

## All India Aakash Test Series for JEE (Advanced)-2021

**TEST - 1A (Paper-1) - Code-E**

Test Date : 17/11/2019

**ANSWERS**

PHYSICS	CHEMISTRY	MATHEMATICS
1. (A, C)	19. (B, C, D)	37. (A, C)
2. (B, D)	20. (A, B, D)	38. (B, C, D)
3. (A, C, D)	21. (B)	39. (B, D)
4. (A, C, D)	22. (A, B, C, D)	40. (A, D)
5. (A, B, C, D)	23. (B, C)	41. (B, C)
6. (A, B)	24. (A, B, C)	42. (B, D)
7. (33)	25. (10)	43. (15)
8. (24)	26. (51)	44. (63)
9. (17)	27. (68)	45. (11)
10. (12)	28. (15)	46. (12)
11. (72)	29. (12)	47. (24)
12. (16)	30. (18)	48. (04)
13. (18)	31. (20)	49. (01)
14. (15)	32. (50)	50. (07)
15. (C)	33. (A)	51. (C)
16. (B)	34. (B)	52. (C)
17. (C)	35. (A)	53. (A)
18. (A)	36. (C)	54. (D)

## HINTS &amp; SOLUTIONS

## PART - I (PHYSICS)

1. Answer (A, C)

**Hint :**  $\Delta \vec{r} = \vec{r}_f - \vec{r}_i$

**Sol. :**

Displacement  $\vec{r} = \vec{r}_f - \vec{r}_i$

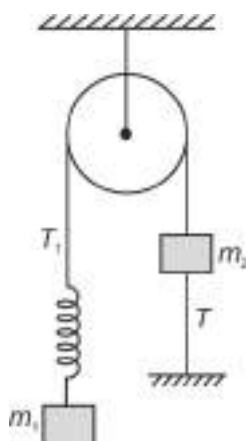
$$\Rightarrow \vec{r} = (3\hat{i} + 4\hat{j} + 5\hat{k}) - (2\hat{i} + 3\hat{j} + 5\hat{k}) = \hat{i} + \hat{j}$$

$\vec{r}$  makes  $45^\circ$  with +ve x-axis in anticlockwise sense.

2. Answer (B, D)

**Hint. :** Sudden impulsive force by spring is zero.

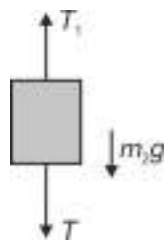
**Sol. :** Let the tension in string BC is  $T$  at equilibrium.



Then for  $m_1$  to be in equilibrium  $K\Delta x = m_1g$

That means spring will be in extended condition and it will transmit  $T_1 = K\Delta x = m_1g$  force on string attached with spring.

So, for ( $m_2$ )



$$\Rightarrow T = m_1g - m_2g$$

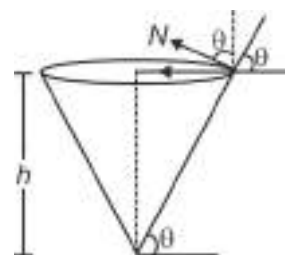
When string BC is burnt suddenly then spring still transmit the same force so acceleration of mass  $m_1$  is zero. And acceleration of mass  $m_2$  is

$$a_2 = \frac{(m_1 - m_2)g}{m_2}$$

3. Answer (A, C, D)

**Hint :**  $v^2 = gr \tan \theta$

**Sol. :**



$$\tan \theta = \frac{h}{r}$$

$$\Rightarrow r = \frac{h}{\tan \theta}$$

Along the plane, with respect to cone the particle is in state of equilibrium.

$$\therefore mg \sin \theta = \frac{mv^2}{r} \cdot \cos \theta$$

$$\Rightarrow gr \cdot \frac{\sin \theta}{\cos \theta} = v^2 \Rightarrow v^2 = \frac{gh}{\tan \theta} \tan \theta$$

$$\therefore v^2 = gh$$

Also,  $N \cos \theta = mg$

And  $N \sin \theta = \frac{mv^2}{r}$

$$\therefore N \sin \theta = \frac{mv^2}{h} \frac{\sin \theta}{\cos \theta} \Rightarrow N \cos \theta = \frac{mv^2}{h}$$

4. Answer (A, C, D)

**Hint :** For velocity to become perpendicular to initial direction  $\theta > \frac{\pi}{4}$ .

**Sol. :**



For velocity to become perpendicular to initial direction  $\theta > \frac{\pi}{4}$ .

For same case,  $m_1 = \tan \theta_1 = \tan \theta$

And  $m_2 = \tan \theta_2 = \frac{(u \sin \theta - gt)}{u \cos \theta}$

$$\therefore (m_1 m_2 = -1) \Rightarrow \frac{(u \sin \theta - gt)}{u \cos \theta} \cdot \frac{\sin \theta}{\cos \theta} = -1$$

$$\Rightarrow u \sin^2 \theta - gt \sin \theta = -u \cos^2 \theta$$

$$\Rightarrow u = gt \sin \theta \therefore t = \frac{u}{g \sin \theta}$$

$$\text{And 1 s before, } \theta_1 = \tan^{-1} \left( \frac{g}{u \cos \theta} \right)$$

So just after 1 s and before 1 s,  $\Delta \theta = 2\theta$

$$\Delta \theta = 2 \tan^{-1} \left( \frac{g}{u \cos \theta} \right)$$

5. Answer (A, B, C, D)

**Hint :** Tangential force will change the speed and perpendicular force will change the direction.

**Sol. :** If  $\vec{v}$  is opposite to  $\vec{F}$  the particle may retrace its path.

If  $\vec{F}$  is perpendicular to  $\vec{v}$  and so  $\vec{F}$  will provide the centripetal force and if  $|\vec{F}|$  is constant, then radius of curvature will be constant. And if at a particular time instant  $\vec{v}$  and  $\vec{F}$  are some angle other than  $0^\circ$  or  $180^\circ$  and  $\vec{F}$  is constant, then it's analogues of projectile motion. Particle will trace the parabolic path.

