART - I (PHYSICS)

1. Answer (C)

Hint : $E_{\text{axis}} = \dfrac{qx}{4\pi\varepsilon_0\left(R^2+x^2\right)^{3/2}}$

Solution:

$$E = \frac{q}{4\pi\varepsilon_0 d^2} - \frac{q\times d}{4\pi\varepsilon_0\left(d^2+R^2\right)^{3/2}} = \frac{3qR^2}{8\pi\varepsilon_0 d^4}$$

2. Answer (D)

Hint: Heat current remains constant

Solution :

$$\frac{\dfrac{(T_1-T)}{L}}{k2\pi a\times\left(\dfrac{a+b}{2}\right)} = \frac{\dfrac{T_1-T_2}{L}}{k\times\pi a\times b}$$

$$\Rightarrow T = \frac{T_1 a + T_2 b}{(a+b)}$$

3. Answer (D)

Hint: Use reverse symmetry concept

Solution:

Using KVL and KCL, we get

$$\text{R}_{\text{eq}} = \frac{2R_1R_2+R_2R_3+R_3R_1}{(R_1+R_2+2R_3)}$$

$$= \frac{2\times(2\times3)+(3\times1)+(1\times2)}{(2+3+1\times2)}$$

$$= \frac{12+3+2}{7} = \frac{17}{7}\Omega$$

4. Answer (B)

Hint : $Q = Q_0\, e^{-t/\tau}$ during discharging

Solution :

$$Q_{.0} = CV_0, C_2\left(\frac{C}{K}\right)$$

$$\therefore V_2 = \frac{CV_0}{\left(\dfrac{C}{K}\right)} = KV_0$$

$$\tau = R\times C_2 = \frac{RC}{K}$$

$$\therefore V = V_2 e^{-t/\tau}$$

$$\Rightarrow \frac{V_0}{2} = KV_0\times e^{-\frac{t}{\tau}}$$

$$\Rightarrow \frac{1}{2K} = e^{-t/\tau}$$

$$\Rightarrow \ln(2K) = \frac{t}{\tau}$$

$$\Rightarrow t = \tau\ln(2K)$$

$$t = \frac{RC}{K}\ln(2K)$$

5. Answer (B)

Hint : At maximum temperature $\dfrac{dT}{dV} = 0$

Solution :

$$\left[P_0 + (1-\alpha)V^2\right]V = nRT$$

$$\Rightarrow T = \frac{P_0V + (1-\alpha)V^3}{nR}$$

$$\therefore \frac{dT}{dV} = 0 \text{ at } V^2 = \frac{P_0}{3(\alpha-1)}$$

$$\therefore P = P_0 + (1-\alpha)\times\frac{P_0}{3(\alpha-1)}$$

$$\Rightarrow P = \frac{2P_0}{3}$$

6. Answer (C)

Hint: Second overtone contains 3 loops

Solution:

$$\frac{\lambda}{2} = \frac{2}{3} \Rightarrow \lambda = \frac{4}{3}\text{m}$$

Amp. = $2A\sin kx = A_{\max}\sin(kx)$

$$\therefore A = (A_{\max})\sin\left(\frac{2\pi\times3}{4}\times\frac{1}{6}\right)$$

$$\Rightarrow A = (2)\times\left(\frac{1}{\sqrt{2}}\right)\text{mm} = \sqrt{2}\text{ mm}$$

7. Answer (B, C, D)

Hint : Use KVL and KCL.

Solution :

$Q_{\text{total}} = 180 - 70 = 110\ \mu$C

$$q_A = \frac{2}{2+6+3}\times(110) = 20\ \mu\text{C}$$

$$q_B = \frac{6}{2+3+6}\times(110) = 60\ \mu\text{C}$$

$$q_C = \frac{3}{2+6+3}\times(110) = 30\ \mu\text{C}$$

$$\Delta q_S = (20+30) - (-70) = 120\ \mu\text{C}$$

8. Answer (B, D)

Hint : $V_{rms}^2 = \dfrac{\int u^2 dN}{N}$

Solution :

$$N = \text{Area} = \frac{1}{2}\times 10\times 10 = 50$$

$$\frac{dN}{du} = u + 10$$

$$\therefore V_{\text{rms}}^2 = \frac{\int u^2\times(10-u)du}{N} = \frac{\int_0^{10}\left(10u^2 - u^3\right)du}{50}$$

$$V_{rms}^2 = \frac{1000\times(4-3)}{12\times 50} = \frac{2500}{3\times 50}$$

$$\Rightarrow V_{rms} = \sqrt{\frac{50}{3}}\,\text{m/s}$$

9. Answer (B, D)

Hint : Apparent wavelength changes when source moves.

