All India Aakash Test Series for JEE (Main)-2020

TEST - 5 - Code-A

Test Date : 01/12/2019

ANSWERS

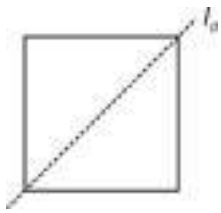
PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(4)	26.	(3)	51.	(1)
2.	(4)	27.	(4)	52.	(3)
3.	(3)	28.	(1)	53.	(4)
4.	(1)	29.	(2)	54.	(3)
5.	(2)	30.	(3)	55.	(1)
6.	(4)	31.	(4)	56.	(2)
7.	(1)	32.	(4)	57.	(2)
8.	(1)	33.	(4)	58.	(4)
9.	(1)	34.	(1)	59.	(1)
10.	(2)	35.	(2)	60.	(1)
11.	(4)	36.	(2)	61.	(4)
12.	(4)	37.	(1)	62.	(4)
13.	(3)	38.	(2)	63.	(3)
14.	(2)	39.	(2)	64.	(2)
15.	(3)	40.	(2)	65.	(2)
16.	(2)	41.	(3)	66.	(2)
17.	(3)	42.	(2)	67.	(2)
18.	(3)	43.	(4)	68.	(4)
19.	(4)	44.	(1)	69.	(2)
20.	(1)	45.	(3)	70.	(4)
21.	(18)	46.	(03)	71.	(61)
22.	(03)	47.	(04)	72.	(16)
23.	(04)	48.	(28)	73.	(02)
24.	(06)	49.	(58)	74.	(04)
25.	(12)	50.	(25)	75.	(03)

PART - A (PHYSICS)

1. Answer (4)

Hint : $I_d = \frac{Ma^2}{12}$

Sol. :



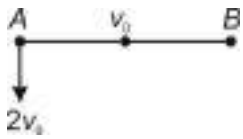
$\Rightarrow I_{AB} = \frac{7}{12}Ma^2$

2. Answer (4)

Hint : Momentum remains conserved.

$\omega = 4v_0/\ell$

Sol. : $v_A = \sqrt{4v_0^2 + v_0^2} = v_0\sqrt{5}$



3. Answer (3)

Hint : Use kinematical equations.

Sol. : On the basis of given information we can easily find

$\frac{s_A}{s_B} = 2$

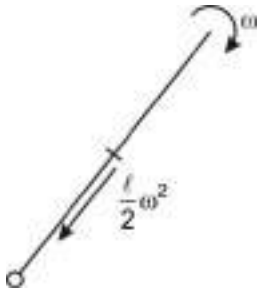
$\Rightarrow s_B = \frac{s}{2}$

4. Answer (1)

Hint and Sol. : $\sum\vec{F} = M\overrightarrow{a_{cm}}$

$-F + mg\cos\theta = (m)\frac{\ell}{2}\omega^2$

$\frac{mg\ell}{2} = \frac{mg\ell}{2}\cos 37° + \frac{1}{2}\frac{m\ell^2}{3}\omega^2$



5. Answer (2)

Hint : $a_{tot} = \sqrt{a_R^2 + a_t^2}$

Sol. : $a_t = 2$ m/s^2

$a_R = \frac{v^2}{R}$

$a_{tot} = \sqrt{a_R^2 + a_t^2} = a\sqrt{1+4\pi^2} = 2\sqrt{1+4\pi^2}$

6. Answer (4)

Hint : Find equation of trajectory.

Sol. : $\left(\frac{X}{4}\right)^2 + \left(\frac{Y}{3}\right)^2 = 1$

$\Rightarrow$ Ellipse

7. Answer (1)

Hint : Draw FBD and write down the equation of motion.

Sol. : If T be the tension in string

$$\left.\begin{aligned} 30 - T &= 20a \\ \Rightarrow\ T - 15 &= 5a \\ F - 30 &= 50a \end{aligned}\right\} \Rightarrow F = 60\text{ N}$$

8. Answer (1)

Hint : $w = pt$

Sol. : $\frac{1}{2}mv^2 = pt$

$\Rightarrow v \propto t^{1/2}$

9. Answer (1)

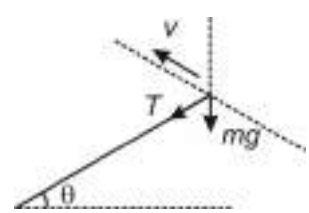
Hint : $w = \int \vec{F}\cdot\vec{dr}$

Sol. : $w = -2\int_{(0,0)}^{(2,2)} d(xy) = -2(2)^2 = -8$ J

10. Answer (2)

Hint : $\sum\vec{F} = m\vec{a}_{cm}$

Sol. : $T + mg\sin\theta = \frac{mv^2}{\ell}$



and $v^2 = u^2 - 2g\ell\sin\theta$

On solving,

$$T = \frac{mg}{2}$$

11. Answer (4)

Hint : $\overrightarrow{v_{cm}} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{(m_1 \times m_2)}$

Sol. : $\vec{v}_{cm} = (\hat{i} + \hat{j})$ m/s

$\vec{a}_{cm} = \frac{1}{2}(\hat{i} + \hat{j})$ m/s^2

$\vec{v} \parallel \vec{a}_{cm}$,

So path is straight line.

12. Answer (4)

Hint : $I_{tot} = \frac{mR^2}{2}$

Sol. : $I + I = \frac{(2M)R^2}{2}$

$\Rightarrow I = \frac{MR^2}{2}$

13. Answer (3)

Hint : At the time of collision both the balls will be at same position.

Sol. : $S_1 = ut - \frac{1}{2}(10)\,t^2$

$S_2 = \frac{1}{2}(10)\,t^2$

$S_1 + S_2 = 100$

$\Rightarrow u = 25$ m/s

14. Answer (2)

Hint : Find $v = f(t)$

$v = 0$ at $t = 2$

Sol. : $\frac{dv}{dt} = -k\sqrt{v}$

$$\Rightarrow \int_{v_0}^{v} \frac{dv}{\sqrt{v}} = \int_0^t -k\,dt$$

$v = 0$ at $t = 2$ s

15. Answer (3)

Hint : Use condition of maximum range.

Sol. : $T = \frac{2u\sin\alpha}{g\cos\theta}$

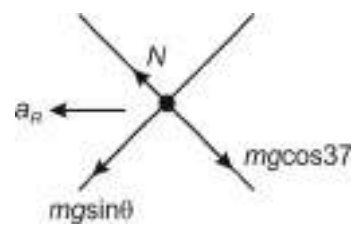
$R = u\cos\alpha \cdot T + \frac{1}{2}g\sin\theta t^2$

$\frac{dR}{d\alpha} = 0$, find α

16. Answer (2)

Hint : Draw FBD, particle is moving in a circular path of radius $x\cos 37$.

Sol. : $a_R = m(x\cos 37)\omega^2$



$N_x = ma_R$

$$\omega^2 = \frac{(g)^{3/5}}{(x)\left(\frac{4}{5}\right)^2} \Rightarrow \omega^2 = \frac{3g}{(5)(x)\frac{16}{25}}$$

$$\omega^2 = \frac{3g \times 5}{16x} \Rightarrow x\omega^2 = \frac{15g}{4 \times 4}$$

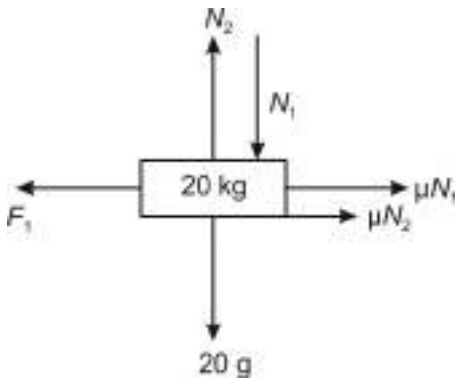
$x = 1$

17. Answer (3)

Hint : Draw FBD

Sol. : $a_{min} = 20\ \mu$

$F_1 - \mu(N_1 + N_2) = 400\ \mu$



18. Answer (3)

Hint : $\overrightarrow{A.I_{cm}} = \overrightarrow{L_f} - \overrightarrow{L_i}$

Sol. : $J \times \frac{L}{2} = I_{cm}\ \omega$

$\Rightarrow \omega = \frac{v}{2}$

$v_{cm} = \left(\frac{v}{2}\right)$

$v_B = v$

$\omega = \frac{v}{L}$

$v_B = (\omega)\frac{L}{2} + \frac{v}{2}$

$= \left(\frac{v}{L}\right)\left(\frac{L}{2}\right) + \frac{v}{2} = v$

19. Answer (4)

Hint : Rigid body will be in pure rotation with respect to any point on the ring.