6. Answer (A, B)

$$\text{Hint : } v = \frac{dx}{dt}; \quad a = \frac{d^2x}{dt^2}$$

$$\text{Sol. : } x = \alpha t^3 + \beta t^2 + \gamma t + \delta$$

$$\therefore \frac{dx}{dt} = v = 3\alpha t^2 + 2\beta t + \gamma$$

$$\frac{d^2x}{dt^2} = a = 6\alpha t + 2\beta$$

$$\therefore 6\alpha t + 2\beta = 3\alpha t^2 + 2\beta t + \gamma$$

$$\Rightarrow 3\alpha t^2 + (2\beta - 6\alpha)t + \gamma - 2\beta = 0$$

$$\text{Here, } 4(3\alpha - \beta)^2 - 4 \times 3\alpha(\gamma - 2\beta) = 0 \text{ for unique } t$$

$$\Rightarrow 9\alpha^2 + \beta^2 - 3\alpha\gamma = 0$$

And for that time instant  $t > 0$

$$\therefore t = \frac{2(3\alpha - \beta)}{2 \times 3\alpha} = \frac{3\alpha - \beta}{3\alpha}$$

$$\therefore 3\alpha - \beta > 0$$

7. Answer (33)

**Hint :** For vertical upward motion,  $y = u_y t - \frac{1}{2}gt^2$

**Sol. :** Let  $v_0$  was the velocity of dropping of 1<sup>st</sup> stone, then



$$\bar{y}_1 = v_0 2 - \frac{1}{2} \times 10 \times 4 = 2v_0 - 20$$

$\Rightarrow |\bar{y}_1| = (20 - 2v_0)$  is the distance from dropping point.

After 1 sec balloon shall have velocity  $v_2 = (v_0 + 1)$

And it must have travelled  $|y| = \left(v_0 + \frac{1}{2}\right)$ .

Then 1 sec after 2<sup>nd</sup> particle will be at

$$y_2 = (v_0 + 1) - \frac{1}{2} \times 10 \times 1$$

$$\Rightarrow |\bar{y}_2| = 5 - v_0 - 1 = 4 - v_0$$

Distance from dropping point

$$\therefore \text{Separation} \Rightarrow s = |\bar{y}_1| + |\bar{y}| - |\bar{y}_2|$$

$$\Rightarrow s = 20 - 2v_0 + v_0 + \frac{1}{2} - 4 + v_0$$

$$\Rightarrow s = 16 + \frac{1}{2} = \frac{33}{2}$$

$$\therefore 2s = 33$$

8. Answer (24)

$$\text{Hint : } v = \frac{dr}{dt}, \quad a = \frac{d^2r}{dt^2}$$

$$\text{Sol. : } \vec{r} = 2t\hat{i} + 4t^2\hat{j} + \hat{k}$$

$$\therefore \vec{v} = \frac{dr}{dt} = 2\hat{i} + 8t\hat{j}$$

$$\text{And } \vec{a} = \frac{d^2r}{dt^2} = 8\hat{j}$$

Acceleration is always along y direction.

So, velocity in y direction at  $t = 3$  s is

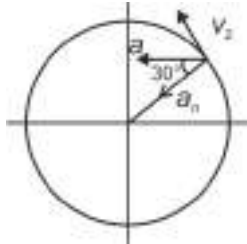
$$|\vec{v}_y| = |8t| \Rightarrow (8 \times 3) = 24$$

9. Answer (17)

**Hint :**  $\tan \theta = \frac{|a_n|}{|a_t|}$

**Sol. :** Angle with velocity vector is  $30^\circ$ .

$$\therefore \tan 30^\circ = \frac{|a_n|}{|a_t|} = \frac{v^2}{Ra}$$



$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{a^2 t^2}{Ra}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{a^2 t^2}{Ra} \Rightarrow \frac{at^2}{R} = \frac{1}{\sqrt{3}}$$

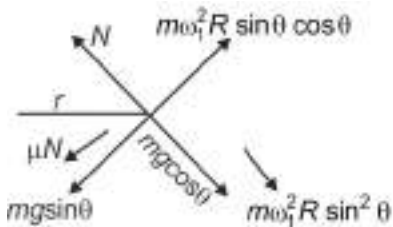
$$\therefore t^2 = \frac{51}{\sqrt{3} \times \sqrt{3}} = 17$$

10. Answer (12)

**Hint :**  $N = mg \cos \theta + m\omega_1^2 R \sin^2 \theta$

$$mg \sin \theta + \mu N = m\omega_1^2 R \sin \theta \cdot \cos \theta$$

**Sol. :** Let  $\omega_1$  be the maximum angular speed and  $\omega_2$  be the minimum angular speed, then



$$N = mg \cos \theta + m\omega_1^2 R \sin^2 \theta$$

And  $mg \sin \theta + \mu N = m\omega_1^2 R \sin \theta \cdot \cos \theta$

$$\Rightarrow mg \sin \theta + \mu mg \cos \theta + \mu m\omega_1^2 R \sin^2 \theta = m\omega_1^2 R \sin \theta \cos \theta$$

$$\Rightarrow \omega_1^2 R \sin \theta (\cos \theta - \mu \sin \theta) = g(\sin \theta + \mu \cos \theta)$$

$$\therefore \omega_1^2 = \frac{g(\sin \theta + \mu \cos \theta)}{R \sin \theta (\cos \theta - \mu \sin \theta)}$$

Similarly,  $\omega_2^2 = \frac{g(\sin \theta - \mu \cos \theta)}{R \sin \theta (\cos \theta + \mu \sin \theta)}$

$$\therefore \frac{\omega_1^2}{\omega_2^2} = \frac{\left(\frac{1}{\sqrt{2}} + \frac{1}{2\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}} + \frac{2}{2\sqrt{2}}\right)}{\left(\frac{1}{\sqrt{2}} - \frac{1}{2\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}} - \frac{1}{2\sqrt{2}}\right)}$$

$$\Rightarrow \frac{\omega_1^2}{\omega_2^2} = \frac{\left(\frac{3}{2\sqrt{2}}\right)^2}{\left(\frac{1}{2\sqrt{2}}\right)^2} = 9$$

$$\therefore X = \frac{\omega_1}{\omega_2} = 3 \quad \therefore 4X = 12$$

11. Answer (72)

**Hint :**  $\frac{mg}{120} x \mu = \frac{mg}{120} (120 - x)$

**Sol. :** For state of impending motion, let  $x$  be the length on the table, then

$$\frac{m}{120} x g \mu = \frac{mg}{120} (120 - x)$$

$$\Rightarrow \frac{2}{3}x = 120 - x \Rightarrow \frac{5x}{3} = 120$$

$$\Rightarrow x = \frac{120 \times 3}{5} = 72$$

12. Answer (16)

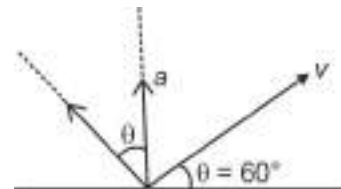
**Hint :**  $a = \frac{dv}{dt}$

**Sol. :**  $v(t) = \frac{8}{\sqrt{3}} \hat{i} + 8t \hat{j}$

$$\therefore \vec{a} = 8 \hat{j}$$

At  $t = 1$  s,  $v = \frac{8}{\sqrt{3}} \hat{i} + 8 \hat{j}$

$$\therefore \tan \theta = \frac{8}{\frac{8}{\sqrt{3}}} \therefore \theta = 60^\circ$$