Solution :

$$f' = \frac{(340-10)}{(340-20)}\times(200) = 206\text{ Hz}$$

$$\lambda' = \lambda_0 = V_s\times T = \frac{340}{200} - 20\times\frac{1}{200} = 1.6\text{ m}$$

10. Answer (B)

Hint : Use Gauss's law

Solution :

σ on outer surface becomes uniform. Potential at outside points is only due to charge on outer surface of shell.

$\therefore V_A = V_B$

11. Answer (A, B, D)

Hint: Use KVL and KCL.

Solution :

$$R_{\text{eq}} = 1 + \frac{20}{9} = \frac{29}{9}\,\Omega$$

$$\therefore I_0 = \frac{58}{(29/9)} = 18\text{ A}$$

$$\therefore 1_{(2\Omega)} = \frac{18}{2} = 9\text{ A}$$

$$1_{(3\Omega)} = \frac{6}{6+3}\times 9 = 6\text{ A}$$

$$I_{(5\Omega)} = \frac{4}{9}\times(18) = 8\text{ A}$$

$$I_{(4\Omega)} = \frac{5}{9}\times(18) = 10\text{ A}$$

$$\therefore V_{(4\Omega)} = 4\times 10 = 40\text{ V}$$

$$P_{(5\Omega)} = 8^2\times 5 = 320\text{ W}$$

12. Answer (A)

Hint: Flux is proportional to charge

Solution :

$$\frac{2\pi(1-\cos\alpha)}{4\pi}\times\left(\frac{q_1}{\varepsilon_0}\right) = \frac{2\pi(1-\cos\beta)}{4\pi}\times\left(\frac{q_2}{\varepsilon_0}\right)$$

$$\Rightarrow \frac{q_1}{q_2} = \frac{1-\cos\beta}{1-\cos\alpha} = \frac{1-0}{1-\frac{1}{2}} = 2$$

13. Answer (C)

Hint : Flux is proportional to charge

Solution:

$q_1 = 3q_2$

$\Rightarrow$ one third of total flux of q_1 will terminate at q_2

$$\therefore \frac{4\pi}{3} = 2\pi(1-\cos\alpha_{\max})$$

$$\Rightarrow \cos(\alpha_{\max}) = 1 - \frac{2}{3} = \frac{1}{3}$$

$$\Rightarrow \tan(\alpha_{\max}) = 2\sqrt{2} \quad \Rightarrow \alpha_{\max} = \tan^{-1}\left(2\sqrt{2}\right)$$

14. Answer (C)

Hint : $A \to B$ Isochoric $C \to D$ Isochoric

$B \to C$ Isothermal $D \to A$ Isothermal

15. Answer (D)

Hint : W isothermal $= nRT_0 \ln\left(\dfrac{V_2}{V_1}\right)$

Solution of Q.Nos. 14 and 15

$$W_{BC} = 2P_0V_0\ln\left(\frac{V_C}{V_B}\right) = -P_0V_0\ln(2)$$

$$\Rightarrow V_C = \frac{V_0}{\sqrt{2}}$$

$$\therefore P_C = \frac{2P_0V_0}{V_C} = 2\sqrt{2}P_0$$

$$\therefore W_{DA} = \left(\sqrt{2}P_0\right)\left(\frac{V_0}{\sqrt{2}}\right)\ln\left(\sqrt{2}\right) = \frac{P_0V_0}{2}\ln(2)$$

$$\therefore W_{ABCDA} = 0 + -P_0V_0\ln(2) + 0 + \frac{1}{2}P_0V_0\ln(2)$$

$$= -\frac{P_0V_0}{2}\ln(2)$$

16. Answer A(P, T); B (Q, R); C(R, S, T); D(Q, R)

Hint : Use Gauss's law.

Solution :

Electric field is uniform in spherical cavity in sphere and in cylindrical cavity in cylinder.

17. Answer A(P, S); B(Q, R); C(P, S); D(P, R)

Hint : Capacitance increases due to slab.

Solution :

Total capacitance increases, so charge on *A* increases as well as voltage increases.

$\therefore$ Voltage on *B* decreases. So, charge on it decreases

$\therefore$ Charge on *C* and *D* increases

18. Answer (29)

Hint: *BC* is isothermal

Solution :

$$(3P_0) \times V_C = P_0 \times V_0$$

$$\Rightarrow V_C = \frac{V_0}{3}$$

$\because$ *CA* is a adiabatic.

$$\therefore \quad (3P_0)\times\left(\frac{V_0}{3}\right)^{r} = \left(\frac{P_0}{2}\right)(V_0)^{r}$$

$$\Rightarrow \quad \gamma = \frac{\ln 6}{\ln 3} = \frac{\ln 2 + \ln 3}{\ln 3} = \frac{18}{11}$$

$\therefore$ $p + q = 18 + 11 = 29$

19. Answer (50)

Hint : Voltmeter are not ideal

Solution :