Sol. : $\vec{v}_A - \vec{v}_C = 2v_0(-\hat{i})$

$|\vec{v}_B - \vec{v}_C| = (R\sqrt{2})v_0$

$|\vec{v}_C - \vec{v}_D| = (R\sqrt{2})v_0$

20. Answer (1)

Hint and Sol. : $\overrightarrow{L_0} = (mv_0)r_\perp$

$r_\perp = 0 \Rightarrow \vec{L}_0 = 0$

21. Answer (18)

Hint : $X_{cm} = \frac{\int x\,dm}{\int dm}$

Sol. : $X_{cm} = \frac{\int_0^3 (x)(\lambda)dx}{\int_0^3 (2+x)dx} = \frac{12}{7}$ cm

Distance from B = $\frac{9}{7}$ cm $= \frac{18}{14}$ cm

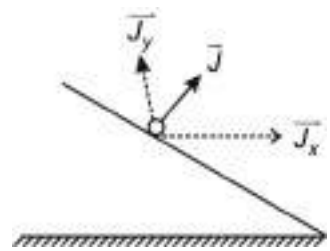
22. Answer (03)

Hint : $\overrightarrow{J_y} = \overrightarrow{\Delta P_y}$

and $\overrightarrow{J_x} = \overrightarrow{\Delta P_x}$

Sol. : $v_f \cos 30° = v_i \cos 60°$

$v_f = \sqrt{3}$ m/s



23. Answer (04)

Hint : $\vec{J} = \vec{P}_f - \vec{P}_i$

Sol. : $12 = 2(v_f - (-2))$

$v_f + 2 = 6$

24. Answer (06)

Hint : $\overrightarrow{p_i^x} = \overrightarrow{p_f^x}$

Sol. : $mv_0 = 6mv$

$\Rightarrow v = \frac{v_0}{6}$

25. Answer (12)

Hint : $\overrightarrow{p_i} = \overrightarrow{p_f}$

Sol. : $5 \times 2 - 2 \times 4 = (2+4)v$

$v = \frac{2}{6} = \frac{1}{3}$ m/s $= \frac{4}{12}$ m/s

PART - B (CHEMISTRY)

26. Answer (3)

Hint : ΔH_{eg} for all noble gases is positive.

Sol. : ΔH_{eg} for hydrogen is negative.

27. Answer (4)

Hint : Electronegativity of an element depends upon the element to which it is bonded.

Sol. : Electronegativity is not equal to the electron affinity.

28. Answer (1)

Hint : Minima on Y-axis gives us the value of bond energy.

Sol. : Abscissa corresponding to minima gives us the value of bond length.

29. Answer (2)

Hint : Direct overlap forms σ bond.

Sol. : Positive overlap forms bonding molecular orbital.

30. Answer (3)

Hint : Greater the bond order, shorter is the bond length.

Sol. : Ethene > Benzene > Graphite (Bond order)

31. Answer (4)

Hint : The electron density is shifted towards benzene ring in only (III).

Sol. : For both benzaldehyde and chlorobenzene, electron displacement vector is away from the benzene ring.

32. Answer (4)

Hint : $\ln K = \left(\frac{-\Delta H}{R}\right)\frac{1}{T} + \frac{\Delta S}{R}$

Sol. : $\Delta H > 0 \Rightarrow \text{slope} < 0$

$\Delta S > 0 \Rightarrow \text{intercept} > 0$

33. Answer (4)

Hint : $L = mvr = \frac{nh}{2\pi}$

Sol. : ΔL is max. for option (4)

34. Answer (1)

Hint : Dalton's law is not applicable for reacting gases.

Sol. : H_2 and He are non-reacting.

35. Answer (2)

Hint : As temperature increases, intensity increases.

Sol. : As temperature increases, the peak of intensity shifts towards lower wavelength.

36. Answer (2)

Hint : Aufbau's Principle is $(n + \ell)$ rule.

Sol. : Zn^{2+} will have the same configuration i.e. $[Ar]3d^{10}$ whether or not the Aufbau's principle is followed.

37. Answer (1)

Hint : For H-atom, degeneracy of an orbit is equal to n^2.

Sol. : Radial nodes > Angular nodes

$\Rightarrow \; n - \ell - 1 > \ell$

$\Rightarrow \; n > 2\ell + 1$

So, S_1 cannot be $3d$

38. Answer (2)

Hint : Magnitude of Ionisation energy is greater than electron gain enthalpy.

Sol. : Electron gain enthalpy of oxygen is less negative than that of sulphur.

39. Answer (2)

Hint : Compare the V_{MP}

Sol. : $(V_{MP})_A < (V_{MP})_B$

$\Rightarrow \; \frac{T_A}{M_A} < \frac{T_B}{M_B}$

$\Rightarrow \; M_B T_A < M_A T_B$

40. Answer (2)

Hint : $\text{Rate} \propto \frac{P}{\sqrt{M}}$

Sol. : $\frac{n_{He}}{n_{N_2}} = 4\sqrt{\frac{28}{4}} = 4\sqrt{7}$

41. Answer (3)

Hint : A → B is isochoric cooling.

Sol. : B → C is isothermal compression.

C → A is isobaric expansion.

42. Answer (2)

Hint : $\Delta S = nR\ln\left(\frac{V_2}{V_1}\right)$ as $\Delta T = 0$

Sol. : Order of volume ratios:

Process (1) > Process (2) > Process (3)

43. Answer (4)

Hint : S_8 has 8 S–S bonds.

P_4 has 6 P–P bonds.

Sol. : P = 8x

Q = 6x

44. Answer (1)

Hint : $d_{solution} = \frac{220}{240} < d_{solvent}$

Sol. : $m = \frac{20}{M_0(0.2)}$

$M = \frac{20}{M_0(0.24)}$

$\therefore\ M < m$

45. Answer (3)

Hint : Moles of H_2SO_4 = 1

So, moles of NaOH = 2

Sol. : eq. of H_3PO_4 + eq. of HCl = eq. of NaOH

$\frac{x}{98}\times 3 + 0.1 = 2$

$x = \frac{(2-0.1)\times 98}{3} \simeq 62\,g$

$\therefore$ % purity = $\frac{62}{70}\times 100 = 88.67\ \%$

46. Answer (03)

Hint : In dimeric $BeCl_2$, Be has a sextet of electrons.

Sol. : $Al_2(CH_3)_6$ has e^- deficient banana bond. SNF_3 has an expanded octet of S.

47. Answer (04)

Hint : There are four lines lying in the Paschen series.

Sol. : 7 → 3, 6 → 3, 5 → 3, 4 → 3

48. Answer (28)

Hint : V_{O_2} unburnt = 44 ml

V_{CO_2} produced = 24 ml

Sol. :

$$C_xH_y + \left(x+\frac{y}{4}\right)O_2 \longrightarrow x\,CO_2 + \frac{y}{2}H_2O$$

12 $\quad 12\left(x+\frac{y}{4}\right) \quad$ 12x

= 24

x = 2

$\therefore\ 12\left(x+\frac{y}{4}\right) = 36$

$\Rightarrow$ y = 4 $\quad\therefore\ C_2H_4$

49. Answer (58)

Hint : Only $NaHCO_3$ decomposes upon heating.

Sol. : $2NaHCO_3 \longrightarrow Na_2CO_3 + H_2O + CO_2$

a $\quad \frac{a}{2} \quad \frac{a}{2} \quad \frac{a}{2}$

Loss in weight = mass of H_2O and CO_2

$= \frac{a}{2}(44+18)$

= 0.31

$\therefore\ a = \frac{0.31\times 2}{62} = 0.01\,\text{moles}$

$\therefore$ Mass of $NaHCO_3$ = a × 84

= 0.84 g

$\therefore$ x = 58%

50. Answer (25)

Hint : A, B, C, D, E are state functions, X = 5

Sol. : B, D, E, F, G are extensive properties, Y = 5

PART - C (MATHEMATICS)

51. Answer (1)

Hint : Put $z = x + iy$

Sol. : $|z - 1 - i| = |z|\sin(\arg z)$

Let $z = x + iy$

$(x - 1)^2 + (y - 1)^2 = y^2$

$\Rightarrow (x - 1)^2 = 2\left(y - \frac{1}{2}\right)$

It is a parabola whose length of latus rectum is 2.