$$\therefore a_n = a \cos 60^\circ = 8 \times \frac{1}{2} = 4$$

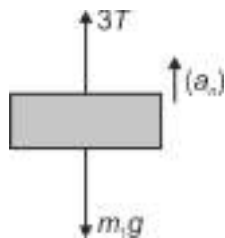
$$\therefore 4|\vec{a}_n| = 16$$

13. Answer (18)

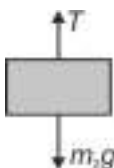
 Hint :  $3T - m_1g = m_1a_1$ 

$$m_2g - T = m_2a_2$$

Sol. :



$$\Rightarrow 3T - m_1g = m_1a_1 \quad \dots(1)$$



$$\Rightarrow m_2g - T = m_2a_2$$

$$\Rightarrow 3m_2g - 3T = 3m_2a_2$$

$$\therefore 3m_2g - m_1g = m_1a_1 + 3m_2a_2$$

$$\text{Also, } a_2 = 3a_1$$

$$\therefore (3m_2 - m_1)g = m_1a_1 + gm_2a_1$$

$$\therefore a_1 = \frac{4g}{20} = 2 \text{ m/s}^2$$

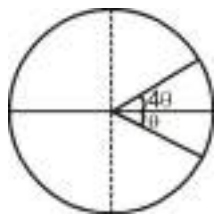
$$\therefore a_2 = 6 \text{ m/s}^2$$

$$\therefore 3a_2 = 18 \text{ m/s}^2$$

14. Answer (15)

 Hint :  $2T\Delta\theta = \Delta m\omega^2 R$ 

Sol. :



$$\lambda = \frac{m}{2\pi R}$$

$$\Delta l = 2R\Delta\theta$$

$$\therefore 2T \sin \Delta\theta = \Delta m \omega^2 R$$

$$\Rightarrow 2T\Delta\theta = \Delta m\omega^2 R$$

$$\Rightarrow 2T\Delta\theta = \frac{m}{2\pi R} 2R\Delta\theta \cdot \omega^2 R$$

$$\Rightarrow T = \frac{m\omega^2 R}{2\pi} = 6.28 \times 5 \times 5 \times \frac{6}{10 \times 2 \times 3.14}$$

$$\Rightarrow T = 15$$

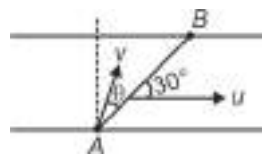
15. Answer (C)

16. Answer (B)

Hint for Q.Nos. 15 and 16 :

For forward and backward, both motion, person must maintain the same angle with line AB.

Solution for Q.Nos. 15 and 16 :



$$\text{Clearly } \sin 30^\circ = \frac{d}{AB}$$

$$\Rightarrow AB = 2d$$

For forward and backward, both motion, person must maintain the same angle with line AB.

$$\therefore v \sin \theta = u \sin 30^\circ \Rightarrow \sin \theta = \frac{u}{4\sqrt{3}}$$

$$\text{From A to B} \Rightarrow v \cos \theta + \frac{u\sqrt{3}}{2} = \frac{2d}{T_1}$$

$$\text{And from B to A} \Rightarrow v \cos \theta - \frac{u\sqrt{3}}{2} = \frac{2d}{T_2}$$

$$\therefore u\sqrt{3} = 2d \left( \frac{1}{T_1} - \frac{1}{T_2} \right)$$

$$\Rightarrow u\sqrt{3} = 2d \left( \frac{T_2 - T_1}{T_1 T_2} \right)$$

$$\therefore u = \frac{2d}{\sqrt{3}} \left[ \frac{T_2 - T_1}{T_1 T_2} \right]$$

$$\text{And } \sin \theta = \frac{1}{4\sqrt{3}} \cdot \frac{2d}{\sqrt{3}} \left( \frac{T_2 - T_1}{T_1 T_2} \right)$$

$$\therefore \theta = \sin^{-1} \left( \frac{d}{6} \left( \frac{T_2 - T_1}{T_1 T_2} \right) \right)$$

17. Answer (C)

18. Answer (A)

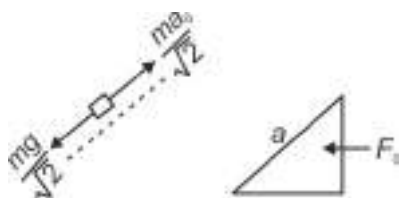
Hint for Q.Nos. 17 and 18 :

Motion is accelerated reference frame.

Solution for Q.Nos. 17 to 18 :

 Let the force be  $F_0$  when small block does not slide with respect to wedge.

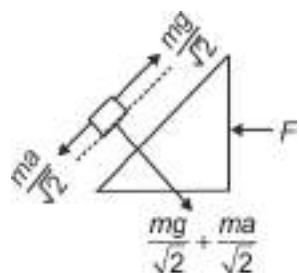
Then



$$\frac{mg}{\sqrt{2}} = \frac{ma_0}{\sqrt{2}} \Rightarrow a_0 = g$$

$$\text{And } F_0 = 2ma_0 = 2mg$$

$$\text{As } F = \frac{3}{2}F_0 \therefore \frac{3}{2} \cdot 2mg \Rightarrow F = 3mg$$