Let resistance of each voltmeter be R_0

$\therefore$ $Ri' = 20\,R_0\,(I - i)$ ……………….(i)

and $2Ri' = 30$ ……………….(ii)

$$\Rightarrow i' = \frac{3}{4}i, \qquad \therefore\ i_{(V_2)} = I - i' = I - \frac{3i}{4}$$

$$\therefore 2Ri' = 30 = R_0(I - i) = R_0\left(I - \frac{3i}{4}\right)$$

$$\Rightarrow i = 400\ \mu\text{A}$$

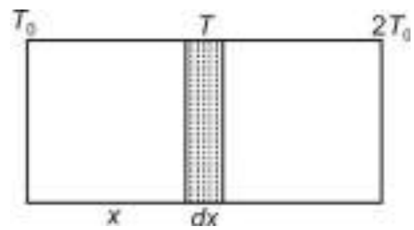
$$\therefore R = \frac{20}{400\times10^{-6}} = 50\times10^{3}\,\Omega = 50\ \text{k}\Omega$$

20. Answer (16)

Hint : Use $PV = nRT$

Solution :

$$T = T_0 + \frac{T_0}{L}x$$



$$\therefore \int dn = \int_{x=0}^{L} \frac{P\times A dx}{R\left(T_0 + \frac{T_0}{L}x\right)}$$

$$\Rightarrow n = \frac{PAL}{RT_0}\ln(2)$$

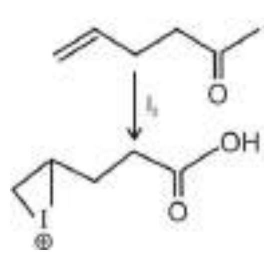
$$\Rightarrow n = \frac{PAL}{4RT_0}\ln(16)$$

$\therefore$ 16

PART - II (CHEMISTRY)

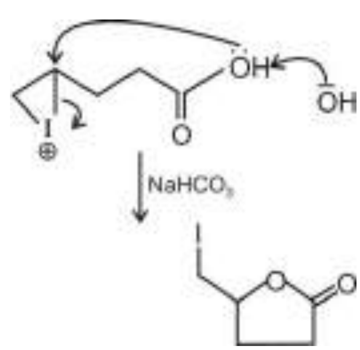
21. Answer (D)

Hint :



intermediate

Solution :



22. Answer (B)

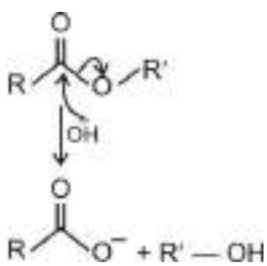
Hint : Compound which are planar, has $(4n + 2)$ πe^- are aromatic

Solution :

Boron has vacant 2*p* orbital hence planar (sp^2) and has $6\pi e^-$

23. Answer (B)

Hint : Hydrolysis of ester under alkaline condition occurs as



Solution :
Greater the extent of electron withdrawing strength of R, greater will be the rate of reaction

24. Answer (B)

Hint : In DMF S_N2 mechanism is favoured during nucleophilic substitution reaction.

Solution :
Electron withdrawing group increases the tendency of S_N2.

25. Answer (A)

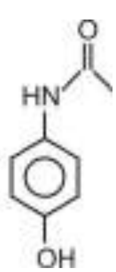
Hint : A paired with T (A = T)

Solution :
G paired with C (G ≡ C)

26. Answer (B)

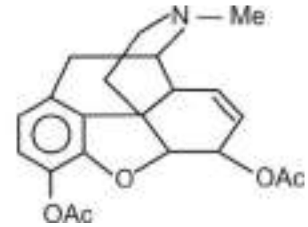
Hint :

Paracetamol

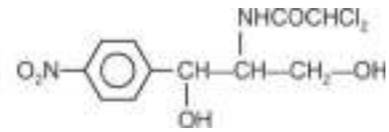


Solution :

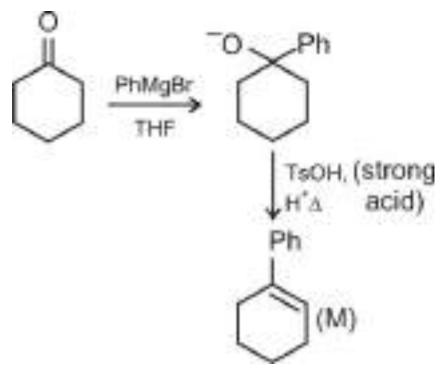
Heroin



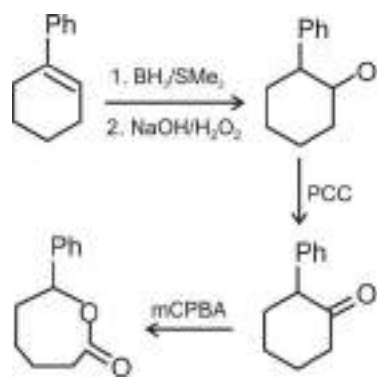
Chloramphenicol



27. Answer (A)
Hint :



Solution :



28. Answer (C)

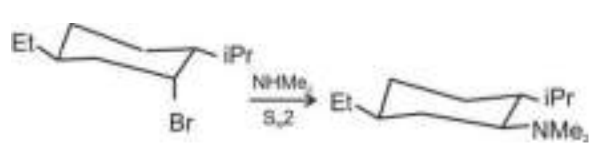
Hint : Hoffmann bromamide reaction.