52. Answer (3)

Hint : For continuous function $f(x)$; if $f(a)\cdot f(b) < 0$ then there exist atleast one root of $f(x)$ in (a, b).

Sol. : Let $f(11) = \alpha$, $f(12) = \beta$ and $f(13) = \gamma$, $\alpha < \beta < \gamma$ because $f(y)$ is increasing function.

$$\frac{1}{x-\alpha} + \frac{1}{x-\beta} + \frac{1}{x-\gamma} = 0$$

$(x - \beta)(x - \gamma) + (x - \alpha)(x - \gamma) + (x - \alpha)(x - \beta) = 0$

Let $g(x) = (x - \beta)(x - \gamma) + (x - \alpha)(x - \gamma) + (x - \alpha)(x - \beta)$

Now, $g(\alpha) = (\alpha - \beta)(\alpha - \gamma) > 0$

$g(\beta) = (\beta - \alpha)(\beta - \gamma) < 0$

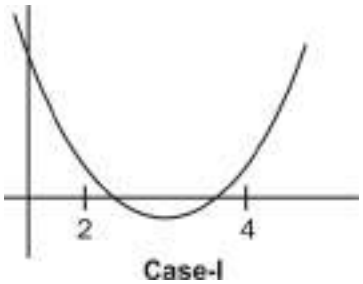
and $g(\gamma) = (\gamma - \alpha)(\gamma - \beta) > 0$

So one root of $g(x) = 0$ lies between α and β and other root lies between β and γ.

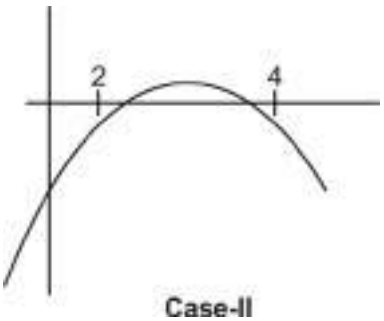
53. Answer (4)

Hint : Draw graph

Sol. : There are two cases possible



Case-I



Case-II

$f(x) = ax^2 + bx + c$

$f(-1) = a - b + c < 0$ (given)

So Case-I cannot be true.

In Case-II, clearly $a < 0$ (downward parabola)

$f(0) < 0 \Rightarrow c < 0$

$f(1) < 0 \Rightarrow a + b + c < 0$

Sum of roots $= -\frac{b}{a} > 0 \Rightarrow b > 0$

54. Answer (3)

Hint : $(1-x)^{10}\left(1+x+\frac{1}{x}\right)^9 = \frac{(1-x)(1-x^3)^9}{x^9}$

Sol. : Coefficient of x^{16} in

$(1-x)^{10}\left(\frac{1+x+x^2}{x}\right)^9$

= coefficient of x^{25} in $(1 - x)(1 - x^3)^9$

$= -{}^9C_8$

$= -9$

55. Answer (1)

Hint : Put $x = 1$, ω and ω^2 in the expansion and add.

Sol. : $(x^2 + x - 1)^{10} = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \ldots$

Put $x = 1$

$1 = a_0 + a_1 + a_2 + a_3 + a_4 + \ldots$ …(i)

Put $x = \omega$

$2^{10} = a_0 + a_1\omega + a_2\omega^2 + a_3 + a_4\omega + \ldots$ …(ii)

Put $x = \omega^2$

$2^{10} = a_0 + a_1\omega^2 + a_2\omega + a_3 + a_4\omega^2 + \ldots$ …(iii)

Adding (i), (ii) and (iii)

$3[a_0 + a_3 + a_6 + \ldots + a_{18}] = 1 + 2^{10} + 2^{10}$

$\Rightarrow a_0 + a_3 + a_6 + \ldots + a_{18} = 683$

56. Answer (2)

Hint : Use expansion

Sol. : $17^{15} = (1 + 16)1^5$

$= 1 + {}^{15}C_1 \cdot 16 + {}^{15}C_2 \cdot 16^2 + {}^{15}C_3 \cdot 16^3 + \ldots$

$15^{17} = -(1 - 16)^{17}$

$= -1 + {}^{17}C_1 \cdot 16 - {}^{17}C_2 \cdot 16^2 + {}^{17}C_3 \cdot 16^3 - \ldots$

$17^{15} + 15^{17} = 16({}^{15}C_1 + {}^{17}C_1) + 16^2({}^{15}C_2 - {}^{17}C_2) + 16^3({}^{15}C_3 + {}^{17}C_3) + \ldots$

$= 16 \times 32 + 16^2 \times (-31) + 16^3({}^{5}C_3 + {}^{17}C_3) + \ldots$

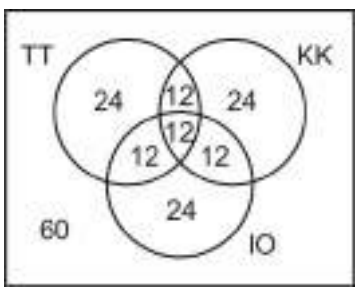
$= 16^2 \times$ odd number

57. Answer (2)

Hint : Principle of inclusion and exclusion.

Sol. : By using principle of inclusion and exclusion, number of such words

= Total words – (TT together + KK together + IO together) + (TT & KK together + KK & IO together + TT & IO together) – (TT, KK and IO together)



$$= \frac{\lfloor 6}{\lfloor 2 \lfloor 2} - \left[\frac{\lfloor 5}{\lfloor 2} + \frac{\lfloor 5}{\lfloor 2} + \frac{\lfloor 5 \cdot \lfloor 2}{\lfloor 2 \lfloor 2}\right] + \left[\lfloor 4 + \frac{\lfloor 4}{\lfloor 2} \cdot \lfloor 2 + \frac{\lfloor 4}{\lfloor 2} \cdot \lfloor 2\right] - [\lfloor 3 \cdot \lfloor 2]$$

= 180 – [3 × 60] + [72] – 12

= 60

58. Answer (4)

Hint : Number formation

Sol. :

Number of odd numbers starting with
1(1 3 4 2) = 24
1 3, 5

Number of odd numbers starting with
2(1 3 4 3) = 36
2 1, 3, 5

Number of odd numbers starting with
3(1 3 4 2) = 24
3 1, 5

Number of odd numbers starting with
41(1 1 3 2) = 6
4 1 3, 5

Number of odd numbers starting with
42(1 1 3 3) = 9
4 2 1, 3, 5

Total 99 numbers

So the number at 100th place will be the first number starting with 43, which is 4315.

59. Answer (1)

Hint : Middle term in the expansion of $(a + b)^n = {}^nC_{n/2}\,(ab)^{n/2}$ if n is even

Sol. : ${}^6C_3 \cdot (2^{\log_2 x})^3 (x^{\log_2 x})^3 = 1280$

$\Rightarrow x^{3+3\log_2 x} = 64$

$\Rightarrow (1+\log_2 x)\cdot \log_2 x = 2$

$\log_2 x = 1$ or -2

$x = 2$ or $\frac{1}{4}$

60. Answer (1)

Hint : Sum of A.G.P. = $\frac{a}{1-r} + \frac{dr}{(1-r)^2}$

Sol. : $f(x) = \left(\sin x - \frac{1}{2}\right)^2 + \frac{3}{4}$

So range of $f(x)$ is $\left[\frac{3}{4}, 3\right]$

$r = \frac{3}{4}, \quad d = 3$

$S = 1 + (1 + d)r + (1 + 2d)r^2 + (1 + 3d)r^3 + \ldots \infty$

$S.r = r + (1 + 2d)r^2 + (1 + 3d)r^3 + \ldots \infty$

$S(1 - r) = 1 + dr + dr^2 + dr^3 + \ldots \infty$

$\Rightarrow S \cdot \frac{1}{4} = 1 + \frac{dr}{1-r} = 1 + \frac{9/4}{1/4}$

$\Rightarrow S = 40$

61. Answer (4)

Hint : $T_n = S_n - S_{n-1}$

Sol. : $\Delta_n = \sum_{k=1}^{n} \Delta_k - \sum_{k=1}^{n-1} \Delta_k = 3^n - 3^{n-1} = 2 \cdot 3^{n-1}$

$$\sum_{k=1}^{\infty} \frac{1}{\Delta_n} = \frac{1}{2}\left[1 + \frac{1}{3} + \frac{1}{3^2} + \ldots\infty\right] = \frac{3}{4}$$