Now as  $F = 3mg$ , let the acceleration of block be  $a_1$  and wedge be  $a$ , then

$$\frac{ma}{\sqrt{2}} - \frac{mg}{\sqrt{2}} = ma_1$$

$$\therefore a_1 = \frac{1}{\sqrt{2}} - \frac{g}{\sqrt{2}}$$

$$\text{And for wedge } 3mg - \left( \frac{mg}{2} + \frac{ma}{2} \right) = ma$$

$$\Rightarrow 6mg - mg - ma = 2ma$$

$$\Rightarrow 5g = 3a \Rightarrow a = \frac{5g}{3}$$

$$\therefore a_1 = \frac{1}{\sqrt{2}} \left( \frac{5}{3} - 1 \right) g = \frac{2g}{3\sqrt{2}} = \frac{\sqrt{2}g}{3}$$

$$\text{So, } \frac{L}{2} = \frac{1}{2} \frac{\sqrt{2}g}{3} \cdot t^2$$

$$\Rightarrow t = \left[ \frac{3L}{\sqrt{2}g} \right]^{\frac{1}{2}}$$

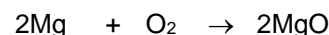
## PART - II (CHEMISTRY)

19. Answer (B, C, D)

**Hint :** Oxygen is the limiting reagent.

**Sol. :** Number of moles of Mg =  $\frac{1}{24}$

Number of moles of  $O_2 = \frac{1}{64}$



$$\text{Initial moles} \quad \frac{1}{24} \quad \frac{1}{64}$$

$$\text{Moles at the end} \quad \left( \frac{1}{24} - \frac{1}{32} \right) \quad 0 \quad \frac{1}{32}$$

of reaction

Mass of Mg left unreacted

$$= \left[ \frac{1}{24} - \frac{1}{32} \right] \times 24 = 0.25 \text{ g}$$

$O_2$  gas is consumed completely.

$$\text{Mass of MgO formed} = \frac{1}{32} \times 40 = 1.25 \text{ g}$$

20. Answer (A, B, D)

**Hint :** Particles in the right zone have greater kinetic energy in distribution curve.

**Sol. :** Greater the kinetic energy, greater would be the tendency to get evaporate  $T_2 > T_1$ .

At higher temperature, vapour phase would exist.

21. Answer (B)

$$\text{Hint : } P_{\text{real}} = \frac{nRT}{V - nb} - a \left( \frac{n}{V} \right)^2$$

$$P_{\text{ideal}} = \frac{nRT}{V}$$

**Sol. :** When cylinder is full

$$P_{\text{real}} = \frac{60 \times 0.08 \times 300}{15 - (60 \times 0.05)} - 0.25 \left( \frac{60}{15} \right)^2$$

$$= 116 \text{ atm}$$

$$P_{\text{ideal}} = \frac{60 \times 0.08 \times 300}{15} = 96 \text{ atm}$$

After prolonged used,

$$P_{\text{real}} = \frac{0.60 \times 0.08 \times 300}{15 - (0.6 \times 0.05)} - (0.25) \left( \frac{0.6}{15} \right)^2$$

$$\approx 0.96 \text{ atm}$$

$$P_{\text{ideal}} = \frac{0.6 \times 0.08 \times 300}{15} = 0.96 \text{ atm}$$

22. Answer (A, B, C, D)

**Hint :** All statements are correct.

**Sol. :** At constant V,  $P = \left( \frac{R}{V-b} \right) T$

At constant P,  $V = b + \left( \frac{R}{P} \right) T$

$$Z = \frac{PV}{RT} = 1 + \frac{Pb}{RT}$$

Since  $Z > 1$ , the repulsive forces dominate over attractive forces.

23. Answer (B, C)

**Hint :** The probability of finding electron,  $\psi^2$  is zero at radial nodes in an orbital.

**Sol. :** The radial wave function for a Bohr atom is given as

$$\psi(\text{radial}) = \frac{1}{16\sqrt{4}} \left[ \frac{Z}{a_0} \right]^3 [(\sigma-1)(\sigma^2-8\sigma+12)] e^{-\frac{\sigma}{2}}$$

where  $\sigma = \frac{2Zr}{a_0}$

At radial nodes,  $\psi^2 = 0$

or  $(\sigma-1)=0$ ;  $\sigma=1 \Rightarrow r = \frac{a_0}{2Z}$

or  $\sigma^2-8\sigma+12=0$ ;  $(\sigma-6)(\sigma-2)=0$

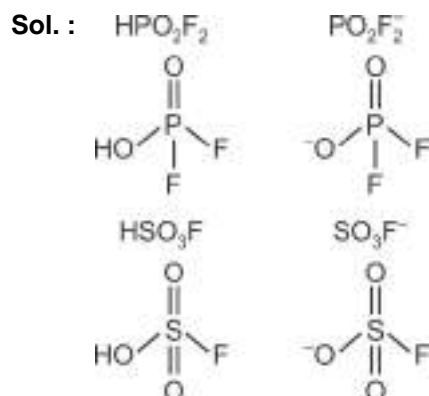
$\Rightarrow \sigma=6$  or  $2$ ;  $r = \frac{3a_0}{Z}$  or  $\frac{a_0}{Z}$

$\therefore$  Minimum position of radial node,  $r = \frac{a_0}{2Z}$

Maximum position of radial node,  $r = \frac{3a_0}{Z}$

24. Answer (A, B, C)

**Hint :** Hybridisation of central atom in all 4 molecules is same.



25. Answer (10)

**Hint :** Average atomic mass =  $\sum X_i M_i$ , where  $X_i$  is the mole fraction of an isotope and  $M_i$  is its atomic mass.

**Sol. :** Let the mole % of  $^{25}\text{Mg}$  be  $x$ . Therefore, mole % of  $^{26}\text{Mg}$  is  $(20-x)\%$ .

$$\therefore 0.80 \times 24 + 0.01x \times 25 + 0.01(20-x) \times 26 = 24.3$$

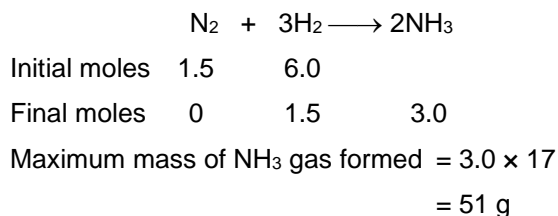
$$\therefore x = 10\%$$

26. Answer (51)

**Hint :**  $\text{N}_2$  is the limiting reagent, that decides the maximum mass of  $\text{NH}_3$ .

**Sol. :** Number of moles of  $\text{N}_2 = \frac{42}{28} = 1.5$

Number of moles of  $\text{H}_2 = \frac{12}{2} = 6.0$



27. Answer (68)

**Hint :** Number of photons

$$= \frac{\text{Total energy absorbed}}{\text{Photon energy}}$$

**Sol. :** Energy of a photon of wavelength,

$$\lambda = 612 \text{ nm}$$

$$E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{612 \times 10^{-9}} = \frac{6.6 \times 10^{-17}}{204}$$