Solution :

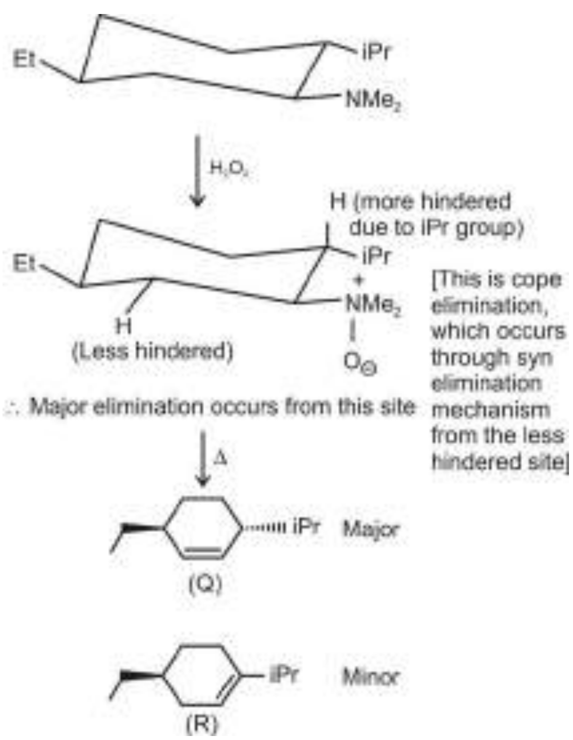
$$\underset{T}{Ph-CH_2-\overset{\overset{O}{\|}}{C}-NH_2} \xrightarrow[NaOH]{Br_2} Ph-CH_2-NH_2$$

29. Answer (A, B, C)

Hint :

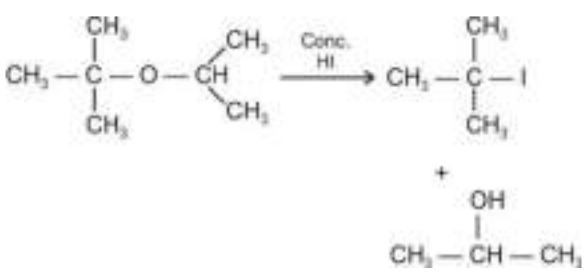


Solution :



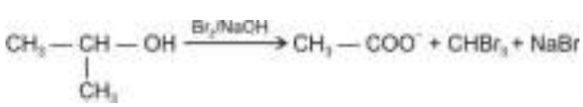
30. Answer (A, B, D)

Hint :



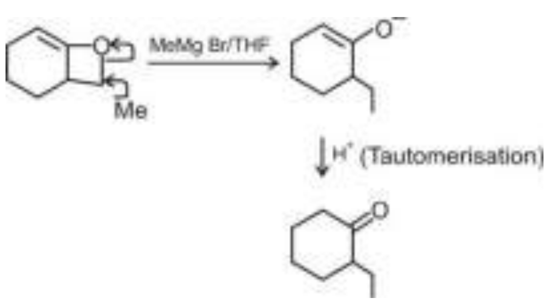
Solution :

Formed alcohol is 2°

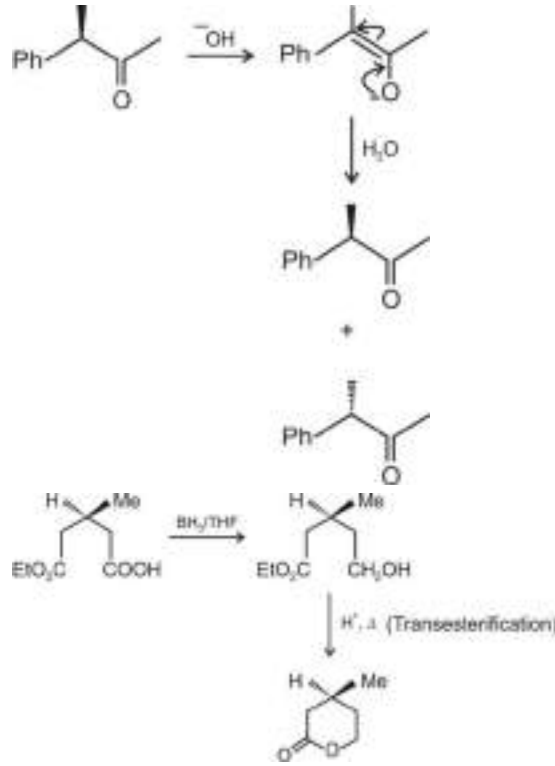


31. Answer (B, D)

Hint :