62. Answer (4)

Hint : General term in the expansion of $(a + b + c)^n$ is $\frac{\lfloor n}{\lfloor p_1 \lfloor p_2 \lfloor p_3} a^{p_1} \cdot b^{p_2} \cdot b^{p_3}$

Sol. : General term in the expansion is

$$\frac{\lfloor 5}{\lfloor p_1 \lfloor p_2 \lfloor p_3}(a^2b)^{p_1}(b^2c)^{p_2}(c^2a)^{p_3}$$

$2p_1 + p_3 = 6$

$2p_2 + p_1 = 4$

$p_2 + 2p_3 = 5$

Also, $p_1 + p_2 + p_3 = 5$

So, $p_1 = 2$, $p_2 = 1$ and $p_3 = 2$

Coeff. of $a^6b^4c^5$ is $\frac{\lfloor 5}{\lfloor 2 \lfloor 1 \lfloor 2} = 30$

63. Answer (3)

Hint : $f(x) = (1 + x)^n - 1 - x^n$

Sol. : $f(x) = (1 + x)^n - 1 - x^n$

So, $f(1) = 2^n - 1 - 1^n$

$f(2) = 3^n - 1 - 2^n$

$f(3) = 4^n - 1 - 3^n$

$f(k) = (k + 1)^n - 1 - k^n$

Adding these equations,

$$\sum_{r=1}^{k} f(r) = (k+1)^n - (k+1)$$

64. Answer (2)

Hint : $T_r = \frac{1}{\lfloor r-2} + \frac{2}{\lfloor r-1} + \frac{1}{\lfloor r}$

Sol. : $T_r = \frac{r^2 + r + 1}{\lfloor r} = \frac{r(r-1) + 2r + 1}{\lfloor r}$

$\Rightarrow T_r = \frac{1}{\lfloor r-2} + 2 \cdot \frac{1}{\lfloor r-1} + \frac{1}{\lfloor r}$

$$\sum_{r=1}^{\infty} T_r = \sum_{r=2}^{\infty} \frac{1}{\lfloor r-2} + 2\sum_{r=1}^{\infty} \frac{1}{\lfloor r-1} + \sum_{r=1}^{\infty} \frac{1}{\lfloor r}$$

$= e + 2e + e - 1 = 4e - 1$

65. Answer (2)

Hint : $T_r = \frac{1}{r} - \frac{1}{r+1}$

Sol. : $T_r = \frac{(r+1)^2 + (r+2)}{(r+1)^4 - (r+1)}$

$\Rightarrow T_r = \frac{r^2 + 3r + 3}{(r+1)[r^3 + 3r^2 + 3r]} = \frac{1}{r(r+1)}$

$\Rightarrow T_r = \frac{1}{r} - \frac{1}{r+1}$

So; $T_1 = \frac{1}{1} - \frac{1}{2}$

$T_2 = \frac{1}{2} - \frac{1}{3}$

$T_3 = \frac{1}{3} - \frac{1}{4}$

$T_{12} = \frac{1}{12} - \frac{1}{13}$

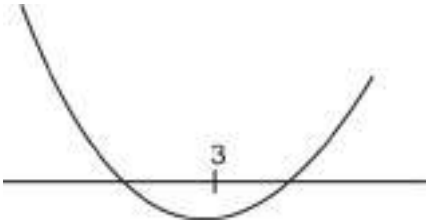
By adding these terms, we get

$S = 1 - \frac{1}{13} = \frac{12}{13}$

66. Answer (2)

Hint : If $a > 0$ and α lies between roots of $ax^2 + bx + c = 0$ then $a\alpha^2 + b\alpha + c < 0$

Sol. : $(k-2)x^2 + (k-1)x + 3 = 0$



$$x^2 + \left(\frac{k-1}{k-2}\right)x + \frac{3}{(k-2)} = 0$$

$\because \quad f(3) < 0$

$$9 + \frac{3k-3}{k-2} + \frac{3}{k-2} < 0$$

$$\frac{12k-18}{k-2} < 0$$

$$k \in \left(\frac{3}{2},\ 2\right)$$

67. Answer (2)

Hint : $\ln\left(\frac{1+x}{1-x}\right) = 2\left[x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \ldots\right]$

Sol. : $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \ldots$$

$$\ln(1+x) - \ln(1-x) = 2\left[x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\right]$$

Put $x = \frac{1}{2}$

$$2\left[\frac{1}{2} + \frac{1}{3.2^3} + \frac{1}{5.2^5} + \ldots\infty\right] = \ln\left(\frac{3}{2}\right) - \ln\left(\frac{1}{2}\right)$$

$$\frac{1}{1\cdot 2} + \frac{1}{3.2^3} + \frac{1}{5.2^5} + \ldots\infty = \ln\sqrt{3}$$

68. Answer (4)

Hint : Change $a + ib$ into $r\cdot e^{i\theta}$

Sol. : $\tan\left(i\ln\left(\frac{\frac{\sqrt{3}}{2} + \frac{1}{2}i}{\frac{\sqrt{3}}{2} - \frac{1}{2}i}\right)\right) = \tan\left(i\ln\left(\frac{e^{i\pi/6}}{e^{-i\pi/6}}\right)\right)$

$$= \tan\left(i\ln\left(e^{2i\pi/6}\right)\right)$$

$$= \tan\left(\frac{\pi}{3}i^2\right) = -\sqrt{3}$$

69. Answer (2)

Hint : $T_r = (1 + r\omega - r\omega^2)(1 - r\omega + r\omega^2) = 1 + 3r^2$

Sol. : $T_r = (1 + r\omega - r\omega^2)(1 - r\omega + r\omega^2)$

$$T_r = 1 + 3r^2$$

$$S_n = \sum_{r=1}^{n} T_r = n + \frac{n(n+1)(2n+1)}{2}$$

$$= \frac{n(2n^2 + 3n + 3)}{2}$$

70. Answer (4)

Hint : Locus of point of intersection of perpendicular tangents to a hyperbola is director circle.

Sol. : $C_2 : |z+5| - |z-5| = 8$ represents a hyperbola and $C_1 : |z| = \sqrt{7}$ is the director circle of given hyperbola.

So angle between tangents will be $\frac{\pi}{2}$.

71. Answer (61)

Hint : Let $x = k$, where $k = 1, 2, 3$ and $y + z + t = 15 - 4k$

Sol. : $y + z + t = 15 - 4x$

No. of solutions = ${}^{14-4x}C_2$

Here $x = 1, 2, 3$

Total number of solutions $= {}^{10}C_2 + {}^{6}C_2 + {}^{2}C_2$

$= 45 + 15 + 1$

$= 61$

72. Answer (16)

Hint : Combinations

Sol. : For each element in S there are 4 options

(i) Element belongs to set P

(ii) Element belongs to set Q

(iii) Element belongs to P and Q both

(iv) Element neither belongs to P nor Q.

$\because n(P \cap Q) = 3$

$\Rightarrow$ For any 3 elements only option (iii) is available and for the remaining elements option (i) and (ii) are available.

So number of ordered pairs $(P, Q) = {}^6C_3 \cdot 2^3 = 160$

73. Answer (02)

Hint : Solve the two curves.

Sol. : $z\bar{z} + z + \bar{z} - 1 = 0$ and $z + \bar{z} + 2 = 0$

Solving the equation of line and circle,

$z(-z-2) + z + (-z-2) - 1 = 0$

$z^2 + 2z + 3 = 0$

$z = -1 \pm \sqrt{2}i$

There are the two points of intersection say z_1 and z_2.

Length of intercept will be $|z_1 - z_2| = 2\sqrt{2}$

74. Answer (04)

Hint : Put $2^x = t$

Sol. : Let $2^x = t$

$t^2 + 2t + a - 5 = 0$

$\Rightarrow (t + 1)^2 = 6 - a$

$(t + 1)^2 \in (1, \infty)$

So the equation has real solutions if

$6 - a > 1$

$a < 5$

Greatest integral value of a is 4.

75. Answer (03)

Hint : $3^{2019} + 4^{2020} = (7k_1 - 1)^{673} + 4(7k_2 + 1)^{673}$

Sol. : $7x$ = remainder when $3^{2019} + 4^{2020}$ is divided by 7

$3^{2019} + 4^{2020} = (28 - 1)^{673} + 4(63 + 1)^{673}$

$= (7\lambda - 1) + 4(7\mu + 1)$

$= 7 \times \text{integer} + 3$

Remainder is 3.