Minimum energy needed to see an object

$$= 2.2 \times 10^{-17} \text{ J}$$

Number of photons required to see an object

$$= \frac{2.2 \times 10^{-17} \times 204}{6.6 \times 10^{-17}} = 68$$

28. Answer (15)

**Hint :** Molality of solution

$$= \frac{\text{Number of moles of solute}}{\text{Mass of solvents in kg}}$$

**Sol. :** Molarity of the given solution = 3.9 M

Volume of solvent in 1 L solution = 1 L

Density of solvent =  $0.26 \text{ g mL}^{-1}$

Mass of 1 L solvent = 260 gm

$$\text{Molality of solution} = \frac{3.9 \times 1000}{260} = 15 \text{ mol kg}^{-1}$$

29. Answer (12)

**Hint :** For  $n=4$ ,  $l=0, 1, 2$ , and  $3$

For  $|m_l| = 1$ ,  $m_l = \pm 1$  and

$$\text{For } |m_s| = \frac{1}{2}, m_s = \pm \frac{1}{2}$$

**Sol. :** For principal quantum number,  $n = 4$ , the possible values of azimuthal quantum number and magnetic quantum number are

$$\ell = 0 \quad m_\ell = 0$$

$$\ell = 1 \quad m_\ell = 0, \pm 1$$

$$\ell = 2 \quad m_\ell = 0, \pm 1, \pm 2$$

$$\ell = 3 \quad m_\ell = 0, \pm 1, \pm 2, \pm 3$$

Given values of magnetic and spin quantum numbers are

$$|m_\ell| = 1; \Rightarrow m_\ell = \pm 1$$

$$|m_s| = \frac{1}{2}; \Rightarrow m_s = \pm \frac{1}{2}$$

There are 6 orbitals which satisfy the given conditions and can accommodate 12 electrons.

30. Answer (18)

**Hint :** Angular momentum of electron in 3<sup>rd</sup> orbit of He<sup>+</sup> ion

$$mv_3 r_3 = 3 \left( \frac{h}{2\pi} \right)$$

Radius of electron in 3<sup>rd</sup> orbit of He<sup>+</sup> ion

$$r_3 = \frac{(3)^2 a_0}{2}$$

$$\text{KE of electron in 3<sup>rd</sup> orbit of He<sup>+</sup> ion} = \frac{(mv_3)^2}{2m}$$

**Sol. :** Angular momentum of an electron in  $n^{\text{th}}$  orbit of a Bohr atom is given by

$$mvr = n \frac{h}{2\pi}$$

For an electron in 3<sup>rd</sup> orbit of He<sup>+</sup> ion,

$$mv_3 r_3 = 3 \frac{h}{2\pi}$$

$$mv_3 = \frac{3h}{2\pi r_3}$$

$$r_3 = \frac{(3)^2 a_0}{2} = \frac{9a_0}{2}$$

$$\therefore mv_3 = \frac{3h \times 2}{2\pi \times 9a_0} = \frac{h}{3\pi a_0}$$

$$\text{KE} = \frac{(mv_3)^2}{2m} = \frac{h^2}{2m \times 9\pi^2 a_0^2} = \left( \frac{h^2}{\pi^2 m a_0^2} \right) \left( \frac{1}{18} \right)$$

$$\therefore x = 18$$

31. Answer (20)

**Hint :** Molarity of stock solution  $\times V$  (ml) =  $0.4 \times 460$

$$\begin{aligned} \text{Sol. : Millimoles of HCl in the final solution} \\ &= 0.4 \times 460 \\ &= 184 \end{aligned}$$

Mass of HCl in stock solution = 29.2 gm

Number of moles of HCl in stock solution

$$= \frac{29.2}{36.5} = 0.8$$

Mass of HCl stock solution = 100 gm

Density of stock solution =  $1.15 \text{ g mL}^{-1}$

$$\text{Volume of 100 g stock solution} = \frac{100}{1.15} \text{ mL}$$

$$\begin{aligned} \text{Molarity of stock solution} &= \frac{0.8 \times 1.15 \times 1000}{100} \\ &= 9.2 \text{ M} \end{aligned}$$

Let  $V$  ml of stock solution is required

$$9.2 \times V = 184$$

$$V = \frac{184}{9.2} = 20 \text{ ml}$$

32. Answer (50)

**Hint :** Number of moles of  $\text{C}_2\text{H}_5\text{Br}$

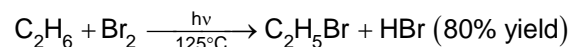
=  $0.80 \times$  Number of moles of  $\text{C}_2\text{H}_6$  consumed

Number of moles of n-butane

$$= \frac{0.56}{2} \times \text{Number of moles of } \text{C}_2\text{H}_5\text{Br consumed}$$

**Sol. :** Let the volume of  $\text{C}_2\text{H}_6$  required at STP be  $x$  L.

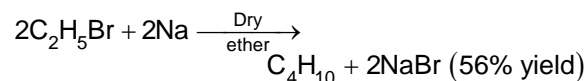
$$\text{Number of moles of } \text{C}_2\text{H}_6 \text{ required} = \frac{x}{22.4}$$



Number of moles of  $\text{C}_2\text{H}_5\text{Br}$  produced

=  $0.80 \times$  Number of moles of  $\text{C}_2\text{H}_6$  consumed

$$= \frac{0.80 \times x}{22.4}$$



Number of moles of  $\text{C}_4\text{H}_{10}$  produced

$$= \frac{0.56}{2} \times \text{number of moles of } \text{C}_2\text{H}_5\text{Br consumed}$$

$$= \frac{0.56 \times 0.80 \times x}{2 \times 22.4}$$



$$\text{Mass of C}_4\text{H}_{10} \text{ produced} = \frac{0.56 \times 0.80 \times x \times 58}{2 \times 22.4}$$

$$= 29 \text{ g}$$

$$x = 50 \text{ L}$$

33. Answer (A)

$$\text{Hint : Rate} = \frac{\text{Volume diffused}}{\text{Time}}$$

$$\text{Sol. : } \frac{r_x}{r_{\text{O}_2}} = \frac{V \times 5.65}{4 \times V} = \sqrt{\frac{32}{M_x}}; M_x = 16$$

34. Answer (B)

$$\text{Hint : Rate} = \frac{\text{Moles diffused}}{\text{Time}}$$

$$\text{Sol. : } \frac{r_{\text{H}_2}}{r_{\text{O}_2}} = \frac{x \times 32 \times 30}{2 \times 60 \times 1} = \sqrt{\frac{32}{2}}; x = 0.50 \text{ g}$$

35. Answer (A)

**Hint & Sol. :** Correct order of dipole moment



36. Answer (C)

**Hint & Sol. :** Compounds (I) and (II) have non-zero dipole moment because the resultant of all the bond dipole moments do not get cancelled.