Solution :



32. Answer (C)

Hint : $SO_2 + O_3 \longrightarrow SO_3 + O_2$

O_3 is consumed by SO_2 only

Solution :

$NO + \dot{O} \longrightarrow NO_2$

So more of O_3 is consumed

33. Answer (C)

Hint : CCl_2F_2 is Freon-12

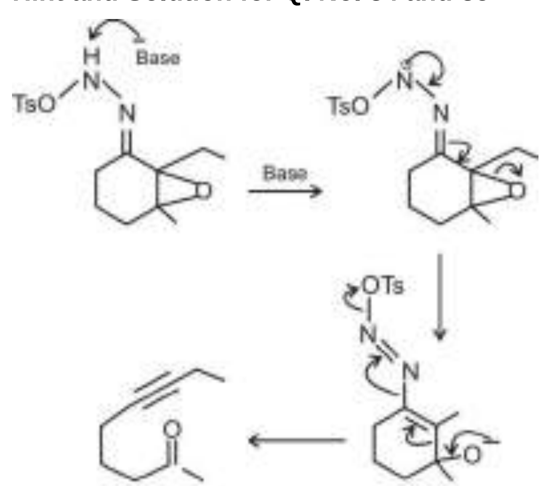
Solution :

Freons initiate radical chain reactions.

34. Answer (B)

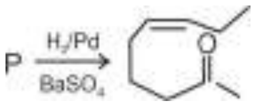
35. Answer (C)

Hint and Solution for Q. No. 34 and 35



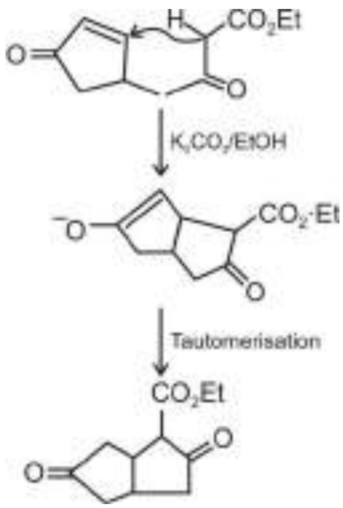
(P)

Solution :



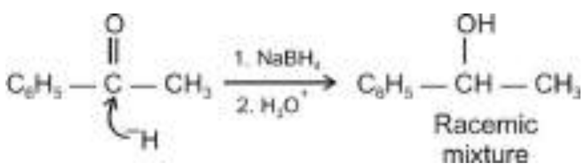
36. Answer A(Q, R, T); B(P); C(R, S, T); D(T)

Hint :



Solution :

In aldol condensation H_2O elimination through E1cB mechanism.



37. Answer A(Q, S, T); B(P, R, S); C(P, R, S, T); D(Q, R, S)

Hint :

Reducing sugars	**Non-reducing Sugars**
Maltose	
Lactose	Cellulose
Glucose	Sucrose
Fructose	

Solution :

Sucrose $\longrightarrow$ α-glucose + β-fructose

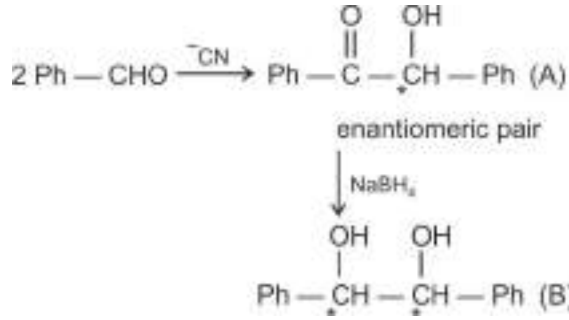
Maltose $\longrightarrow$ 2 α-glucose

Lactose $\longrightarrow$ β-galactose + β-glucose

Cellulose $\longrightarrow$ β-glucose

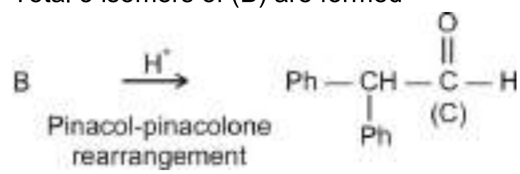
38. Answer (12)

Hint :



Solution :

Total 3 isomers of (B) are formed



Degree of unsaturation of (C) is 9

39. Answer (04)

Hint : Since, six 1° H′s contribute to the 42% yield of 1-chloro propane, we can say that one 1° H leads to 7% (42/6) of this product. Similarly each 2° hydrogen contributes 28% (56/2) yield to the 2-chloro propane product.

Solution :

So the relative rate of the reaction of each 2°H compared to 1° H is $\frac{28}{7} = 4$

40. Answer (25)

Hint : Since, the sample has [α] to be +4.25 it means (+) alanine is present in excess.