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All India Aakash Test Series for JEE (Main)-2020

TEST - 5 - Code-B

Test Date : 01/12/2019

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(1)	26.	(3)	51.	(4)
2.	(4)	27.	(1)	52.	(2)
3.	(3)	28.	(4)	53.	(4)
4.	(3)	29.	(2)	54.	(2)
5.	(2)	30.	(3)	55.	(2)
6.	(3)	31.	(2)	56.	(2)
7.	(2)	32.	(2)	57.	(2)
8.	(3)	33.	(2)	58.	(3)
9.	(4)	34.	(1)	59.	(4)
10.	(4)	35.	(2)	60.	(4)
11.	(2)	36.	(2)	61.	(1)
12.	(1)	37.	(1)	62.	(1)
13.	(1)	38.	(4)	63.	(4)
14.	(1)	39.	(4)	64.	(2)
15.	(4)	40.	(4)	65.	(2)
16.	(2)	41.	(3)	66.	(1)
17.	(1)	42.	(2)	67.	(3)
18.	(3)	43.	(1)	68.	(4)
19.	(4)	44.	(4)	69.	(3)
20.	(4)	45.	(3)	70.	(1)
21.	(12)	46.	(25)	71.	(03)
22.	(06)	47.	(58)	72.	(04)
23.	(04)	48.	(28)	73.	(02)
24.	(03)	49.	(04)	74.	(16)
25.	(18)	50.	(03)	75.	(61)

PART - A (PHYSICS)

1. Answer (1)

Hint and Sol. : $\overrightarrow{L_0} = (mv_0)r_\perp$

$r_\perp = 0 \Rightarrow \vec{L}_0 = 0$

2. Answer (4)

Hint : Rigid body will be in pure rotation with respect to any point on the ring.

Sol. : $\vec{v}_A - \vec{v}_C = 2v_0(-\hat{i})$

$|\vec{v}_B - \vec{v}_C| = (R\sqrt{2})v_0$

$|\vec{v}_C - \vec{v}_D| = (R\sqrt{2})v_0$

3. Answer (3)

Hint : $\overrightarrow{A.I_{cm}} = \overrightarrow{L_f} - \overrightarrow{L_i}$

Sol. : $J \times \frac{L}{2} = I_{cm}\ \omega$

$\Rightarrow \omega = \frac{v}{2}$

$v_{cm} = \left(\frac{v}{2}\right)$

$v_B = v$

$\omega = \frac{v}{L}$

$v_B = (\omega)\frac{L}{2} + \frac{v}{2}$

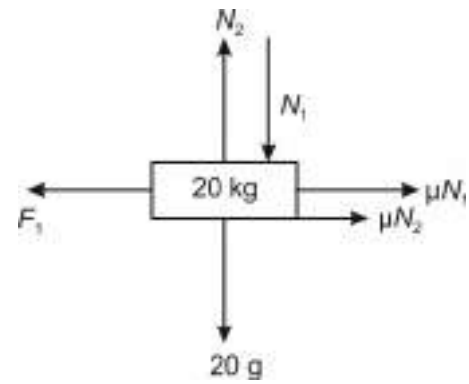
$= \left(\frac{v}{L}\right)\left(\frac{L}{2}\right) + \frac{v}{2} = v$

4. Answer (3)

Hint : Draw FBD

Sol. : $a_{min} = 20\ \mu$

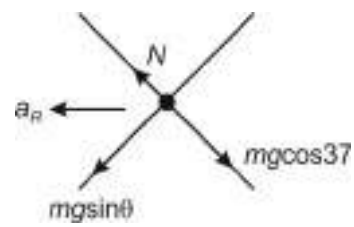
$F_1 - \mu(N_1 + N_2) = 400\ \mu$



5. Answer (2)

Hint : Draw FBD, particle is moving in a circular path of radius $x\cos 37$.

Sol. : $a_R = m(x\cos 37)\omega^2$



$N_x = ma_R$

$\omega^2 = \frac{(g)^{3/5}}{(x)\left(\frac{4}{5}\right)^2} \Rightarrow \omega^2 = \frac{3g}{(5)(x)\frac{16}{25}}$

$\omega^2 = \frac{3g \times 5}{16x} \Rightarrow x\omega^2 = \frac{15g}{4 \times 4}$

$x = 1$

6. Answer (3)

Hint : Use condition of maximum range.

Sol. : $T = \frac{2u\sin\alpha}{g\cos\theta}$

$R = u\cos\alpha \cdot T + \frac{1}{2}g\sin\theta t^2$

$\frac{dR}{d\alpha} = 0$, find α

7. Answer (2)

Hint : Find $v = f(t)$

$v = 0$ at $t = 2$

Sol. : $\frac{dv}{dt} = -k\sqrt{v}$

$\Rightarrow \int_{v_0}^{v} \frac{dv}{\sqrt{v}} = \int_{0}^{t} -k\, dt$

$v = 0$ at $t = 2$ s

8. Answer (3)

Hint : At the time of collision both the balls will be at same position.

Sol. : $S_1 = ut - \frac{1}{2}(10)\, t^2$

$S_2 = \frac{1}{2}(10)\, t^2$

$S_1 + S_2 = 100$

$\Rightarrow u = 25$ m/s

9. Answer (4)

Hint : $I_{tot} = \frac{mR^2}{2}$

Sol. : $I + I = \frac{(2M)R^2}{2}$

$\Rightarrow I = \frac{MR^2}{2}$

10. Answer (4)

Hint : $\overrightarrow{v_{cm}} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{(m_1 \times m_2)}$

Sol. : $\vec{v}_{cm} = (\hat{i} + \hat{j})$ m/s

$\vec{a}_{cm} = \frac{1}{2}(\hat{i} + \hat{j})$ m/s^2

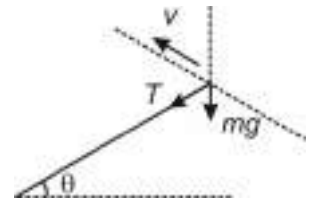
$\vec{v} \parallel \vec{a}_{cm}$,

So path is straight line.

11. Answer (2)

Hint : $\sum \vec{F} = m\vec{a}_{cm}$

Sol. : $T + mg\sin\theta = \frac{mv^2}{\ell}$



and $v^2 = u^2 - 2g\ell\sin\theta$

On solving,

$T = \frac{mg}{2}$

12. Answer (1)

Hint : $w = \int \vec{F} \cdot \overrightarrow{dr}$

Sol. : $w = -2\int_{(0,0)}^{(2,2)} d(xy) = -2(2)^2 = -8$ J

13. Answer (1)

Hint : $w = pt$

Sol. : $\frac{1}{2}mv^2 = pt$

$\Rightarrow v \propto t^{1/2}$

14. Answer (1)

Hint : Draw FBD and write down the equation of motion.

Sol. : If T be the tension in string

$$\left.\begin{aligned} 30 - T &= 20a \\ \Rightarrow\ T - 15 &= 5a \\ F - 30 &= 50a \end{aligned}\right\} \Rightarrow F = 60\text{ N}$$

15. Answer (4)

Hint : Find equation of trajectory.

Sol. : $\left(\frac{X}{4}\right)^2 + \left(\frac{Y}{3}\right)^2 = 1$

$\Rightarrow$ Ellipse

16. Answer (2)

Hint : $a_{tot} = \sqrt{a_R^2 + a_t^2}$

Sol. : $a_t = 2$ m/s^2

$a_R = \frac{v^2}{R}$

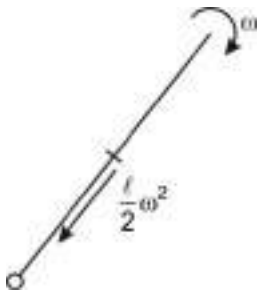
$a_{tot} = \sqrt{a_R^2 + a_t^2} = a\sqrt{1 + 4\pi^2} = 2\sqrt{1 + 4\pi^2}$

17. Answer (1)

Hint and Sol. : $\sum \vec{F} = M\overrightarrow{a_{cm}}$

$$-F + mg\cos\theta = (m)\frac{\ell}{2}\omega^2$$

$$\frac{mg\ell}{2} = \frac{mg\ell}{2}\cos 37° + \frac{1}{2}\frac{m\ell^2}{3}\omega^2$$



18. Answer (3)

Hint : Use kinematical equations.