### PART - III (MATHEMATICS)

37. Answer (A, C)

**Hint :** Form an equation whose roots are  $\frac{\alpha_i}{1+\alpha_i}$ ,

where  $i = 1, 2, 3, 4$ .

**Sol. :**  $x^4 - 7x + 1 = 0$  has roots  $\alpha_1, \alpha_2, \alpha_3$  and  $\alpha_4$ .

$$\text{Let } y = \frac{x}{1+x} \Rightarrow x = \frac{y}{1-y}$$

$$\left(\frac{y}{1-y}\right)^4 - 7\left(\frac{y}{1-y}\right) + 1 = 0$$

$$\Rightarrow y^4 - 7y(1-y)^3 + (1-y)^4 = 0$$

$$\Rightarrow 9y^4 - 25y^3 + 27y^2 - 11y + 1 = 0 \quad \dots(i)$$

The roots of equation (i) are  $\frac{\alpha_i}{1+\alpha_i}; i = 1, 2, 3, 4$

$$\sum_{i=1}^4 \frac{\alpha_i}{1+\alpha_i} = \text{Sum of roots of (i)} = \frac{25}{9}$$

$$\prod_{i=1}^4 \frac{\alpha_i}{1+\alpha_i} = \text{Product of roots of (i)} = \frac{1}{9}$$

38. Answer (B, C, D)

**Hint :** Put  $x - 2 = t$

**Sol. :** Let  $x - 2 = t$

$$\Rightarrow (t+1)^4 + (t-1)^4 = k$$

$$\Rightarrow t^4 + 6t^2 + 1 = \frac{k}{2}$$

$$\Rightarrow (t^2 + 3)^2 = 8 + \frac{k}{2}$$

$$\Rightarrow t^2 = -3 \pm \sqrt{8 + \frac{k}{2}} \quad \dots(i)$$

When  $t^2 > 0 \Rightarrow$  Two distinct real values of  $x$

$t^2 < 0 \Rightarrow$  Two imaginary values of  $x$ .

From (i) at least one value of  $t^2$  is negative, while other value may be positive if  $k > 2$ .

39. Answer (B, D)

**Hint :** Put  $z = x + iy$  and solve for  $x$  and  $y$ .

**Sol. :** Let  $z = x + iy$

$$x + iy + 1 + i = \sqrt{x^2 + y^2}$$

$$\Rightarrow (x+1) + i(y+1) = \sqrt{x^2 + y^2}$$

$$\Rightarrow y+1 = 0 \text{ and } x+1 = \sqrt{x^2 + y^2}$$

$$\Rightarrow y = -1 \text{ and } x = 0$$

$$\text{So, } z = -i$$

40. Answer (A, D)

**Hint :** Range of  $f(x)$  is  $\left[-\frac{1}{5}, \frac{1}{3}\right]$ .

**Sol. :** Domain of  $f(x)$  is  $R$  as  $x^2 + x + 4 \neq 0$ .

$$\text{Let } y = \frac{x+1}{x^2+x+4} = yx^2 + x(y-1) + (4y-1) = 0$$

$$\therefore x \in R, (y-1)^2 - 4y(4y-1) \geq 0$$

$$\Rightarrow 15y^2 - 2y - 1 \leq 0$$

$$y \in \left[-\frac{1}{5}, \frac{1}{3}\right]$$

41. Answer (B, C)

**Hint :** Use properties.

**Sol. :**  $\because 1 \notin A \cup (B \cap \{1, 2, 3\})$

$$\Rightarrow 1 \notin A \text{ and } 1 \notin B \cap \{1, 2, 3\}$$

$$\Rightarrow 1 \notin A \text{ and } 1 \notin B$$

$$\Rightarrow 1 \notin A \cup B$$

$$\Rightarrow 1 \in (A \cup B)'$$

$$\therefore 4 \notin B \cap \{1, 2, 3\} \text{ and } 5 \notin B \cap \{1, 2, 3\}$$

So, the smallest possible set  $A = \{4, 5\}$

Also, smallest possible set  $B = \phi$  (when  $A = \{2, 3, 4, 5\}$ )

42. Answer (B, D)

**Hint :** Use condition for common root.

**Sol. :**  $(a_1b_2 - a_2b_1)(b_1c_2 - b_2c_1) = (a_1c_2 - a_2c_1)^2$

$$\Rightarrow 3(-2\lambda) = (-\lambda)^2$$

$$\Rightarrow \lambda = 0, -6$$

43. Answer (15)

**Hint :** Use  $\tan\theta \cdot \tan(60^\circ - \theta) \cdot \tan(60^\circ + \theta) = \tan 3\theta$ 

**Sol. :**  $\tan 4^\circ \cdot \tan 8^\circ \cdot \tan 12^\circ \dots \tan 88^\circ$

$$= (\tan 4^\circ \cdot \tan 56^\circ \cdot \tan 64^\circ)(\tan 8^\circ \cdot \tan 52^\circ \cdot \tan 68^\circ) \dots$$

$$(\tan 28^\circ \cdot \tan 32^\circ \cdot \tan 88^\circ) \cdot \tan 60^\circ$$

$$= (\tan 12^\circ \cdot \tan 24^\circ \cdot \tan 36^\circ \cdot \tan 48^\circ \cdot \tan 60^\circ \cdot \tan 72^\circ \cdot$$

$$\tan 84^\circ) \sqrt{3}$$

$$= 3[(\tan 12^\circ \cdot \tan 48^\circ \cdot \tan 72^\circ)(\tan 24^\circ \cdot \tan 36^\circ \cdot \tan 84^\circ)]$$

$$= 3 \tan 36^\circ \cdot \tan 72^\circ$$

$$= 3 \cdot \frac{\sin 36^\circ \cdot \cos 18^\circ}{\cos 36^\circ \cdot \sin 18^\circ}$$

$$= 3 \left[ \frac{\sqrt{10-2\sqrt{5}} \cdot \sqrt{10+2\sqrt{5}}}{(\sqrt{5}+1)(\sqrt{5}-1)} \right]$$