Solution :

Optical purity = $\frac{4.25}{8.5} \times 100 = 50\%$. This means that 50% of the sample is pure (+) alanine and the other 50% is racemic. In which equal amount (i.e. 25% each) of (+) and (–) alanine is present.

PART - III (MATHEMATICS)

41. Answer (B)

Hint : Tangency condition.

Solution :

Let the line is $y = mx + 5$

$\because$ $m > 0$ and is least $\therefore$ the line should touch the ellipse

$\Rightarrow$ $25 = 16m^2 + 9$

$\Rightarrow$ $16m^2 = 16$

$\Rightarrow$ $m = \pm 1$ $\Rightarrow$ $m = 1$

42. Answer (A)

Hint : Think of quadratic equation to solve.

Solution :

Let equation of circle is

$(x-r)^2 + y^2 = r^2$ ...(1)

$\Rightarrow$ $(at^2 - r)^2 + 4a^2t^2 \geq r^2$

$\Rightarrow$ $a^2t^4 + r^2 - 2rat^2 + 4a^2t^2 \geq r^2$

$\Rightarrow$ $a^2t^4 - 2art^2 + 4a^2t^2 \geq 0$

$\Rightarrow$ $at^2 - 2r + 4a \geq 0$

$\Rightarrow$ $r \leq \frac{a}{2}(t^2+4) \leq 2a$

$\therefore$ Maximum value of $r = 2a$

43. Answer (D)

Hint : Find chord of contact equation.

Solution :

Equation of tangent at (1, 2) to C_1 is

$x + 2y - 5 = 0$ ...(1)

Let point T is (h, k)

$\therefore$ Equation of C.O.C. w.r.t. C_2 is

$xh + yk - 9 = 0$ ...(2)

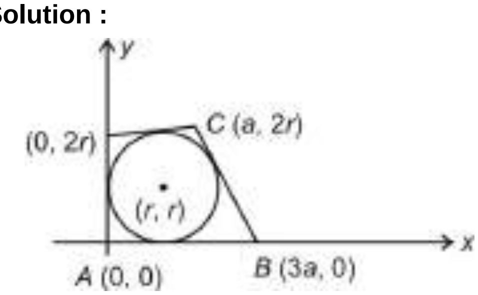
$\Rightarrow$ $\frac{h}{1} = \frac{k}{2} = \frac{9}{5}$

$\Rightarrow$ $h = \frac{9}{5},\ k = \frac{18}{5}$

44. Answer (B)

Hint : Use condition for tangency.

Solution :



Area of trapezium = $\frac{1}{2}(a+3a)(2r) = 4$

$\Rightarrow$ $ar = 1$

Equation of BC is $y = -r^2\left(x - \frac{3}{r}\right)$

$\Rightarrow$ $y + r^2x - 3r = 0$

As BC is a tangent

$\Rightarrow$ $\frac{\left|r + r^3 - 3r\right|}{\sqrt{1+r^4}} = r$

$\Rightarrow$ $r = \frac{\sqrt{3}}{2}$

45. Answer (C)

Hint : First find point of intersection of lines.

Solution :

The vertices of the triangle are

$O(0, 0),\ A\left(\frac{1}{\ell+m}, \frac{1}{\ell+m}\right), B\left(\frac{1}{\ell-m}, \frac{-1}{\ell-m}\right)$

Let circumcenter is (h, k)

$\therefore$ $h = \frac{\ell}{\ell^2 - m^2},\ k = \frac{-m}{\ell^2-m^2}$

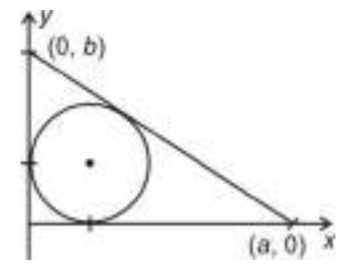
$\Rightarrow$ $h^2 + k^2 = \frac{1}{\left(\ell^2-m^2\right)^2}$ and $h^2 - k^2 = \frac{1}{\ell^2 - m^2}$

$\Rightarrow$ Required locus $x^2 + y^2 = \left(x^2-y^2\right)^2$

46. Answer (B)

Hints : Circumcentre is mid point of hypotenuse.

Solution :



Clearly $a > 2$, $b > 2$

$\Rightarrow$ $\frac{1}{a} < \frac{1}{2},\ \frac{1}{b} < 2$

$\Rightarrow$ $\frac{1}{a} + \frac{1}{b} < 1$

Also, $rS = \Delta$

$\Rightarrow$ $1\left(\frac{a+b+\sqrt{a^2+b^2}}{2}\right) = \frac{1}{2}ab$

$\Rightarrow$ $\frac{1}{a} + \frac{1}{b} + \sqrt{\frac{a^2+b^2}{a^2b^2}} = 1$

47. Answer (A, D)

Hint : Form family of circles.