Sol. : On the basis of given information we can easily find

$$\frac{s_A}{s_B} = 2$$

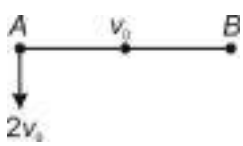
$$\Rightarrow s_B = \frac{s}{2}$$

19. Answer (4)

Hint : Momentum remains conserved.

$\omega = 4v_0/\ell$

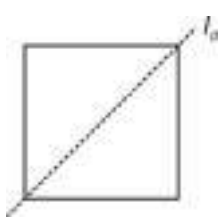
Sol. : $v_A = \sqrt{4v_0^2 + v_0^2} = v_0\sqrt{5}$



20. Answer (4)

Hint : $I_d = \frac{Ma^2}{12}$

Sol. :



$$\Rightarrow I_{AB} = \frac{7}{12}Ma^2$$

21. Answer (12)

Hint : $\overrightarrow{p_i} = \overrightarrow{p_f}$

Sol. : $5 \times 2 - 2 \times 4 = (2+4)v$

$$v = \frac{2}{6} = \frac{1}{3}\text{ m/s} = \frac{4}{12}\text{ m/s}$$

22. Answer (06)

Hint : $\overrightarrow{p_i^x} = \overrightarrow{p_f^x}$

Sol. : $mv_0 = 6mv$

$$\Rightarrow v = \frac{v_0}{6}$$

23. Answer (04)

Hint : $\vec{J} = \vec{P}_f - \vec{P}_i$

Sol. : $12 = 2(v_f - (-2))$

$v_f + 2 = 6$

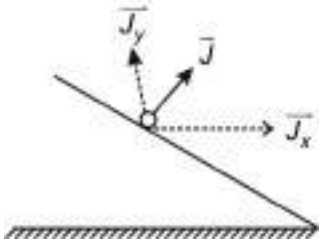
24. Answer (03)

Hint : $\overrightarrow{J_y} = \overrightarrow{\Delta P_y}$

and $\overrightarrow{J_x} = \overrightarrow{\Delta P_x}$

Sol. : $v_f\cos 30° = v_i\cos 60°$

$v_f = \sqrt{3}$ m/s



25. Answer (18)

Hint : $X_{cm} = \frac{\int x\,dm}{\int dm}$

Sol. : $X_{cm} = \frac{\int_0^3 (x)(\lambda)dx}{\int_0^3 (2+x)dx} = \frac{12}{7}$ cm

Distance from B = $\frac{9}{7}$ cm $= \frac{18}{14}$ cm

PART - B (CHEMISTRY)

26. Answer (3)

Hint : Moles of H_2SO_4 = 1

So, moles of NaOH = 2

Sol. : eq. of H_3PO_4 + eq. of HCl = eq. of NaOH

$$\frac{x}{98}\times 3+0.1=2$$

$$x=\frac{(2-0.1)\times 98}{3}\simeq 62\,g$$

$\therefore$ % purity = $\frac{62}{70}\times 100=88.67$ %

27. Answer (1)

Hint : $d_{solution} = \frac{220}{240} < d_{solvent}$

Sol. : $m=\frac{20}{M_0(0.2)}$

$$M=\frac{20}{M_0(0.24)}$$

$\therefore$ $M<m$

28. Answer (4)

Hint : S_8 has 8 S–S bonds.

P_4 has 6 P–P bonds.

Sol. : P = 8x

Q = 6x

29. Answer (2)

Hint : $\Delta S=nR\ln\left(\frac{V_2}{V_1}\right)$ as ΔT = 0

Sol. : Order of volume ratios:

Process (1) > Process (2) > Process (3)

30. Answer (3)

Hint : A → B is isochoric cooling.

Sol. : B → C is isothermal compression.

C → A is isobaric expansion.

31. Answer (2)

Hint : Rate $\propto \frac{P}{\sqrt{M}}$

Sol. : $\frac{n_{He}}{n_{N_2}}=4\sqrt{\frac{28}{4}}=4\sqrt{7}$

32. Answer (2)

Hint : Compare the V_{MP}

Sol. : $(V_{MP})_A < (V_{MP})_B$

$\Rightarrow \frac{T_A}{M_A}<\frac{T_B}{M_B}$

$\Rightarrow M_BT_A < M_AT_B$

33. Answer (2)

Hint : Magnitude of Ionisation energy is greater than electron gain enthalpy.

Sol. : Electron gain enthalpy of oxygen is less negative than that of sulphur.

34. Answer (1)

Hint : For H-atom, degeneracy of an orbit is equal to n^2.

Sol. : Radial nodes > Angular nodes

$\Rightarrow n-\ell-1>\ell$

$\Rightarrow n>2\ell+1$

So, S_1 cannot be 3*d*

35. Answer (2)

Hint : Aufbau's Principle is $(n+\ell)$ rule.

Sol. : Zn^{2+} will have the same configuration i.e. $[Ar]3d^{10}$ whether or not the Aufbau's principle is followed.

36. Answer (2)

Hint : As temperature increases, intensity increases.

Sol. : As temperature increases, the peak of intensity shifts towards lower wavelength.

37. Answer (1)

Hint : Dalton's law is not applicable for reacting gases.

Sol. : H_2 and He are non-reacting.

38. Answer (4)

Hint : $L = mvr = \frac{nh}{2\pi}$

Sol. : ΔL is max. for option (4)

39. Answer (4)

Hint : $\ln K = \left(\frac{-\Delta H}{R}\right)\frac{1}{T} + \frac{\Delta S}{R}$

Sol. : $\Delta H > 0 \Rightarrow$ slope < 0

$\Delta S > 0 \Rightarrow$ intercept > 0

40. Answer (4)

Hint : The electron density is shifted towards benzene ring in only (III).

Sol. : For both benzaldehyde and chlorobenzene, electron displacement vector is away from the benzene ring.

41. Answer (3)

Hint : Greater the bond order, shorter is the bond length.

Sol. : Ethene > Benzene > Graphite (Bond order)

42. Answer (2)

Hint : Direct overlap forms σ bond.

Sol. : Positive overlap forms bonding molecular orbital.

43. Answer (1)

Hint : Minima on Y-axis gives us the value of bond energy.

Sol. : Abscissa corresponding to minima gives us the value of bond length.

44. Answer (4)

Hint : Electronegativity of an element depends upon the element to which it is bonded.

Sol. : Electronegativity is not equal to the electron affinity.

45. Answer (3)

Hint : ΔH_{eg} for all noble gases is positive.

Sol. : ΔH_{eg} for hydrogen is negative.

46. Answer (25)

Hint : A, B, C, D, E are state functions, X = 5

Sol. : B, D, E, F, G are extensive properties, Y = 5

47. Answer (58)

Hint : Only $NaHCO_3$ decomposes upon heating.

Sol. : $2NaHCO_3 \longrightarrow Na_2CO_3 + H_2O + CO_2$

$a \qquad \frac{a}{2} \qquad \frac{a}{2} \qquad \frac{a}{2}$

Loss in weight = mass of H_2O and CO_2

$= \frac{a}{2}(44 + 18)$

$= 0.31$

$\therefore \; a = \frac{0.31 \times 2}{62} = 0.01$ moles

$\therefore$ Mass of $NaHCO_3$ = a × 84

= 0.84 g

$\therefore$ x = 58%

48. Answer (28)

Hint : V_{O_2} unburnt = 44 ml

V_{CO_2} produced = 24 ml

Sol. :

$C_xH_y + \left(x + \frac{y}{4}\right)O_2 \longrightarrow x\,CO_2 + \frac{y}{2}H_2O$

$12 \qquad 12\left(x + \frac{y}{4}\right) \qquad 12x$

$= 24$

x = 2

$\therefore \; 12\left(x + \frac{y}{4}\right) = 36$

$\Rightarrow$ y = 4 $\quad\therefore\quad C_2H_4$

49. Answer (04)

Hint : There are four lines lying in the Paschen series.

Sol. : $7 \to 3,\ 6 \to 3,\ 5 \to 3,\ 4 \to 3$

50. Answer (03)

Hint : In dimeric $BeCl_2$, Be has a sextet of electrons.

Sol. : $Al_2(CH_3)_6$ has e^- deficient banana bond. SNF_3 has an expanded octet of S.

PART - C (MATHEMATICS)

51. Answer (4)

Hint : Locus of point of intersection of perpendicular tangents to a hyperbola is director circle.

Sol. : $C_2 : |z+5| - |z-5| = 8$ represents a hyperbola and $C_1 : |z| = \sqrt{7}$ is the director circle of given hyperbola.