$$= 3 \left[ \frac{\sqrt{100-20}}{4} \right]$$

$$= 3\sqrt{5}$$

44. Answer (63)

**Hint :**  $\tan C = -\tan(A+B) = -\frac{2 \tan\left(\frac{A+B}{2}\right)}{1 - \tan^2\left(\frac{A+B}{2}\right)}$

**Sol. :**  $\therefore C = \pi - (A+B)$

$$\Rightarrow \tan C = -\tan(A+B)$$

$$\Rightarrow \tan C = -\frac{2 \tan\left(\frac{A+B}{2}\right)}{1 - \tan^2\left(\frac{A+B}{2}\right)}$$

$$\text{Now, } \tan\left(\frac{A+B}{2}\right) = \frac{\tan \frac{A}{2} + \tan \frac{B}{2}}{1 - \tan \frac{A}{2} \cdot \tan \frac{B}{2}}$$

$$= 8$$

$$\text{So, } \tan C = \frac{16}{63}$$

$$\cos C = \frac{63}{65}$$

45. Answer (11)

**Hint :** Find the range of both trigonometric functions.

**Sol. :**  $\therefore 2k+1 \in [-\sqrt{193}, \sqrt{193}] \dots (i)$

$$\text{Also, } 2k = 4\sec^2 y + \operatorname{cosec}^2 y$$

$$2k = 5 + 4\tan^2 y + \frac{1}{\tan^2 y}$$

$$2k \in [9, \infty] \dots (ii)$$

From (i) and (ii),

$$k = 5 \text{ or } 6$$

46. Answer (12)

**Hint :**  $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$

**Sol. :**  $\arg(z) = \arg(\sqrt{i}) - \arg(\sqrt{3} + i)$

$$= \frac{1}{2} \arg(i) - \frac{\pi}{6}$$

$$= \frac{\pi}{4} - \frac{\pi}{6}$$

$$= \frac{\pi}{12}$$

47. Answer (24)

**Hint :**  $(A \times B \times B) \cap (A \times A \times B) = A \times (A \cap B) \times B$

**Sol. :** If  $(x, y, z) \in (A \times B \times B) \cap (A \times A \times B)$ , then  $x \in A, y \in A$  and  $y \in B, z \in B$

 Possible number of values of  $x = 3$ 

 Possible number of values of  $y = 2$ 

 Possible number of values of  $z = 4$ 

$$\therefore n((A \times B \times B) \cap (A \times A \times B)) = 24$$

48. Answer (04)

**Hint :** Put  $\log_2 3 = a$  to simplify  $X$  and use

$$(\sqrt{3}-1)^2 = 2(2-\sqrt{3}) \text{ to simplify } Y.$$

**Sol. :**

$$X = (4 + \log_2 3)(5 + \log_2 3) - (3 + \log_2 3)(6 + \log_2 3)$$

 Put  $\log_2 3 = a$ 

$$\Rightarrow X = (4+a)(5+a) - (3+a)(6+a) = 2$$

$$Y = \frac{1 + \log_2(2-\sqrt{3})}{\log_2(\sqrt{3}-1)} = \frac{\log_2(4-2\sqrt{3})}{\log_2(\sqrt{3}-1)} = 2$$

49. Answer (01)

**Hint :**  $x \in \left[ 2n\pi + \frac{\pi}{4}, 2n\pi + \frac{\pi}{2} \right)$

**Sol. :**  $\log_{\frac{\sqrt{3}}{2}}(\sin x) \leq \log_{\frac{\sqrt{3}}{2}}(\cos x)$

$\Rightarrow \sin x \geq \cos x$  also  $\sin x > 0 \cap \cos x > 0$

$\Rightarrow x \in \left[ 2n\pi + \frac{\pi}{4}, 2n\pi + \frac{\pi}{2} \right)$

$\therefore x \in [0, 12]$ , then  $x \in \left[ \frac{\pi}{4}, \frac{\pi}{2} \right) \cup \left[ \frac{9\pi}{4}, \frac{5\pi}{2} \right)$

$\therefore x$  is an integer, then  $x = 1$  only.

50. Answer (07)

**Hint :**  $f(x) = 3 + 2(\tan^2 x + \cot^2 x)$

**Sol. :**  $f(x) = \sin^2 x + \cos^2 x + \tan^2 x + \cot^2 x + \sec^2 x + \operatorname{cosec}^2 x$

$\Rightarrow f(x) = 3 + 2(\tan^2 x + \cot^2 x)$

$\Rightarrow f(x) = 7 + 2(\tan x - \cot x)^2$

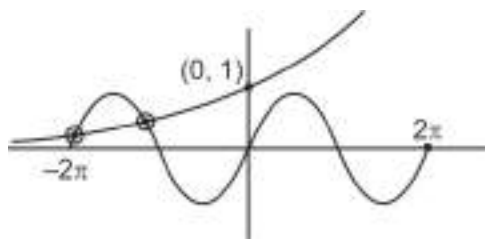
$\therefore$  Minimum value of  $f(x) = 7$

51. Answer (C)

**Hint :**  $e^x = \sin x$ ; draw the graphs of LHS and RHS

**Sol. :**  $\therefore e^x = \sin x$

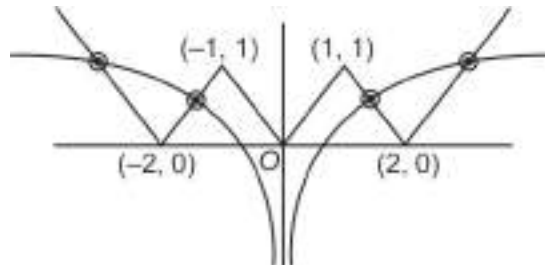
From the graph, there are two points of intersection.



52. Answer (C)

**Hint :** Draw graphs of LHS and RHS.

**Sol. :**  $\log_2 |x| = ||x| - 1| - 1|$



From the graph; we get 4 solutions.