Solution :

Circle with points $\left(2t_1, \frac{2}{t_1}\right)$ and $\left(2t_2, \frac{2}{t_2}\right)$ as diameter is

$$(x-2t_1)(x-2t_2)+\left(y-\frac{2}{t_1}\right)\left(y-\frac{2}{t_2}\right)=0$$

Also $t_1t_2 = -1$

Hence the equation of circle is $(x^2 + y^2 - 8) - 2(t_1 + t_2)(x - y) = 0$

The point of intersection of $x^2 + y^2 = 8$ and $x - y = 0$ are (2, 2) and (–2, –2)

48. Answer (C, D)

Hint : Property of normal.

Solution :

$\because$ Normal intersects the parabola $y^2 = 4ax$ again

$\therefore x_1 x_2 = 4a^2$ and $y_1 y_2 = 8a^2$

$\because a = 2 \quad \Rightarrow x_1 x_2 = 16$ and $y_1 y_2 = 32$

49. Answer (A, B, C)

Hint : Concept of orthogonality of two curves.

Solution :

Due to orthogonal intersection of ellipse and hyperbola

$a^2 + b^2 = 16$

$\Rightarrow \quad a^2e^2 = 16$

$\Rightarrow \quad a^2 = 4 \quad \Rightarrow \quad b^2 = 12$

$\therefore$ No director circle of hyperbola is possible.

50. Answer (A, B)

Hint : Conversion into trigonometric function values.

Solution :

$$\because \tan\alpha = \frac{36}{77}, \qquad \tan\beta = \frac{3}{4}, \qquad \tan\gamma = \frac{8}{15}$$

$$\tan(\alpha+\beta+\gamma) = \frac{\Sigma(\tan\alpha) - \pi(\tan\alpha)}{1 - \Sigma\tan\alpha\tan\beta} = \infty$$

$$\Rightarrow \quad \alpha+\beta+\gamma = \frac{\pi}{2}$$

$\therefore$ Option (A) and (B) are correct.

51. Answer (B, C, D)

Hint : A.M $\geq$ G.M

Solution :

$$\therefore uv < 0 \Rightarrow u + \frac{1}{u} \geq 2, \quad v + \frac{1}{v} \leq -2$$

$$\text{or} \quad u + \frac{1}{u} \leq -2 \quad \text{or} \quad v + \frac{1}{v} \geq 2$$

$$\Rightarrow \quad \sec^{-1}\left(u + \frac{1}{u}\right) \in \left[\frac{\pi}{3}, \frac{\pi}{2}\right)$$

$$\sec^{-1}\left(v + \frac{1}{v}\right) \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right]$$

$$\therefore \quad t \in \left(\frac{5\pi}{6}, \frac{7\pi}{6}\right)$$

52. Answer (A)

53. Answer (D)

Hint for Q. No. 52 and 53

Family of circles.

Solution for Q. No. 52 and 53

Let Σ is $x^2 + y^2 - 9x - 12y + 53 + \lambda(2x + 3y - 27) = 0$

Given circle $x^2 + y^2 - 4x - 6y - 3 = 0$

$\therefore$ Equation of common chord

$-5x + 6y + 56 + \lambda(2x + 3y - 27) = 0$

$\therefore$ Chord passes through the point of intersection of $5x + 6y - 56 = 0$ and $2x + 3y - 27 = 0$

i.e. $\left(2, \frac{23}{3}\right)$

$\because$ Σ intersects $x^2 + y^2 = 29$ orthogonally.

$53 - 27\lambda - 29 = 0$

$$\lambda = \frac{24}{27} = \frac{8}{9}$$

$\therefore$ Circle is

$$x^2 + y^2 + \left(\frac{16}{9} - 9\right)x + \left(\frac{29}{9} - 12\right)y + 29 = 0$$

$\therefore$ Center is $\left(\frac{65}{18}, \frac{14}{3}\right)$

54. Answer (B)

55. Answer (C)

Hind for Q. No. 54 and 55

Mathematical induction approach.

Solution for Q. No. 54 and 55

Put $n = 2$

$\Rightarrow \quad f(1) + 2f(2) = 6f(2)$

$\Rightarrow \quad 4f(2) = f(1)$

$\Rightarrow \quad f(2) = \frac{1}{8}$

Similarly $f(3) = \frac{1}{12}$, $f(4) = \frac{1}{16}$ … and so on

$\therefore \quad f(n) = \frac{1}{4n} \qquad \therefore \ f(1010) = \frac{1}{4040}$

56. Answer A(S); B(Q, R, S, T); C(R); D(P, Q, R, S, T)

Hint : Eccentricity formula for conic.