So angle between tangents will be $\frac{\pi}{2}$.

52. Answer (2)

Hint : $T_r = (1 + r\omega - r\omega^2)(1 - r\omega + r\omega^2) = 1 + 3r^2$

Sol. : $T_r = (1 + r\omega - r\omega^2)(1 - r\omega + r\omega^2)$

$T_r = 1 + 3r^2$

$$S_n = \sum_{r=1}^{n} T_r = n + \frac{n(n+1)(2n+1)}{2}$$

$$= \frac{n(2n^2+3n+3)}{2}$$

53. Answer (4)

Hint : Change $a + ib$ into $r \cdot e^{i\theta}$

Sol. : $$\tan\left(i\ln\left(\frac{\frac{\sqrt{3}}{2}+\frac{1}{2}i}{\frac{\sqrt{3}}{2}-\frac{1}{2}i}\right)\right) = \tan\left(i\ln\left(\frac{e^{i\pi/6}}{e^{-i\pi/6}}\right)\right)$$

$$= \tan\left(i\ln\left(e^{2i\pi/6}\right)\right)$$

$$= \tan\left(\frac{\pi}{3}i^2\right) = -\sqrt{3}$$

54. Answer (2)

Hint : $\ln\left(\frac{1+x}{1-x}\right) = 2\left[x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \ldots\right]$

Sol. : $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$

$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \ldots$

$\ln(1+x) - \ln(1-x) = 2\left[x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\right]$

Put $x = \frac{1}{2}$

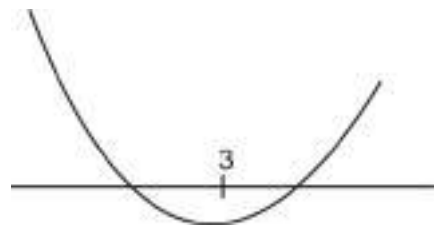
$$2\left[\frac{1}{2} + \frac{1}{3.2^3} + \frac{1}{5.2^5} + \ldots\infty\right] = \ln\left(\frac{3}{2}\right) - \ln\left(\frac{1}{2}\right)$$

$$\frac{1}{1\cdot 2} + \frac{1}{3.2^3} + \frac{1}{5.2^5} + \ldots\infty = \ln\sqrt{3}$$

55. Answer (2)

Hint : If $a > 0$ and α lies between roots of $ax^2 + bx + c = 0$ then $a\alpha^2 + b\alpha + c < 0$

Sol. : $(k-2)x^2 + (k-1)x + 3 = 0$



$$x^2 + \left(\frac{k-1}{k-2}\right)x + \frac{3}{(k-2)} = 0$$

$\because \; f(3) < 0$

$$9 + \frac{3k-3}{k-2} + \frac{3}{k-2} < 0$$

$$\frac{12k-18}{k-2} < 0$$

$$k \in \left(\frac{3}{2}, 2\right)$$

56. Answer (2)

Hint : $T_r = \frac{1}{r} - \frac{1}{r+1}$

Sol. : $T_r = \frac{(r+1)^2 + (r+2)}{(r+1)^4 - (r+1)}$

$$\Rightarrow T_r = \frac{r^2+3r+3}{(r+1)[r^3+3r^2+3r]} = \frac{1}{r(r+1)}$$

$$\Rightarrow T_r = \frac{1}{r} - \frac{1}{r+1}$$

So; $T_1 = \frac{1}{1} - \frac{1}{2}$

$T_2 = \frac{1}{2} - \frac{1}{3}$

$T_3 = \frac{1}{3} - \frac{1}{4}$

$T_{12} = \frac{1}{12} - \frac{1}{13}$

By adding these terms, we get

$S = 1 - \frac{1}{13} = \frac{12}{13}$

57. Answer (2)

Hint : $T_r = \frac{1}{\lfloor r-2} + \frac{2}{\lfloor r-1} + \frac{1}{\lfloor r}$

Sol. : $T_r = \frac{r^2+r+1}{\lfloor r} = \frac{r(r-1)+2r+1}{\lfloor r}$

$\Rightarrow T_r = \frac{1}{\lfloor r-2} + 2\cdot\frac{1}{\lfloor r-1} + \frac{1}{\lfloor r}$

$\sum_{r=1}^{\infty} T_r = \sum_{r=2}^{\infty}\frac{1}{\lfloor r-2} + 2\sum_{r=1}^{\infty}\frac{1}{\lfloor r-1} + \sum_{r=1}^{\infty}\frac{1}{\lfloor r}$

$= e + 2e + e - 1 = 4e - 1$

58. Answer (3)

Hint : $f(x) = (1 + x)^n - 1 - x^n$

Sol. : $f(x) = (1 + x)^n - 1 - x^n$

So, $f(1) = 2^n - 1 - 1^n$

$f(2) = 3^n - 1 - 2^n$

$f(3) = 4^n - 1 - 3^n$

$f(k) = (k + 1)^n - 1 - k^n$

Adding these equations,

$\sum_{r=1}^{k} f(r) = (k+1)^n - (k+1)$

59. Answer (4)

Hint : General term in the expansion of $(a + b + c)^n$ is $\frac{\lfloor n}{\lfloor p_1 \lfloor p_2 \lfloor p_3} a^{p_1}\cdot b^{p_2}\cdot b^{p_3}$

Sol. : General term in the expansion is

$\frac{\lfloor 5}{\lfloor p_1 \lfloor p_2 \lfloor p_3}(a^2b)^{p_1}(b^2c)^{p_2}(c^2a)^{p_3}$

$2p_1 + p_3 = 6$

$2p_2 + p_1 = 4$

$p_2 + 2p_3 = 5$

Also, $p_1 + p_2 + p_3 = 5$

So, $p_1 = 2$, $p_2 = 1$ and $p_3 = 2$

Coeff. of $a^6b^4c^5$ is $\frac{\lfloor 5}{\lfloor 2 \lfloor 1 \lfloor 2} = 30$

60. Answer (4)

Hint : $T_n = S_n - S_{n-1}$

Sol. : $\Delta_n = \sum_{k=1}^{n}\Delta_k - \sum_{k=1}^{n-1}\Delta_k = 3^n - 3^{n-1} = 2\cdot 3^{n-1}$

$\sum_{k=1}^{\infty}\frac{1}{\Delta_n} = \frac{1}{2}\left[1 + \frac{1}{3} + \frac{1}{3^2} + \ldots\infty\right] = \frac{3}{4}$

61. Answer (1)

Hint : Sum of A.G.P. = $\frac{a}{1-r} + \frac{dr}{(1-r)^2}$

Sol. : $f(x) = \left(\sin x - \frac{1}{2}\right)^2 + \frac{3}{4}$

So range of $f(x)$ is $\left[\frac{3}{4}, 3\right]$

$r = \frac{3}{4}, \quad d = 3$

$S = 1 + (1 + d)r + (1 + 2d)r^2 + (1 + 3d)r^3 + \ldots \infty$

$S.r = r + (1 + 2d)r^2 + (1 + 3d)r^3 + \ldots \infty$

$S(1 - r) = 1 + dr + dr^2 + dr^3 + \ldots \infty$

$\Rightarrow S\cdot\frac{1}{4} = 1 + \frac{dr}{1-r} = 1 + \frac{9/4}{1/4}$

$\Rightarrow S = 40$

62. Answer (1)

Hint : Middle term in the expansion of $(a + b)^n = {}^nC_{n/2}(ab)^{n/2}$ if n is even

Sol. : $^6C_3 \cdot (2^{\log_2 x})^3 (x^{\log_2 x})^3 = 1280$

$\Rightarrow x^{3+3\log_2 x} = 64$

$\Rightarrow (1+\log_2 x)\cdot \log_2 x = 2$

$\log_2 x = 1$ or -2

$x = 2$ or $\frac{1}{4}$

63. Answer (4)

Hint : Number formation

Sol. :

Number of odd numbers starting with
1(1 3 4 2) = 24
1 3, 5

Number of odd numbers starting with
2(1 3 4 3) = 36
2 1, 3, 5

Number of odd numbers starting with
3(1 3 4 2) = 24
3 1, 5

Number of odd numbers starting with
41(1 1 3 2) = 6
4 1 3, 5

Number of odd numbers starting with
42(1 1 3 3) = 9
4 2 1, 3, 5

Total 99 numbers

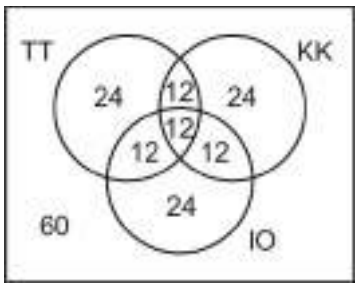
So the number at 100th place will be the first number starting with 43, which is 4315.