Solution :

(A) $\because \sqrt{c^2+d^2}=a; \quad \sqrt{a^2-b^2}=c$

$\Rightarrow c^2+d^2=a^2$ and $a^2-b^2=c^2$

$\Rightarrow d=b \quad \Rightarrow \dfrac{d}{b}=1$

(B) Now $e_1=1-\dfrac{b^2}{a^2} \quad e_2=1+\dfrac{d^2}{c^2}$

$\Rightarrow e_1^2+e_2^2=2+b^2\left(\dfrac{a^2-c^2}{a^2c^2}\right)$

$e_1+e_2=e_1^2+\dfrac{1}{e_1^2}>2$

(C) $2\tan^{-1}\left(\dfrac{d}{c}\right)=\dfrac{2\pi}{3} \Rightarrow d=\sqrt{3}c \Rightarrow d^2=3c^2$

$\Rightarrow a^2=4c^2 \quad \Rightarrow a=2c$

$\therefore 4e_1=4\sqrt{1-\dfrac{b^2}{a^2}}=4\sqrt{1-\dfrac{3c^2}{4c^2}}=2$

(D) $b^2=a^2\left(1-e_1^2\right)$

$\Rightarrow a^2=2b^2 \quad \Rightarrow c^2=b^2$

For P.O.I. $\dfrac{h^2}{b^2}-\dfrac{k^2}{b^2}=\dfrac{h^2}{a^2}+\dfrac{k^2}{b^2}$

$\Rightarrow h^2\left(\dfrac{a^2-b^2}{a^2b^2}\right)=\dfrac{2k^2}{b^2}$

$\Rightarrow \dfrac{h^2}{k^2}=\dfrac{2a^2}{a^2e_1^2}=4$

57. Answer A(Q, R, S); B(Q); C(R, S); D(P, T)

Hint : Equality hold conditions for I.T.F.

Solution :

(A) $(\sin^{-1}x)^2=(\sin^{-1}y)^2=\dfrac{\pi^2}{4}$

$\Rightarrow x=\pm 1$ and $y=\pm 1$

$\therefore x^3+y^3=-2, 0, 2$

(B) $(\cos^{-1}x)^2=(\cos^{-1}y)^2=\pi^2$

$\Rightarrow x=y=-1$

$\therefore x^5+y^5=-2$

(C) $(\sin^{-1}x)^2=\dfrac{\pi^2}{4}$ and $(\cos^{-1}y)^2=\pi^2$

$\Rightarrow \sin^{-1}x=\pm\dfrac{\pi}{2}$ and $\cos^{-1}y=\pi$

$\Rightarrow x=\pm 1$ and $y=-1$

(D) $|\sin^{-1}x-\sin^{-1}y|=\pi$

$\Rightarrow$ either $\sin^{-1}x=-\dfrac{\pi}{2}$ and $\sin^{-1}y=\dfrac{\pi}{2}$

or $\sin^{-1}x=\dfrac{\pi}{2}$ and $\sin^{-1}y=-\dfrac{\pi}{2}$

$x=-1$ and $y=1$ or $x=1$ and $y=-1$

$\therefore x^y=(-1)^1$ or $(1)^{-1}$

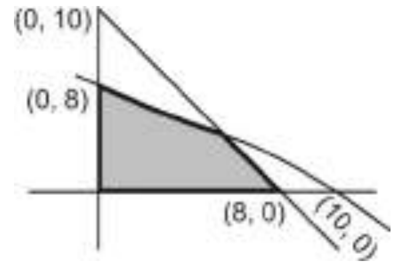
$=-1$ or 1

58. Answer (45)

Hint : Linear inequalities of two variables.

Solution :

Total number of integral co-ordinates in shaded region are 45



59. Answer (16)

Hint : Monotonicity of function.

Solution :

$\because x\in[-1, 1]$

Also $f(x)$ is an increasing function in domain

$\therefore p=f(-1)$ and $q=f(1)$

$\Rightarrow p=-\dfrac{\pi}{2}-\dfrac{\pi}{2}+(-2)=-\pi-2$

and $q=\dfrac{\pi}{2}+\dfrac{\pi}{2}+6=\pi+6$

$\therefore p+q=4 \quad \Rightarrow (p+q)^2=16$

60. Answer (36)

Hint : Point of intersection of two normals.

Solution :

Let $P(t_1)$ and $Q(t_2)$ are points

$\therefore t_2=2t_1$

$\because$ P.O.I of normals

$R\left(2a+a\left(t_1^2+t_1t_2+t_2^2\right), -t_1t_2\left(t_1+t_2\right)\right)$

$R\left(2+t_1^2+t_1t_2+t_2^2, -t_1t_2\left(t_1+t_2\right)\right)$

$\therefore x=2+7t_1^2, \quad y=-6t_1^3$

$\left(\dfrac{x-2}{7}\right)^3=t_1^6=\left(\dfrac{-y}{6}\right)^2=\dfrac{y^2}{36}$

$\therefore$ Locus is $y^2=\dfrac{36}{343}(x-2)^3$

$\therefore k=36$

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