64. Answer (2)

Hint : Principle of inclusion and exclusion.

Sol. : By using principle of inclusion and exclusion, number of such words

= Total words – (TT together + KK together + IO together) + (TT & KK together + KK & IO together + TT & IO together) – (TT, KK and IO together)



$$= \frac{\lfloor 6}{\lfloor 2 \lfloor 2} - \left[\frac{\lfloor 5}{\lfloor 2} + \frac{\lfloor 5}{\lfloor 2} + \frac{\lfloor 5 \cdot \lfloor 2}{\lfloor 2 \lfloor 2}\right]$$
$$+ \left[\lfloor 4 + \frac{\lfloor 4}{\lfloor 2}\cdot \lfloor 2 + \frac{\lfloor 4}{\lfloor 2}\cdot \lfloor 2\right]$$
$$-[\lfloor 3 \cdot \lfloor 2]$$

= 180 – [3 × 60] + [72] – 12

= 60

65. Answer (2)

Hint : Use expansion

Sol. : $17^{15} = (1 + 16)^{15}$

$= 1 + {}^{15}C_1 \cdot 16 + {}^{15}C_2 \cdot 16^2 + {}^{15}C_3 \cdot 16^3 + \ldots$

$15^{17} = -(1 - 16)^{17}$

$= -1 + {}^{17}C_1 \cdot 16 - {}^{17}C_2 \cdot 16^2 + {}^{17}C_3 \cdot 16^3 - \ldots$

$17^{15} + 15^{17} = 16({}^{15}C_1 + {}^{17}C_1) + 16^2({}^{15}C_2 - {}^{17}C_2) + 16^3({}^{15}C_3 + {}^{17}C_3) + \ldots$

$= 16 \times 32 + 16^2 \times (-31) + 16^3({}^5C_3 + {}^{17}C_3) + \ldots$

$= 16^2 \times$ odd number

66. Answer (1)

Hint : Put $x = 1$, ω and ω^2 in the expansion and add.

Sol. : $(x^2 + x - 1)^{10} = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \ldots$

Put $x = 1$

$1 = a_0 + a_1 + a_2 + a_3 + a_4 + \ldots$ …(i)

Put $x = \omega$

$2^{10} = a_0 + a_1\omega + a_2\omega^2 + a_3 + a_4\omega + \ldots$ …(ii)

Put $x = \omega^2$

$2^{10} = a_0 + a_1\omega^2 + a_2\omega + a_3 + a_4\omega^2 + \ldots$ …(iii)

Adding (i), (ii) and (iii)

$3[a_0 + a_3 + a_6 + \ldots + a_{18}] = 1 + 2^{10} + 2^{10}$

$\Rightarrow a_0 + a_3 + a_6 + \ldots + a_{18} = 683$

67. Answer (3)

Hint : $(1-x)^{10}\left(1+x+\frac{1}{x}\right)^9 = \frac{(1-x)(1-x^3)^9}{x^9}$

Sol. : Coefficient of x^{16} in

$(1-x)^{10}\left(\frac{1+x+x^2}{x}\right)^9$

= coefficient of x^{25} in $(1-x)(1-x^3)^9$

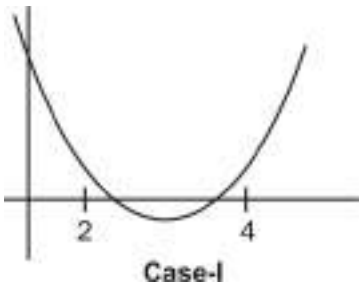
$= -{}^9C_8$

$= -9$

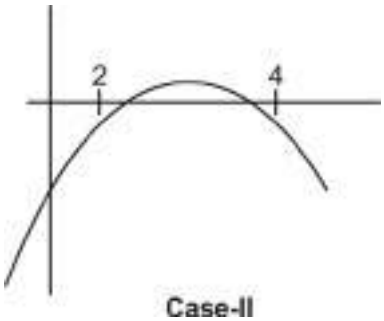
68. Answer (4)

Hint : Draw graph

Sol. : There are two cases possible



Case-I



Case-II

$f(x) = ax^2 + bx + c$

$f(-1) = a - b + c < 0$ (given)

So Case-I cannot be true.

In Case-II, clearly $a < 0$ (downward parabola)

$f(0) < 0 \Rightarrow c < 0$

$f(1) < 0 \Rightarrow a + b + c < 0$

Sum of roots $= -\frac{b}{a} > 0 \Rightarrow b > 0$

69. Answer (3)

Hint : For continuous function $f(x)$; if $f(a)\cdot f(b) < 0$ then there exist atleast one root of $f(x)$ in (a, b).

Sol. : Let $f(11) = \alpha$, $f(12) = \beta$ and $f(13) = \gamma$, $\alpha < \beta < \gamma$ because $f(y)$ is increasing function.

$$\frac{1}{x-\alpha} + \frac{1}{x-\beta} + \frac{1}{x-\gamma} = 0$$

$(x-\beta)(x-\gamma) + (x-\alpha)(x-\gamma) + (x-\alpha)(x-\beta) = 0$

Let $g(x) = (x-\beta)(x-\gamma) + (x-\alpha)(x-\gamma) + (x-\alpha)(x-\beta)$

Now, $g(\alpha) = (\alpha-\beta)(\alpha-\gamma) > 0$

$g(\beta) = (\beta-\alpha)(\beta-\gamma) < 0$

and $g(\gamma) = (\gamma-\alpha)(\gamma-\beta) > 0$

So one root of $g(x) = 0$ lies between α and β and other root lies between β and γ.

70. Answer (1)

Hint : Put $z = x + iy$

Sol. : $|z - 1 - i| = |z|\sin(\arg z)$

Let $z = x + iy$

$(x-1)^2 + (y-1)^2 = y^2$

$\Rightarrow (x-1)^2 = 2\left(y - \frac{1}{2}\right)$

It is a parabola whose length of latus rectum is 2.

71. Answer (03)

Hint : $3^{2019} + 4^{2020} = (7k_1 - 1)^{673} + 4(7k_2 + 1)^{673}$

Sol. : $7x$ = remainder when $3^{2019} + 4^{2020}$ is divided by 7

$3^{2019} + 4^{2020} = (28-1)^{673} + 4(63+1)^{673}$

$= (7\lambda - 1) + 4(7\mu + 1)$

$= 7 \times \text{integer} + 3$

Remainder is 3.

72. Answer (04)

Hint : Put $2^x = t$

Sol. : Let $2^x = t$

$t^2 + 2t + a - 5 = 0$

$\Rightarrow (t + 1)^2 = 6 - a$

$(t + 1)^2 \in (1, \infty)$

So the equation has real solutions if

$6 - a > 1$

$a < 5$

Greatest integral value of a is 4.

73. Answer (02)

Hint : Solve the two curves.

Sol. : $z\bar{z} + z + \bar{z} - 1 = 0$ and $z + \bar{z} + 2 = 0$

Solving the equation of line and circle,

$z(-z-2) + z + (-z-2) - 1 = 0$

$z^2 + 2z + 3 = 0$

$z = -1 \pm \sqrt{2}i$

There are the two points of intersection say z_1 and z_2.

Length of intercept will be $|z_1 - z_2| = 2\sqrt{2}$

74. Answer (16)

Hint : Combinations

Sol. : For each element in S there are 4 options

(i) Element belongs to set P

(ii) Element belongs to set Q

(iii) Element belongs to P and Q both

(iv) Element neither belongs to P nor Q.

$\because n(P \cap Q) = 3$

$\Rightarrow$ For any 3 elements only option (iii) is available and for the remaining elements option (i) and (ii) are available.

So number of ordered pairs $(P, Q) = {}^6C_3 \cdot 2^3 = 160$

75. Answer (61)

Hint : Let $x = k$, where $k = 1, 2, 3$ and $y + z + t = 15 - 4k$

Sol. : $y + z + t = 15 - 4x$

No. of solutions $= {}^{14-4x}C_2$

Here $x = 1, 2, 3$

Total number of solutions $= {}^{10}C_2 + {}^6C_2 + {}^2C_2$

$= 45 + 15 + 1 = 61$

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