53. Answer (A)

**Hint :** Put  $x = \sin \theta \Rightarrow \sin 3\theta = \frac{1}{\sqrt{2}}$

**Sol. :** Let  $x = \sin \theta$

$\Rightarrow \sin 3\theta = \frac{1}{\sqrt{2}} = \sin 45^\circ$

So, possible value of  $\theta$  is  $15^\circ$ , then

$x = \sin 15^\circ = \frac{\sqrt{3} - 1}{2\sqrt{2}}$

54. Answer (D)

**Hint :** Put  $x = \sin \theta \Rightarrow \sin 2\theta = \frac{\sqrt{10 - 2\sqrt{5}}}{4}$

**Sol. :** Let  $x = \sin \theta$

$\Rightarrow \sin 2\theta = \frac{\sqrt{10 - 2\sqrt{5}}}{4}$

$\Rightarrow \sin 2\theta = \sin 36^\circ$

So, possible value of  $\theta$  is  $18^\circ$ , then

$x = \sin 18^\circ = \frac{\sqrt{5} - 1}{4}$



## All India Aakash Test Series for JEE (Advanced)-2021

**TEST - 1A (Paper-1) - Code-F**

Test Date : 17/11/2019

**ANSWERS**

PHYSICS	CHEMISTRY	MATHEMATICS
1. (A, B)	19. (A, B, C)	37. (B, D)
2. (A, B, C, D)	20. (B, C)	38. (B, C)
3. (A, C, D)	21. (A, B, C, D)	39. (A, D)
4. (A, C, D)	22. (B)	40. (B, D)
5. (B, D)	23. (A, B, D)	41. (B, C, D)
6. (A, C)	24. (B, C, D)	42. (A, C)
7. (15)	25. (50)	43. (07)
8. (18)	26. (20)	44. (01)
9. (16)	27. (18)	45. (04)
10. (72)	28. (12)	46. (24)
11. (12)	29. (15)	47. (12)
12. (17)	30. (68)	48. (11)
13. (24)	31. (51)	49. (63)
14. (33)	32. (10)	50. (15)
15. (C)	33. (A)	51. (C)
16. (B)	34. (B)	52. (C)
17. (C)	35. (A)	53. (A)
18. (A)	36. (C)	54. (D)

## HINTS &amp; SOLUTIONS

## PART - I (PHYSICS)

1. Answer (A, B)

**Hint :**  $v = \frac{dx}{dt}$ ;  $a = \frac{d^2x}{dt^2}$

**Sol. :**  $x = \alpha t^3 + \beta t^2 + \gamma t + \delta$

$$\therefore \frac{dx}{dt} = v = 3\alpha t^2 + 2\beta t + \gamma$$

$$\frac{d^2x}{dt^2} = a = 6\alpha t + 2\beta$$

$$\therefore 6\alpha t + 2\beta = 3\alpha t^2 + 2\beta t + \gamma$$

$$\Rightarrow 3\alpha t^2 + (2\beta - 6\alpha)t + \gamma - 2\beta = 0$$

Here,  $4(3\alpha - \beta)^2 - 4 \times 3\alpha(\gamma - 2\beta) = 0$  for unique  $t$

$$\Rightarrow 9\alpha^2 + \beta^2 - 3\alpha\gamma = 0$$

And for that time instant  $t > 0$

$$\therefore t = \frac{2(3\alpha - \beta)}{2 \times 3\alpha} = \frac{3\alpha - \beta}{3\alpha}$$

$$\therefore 3\alpha - \beta > 0$$

2. Answer (A, B, C, D)

**Hint :** Tangential force will change the speed and perpendicular force will change the direction.

**Sol. :** If  $\vec{v}$  is opposite to  $\vec{F}$  the particle may retrace its path.

If  $\vec{F}$  is perpendicular to  $\vec{v}$  and so  $\vec{F}$  will provide the centripetal force and if  $|\vec{F}|$  is constant, then radius of curvature will be constant. And if at a particular time instant  $\vec{v}$  and  $\vec{F}$  are some angle other than  $0^\circ$  or  $180^\circ$  and  $\vec{F}$  is constant, then it's analogues of projectile motion. Particle will trace the parabolic path.

3. Answer (A, C, D)

**Hint :** For velocity to become perpendicular to initial direction  $\theta > \frac{\pi}{4}$ .

**Sol. :**



For velocity to become perpendicular to initial direction  $\theta > \frac{\pi}{4}$ .

For same case,  $m_1 = \tan\theta_1 = \tan\theta$

$$\text{And } m_2 = \tan\theta_2 = \frac{(u\sin\theta - gt)}{u\cos\theta}$$

$$\therefore (m_1 m_2 = -1) \Rightarrow \frac{(u\sin\theta - gt)}{u\cos\theta} \cdot \frac{\sin\theta}{\cos\theta} = -1$$

$$\Rightarrow u\sin^2\theta - gt\sin\theta = -u\cos^2\theta$$

$$\Rightarrow u = gt\sin\theta \therefore t = \frac{u}{g\sin\theta}$$

$$\text{And 1 s before, } \theta_1 = \tan^{-1}\left(\frac{g}{u\cos\theta}\right)$$

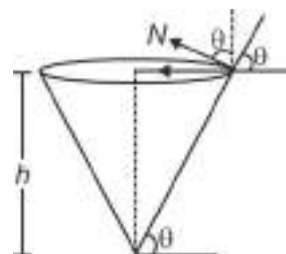
So just after 1 s and before 1 s,  $\Delta\theta = 2\theta$

$$\Delta\theta = 2 \tan^{-1}\left(\frac{g}{u\cos\theta}\right)$$

4. Answer (A, C, D)

**Hint :**  $v^2 = gr \tan\theta$

**Sol. :**



$$\tan\theta = \frac{h}{r}$$

$$\Rightarrow r = \frac{h}{\tan\theta}$$

Along the plane, with respect to cone the particle is in state of equilibrium.

$$\therefore mg\sin\theta = \frac{mv^2}{r} \cdot \cos\theta$$

$$\Rightarrow gr \cdot \frac{\sin\theta}{\cos\theta} = v^2 \Rightarrow v^2 = \frac{gh}{\tan\theta} \tan\theta$$

$$\therefore v^2 = gh$$

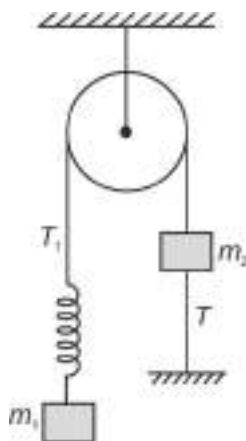
$$\text{Also, } N\cos\theta = mg$$

$$\text{And } N\sin\theta = \frac{mv^2}{r}$$

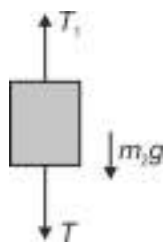
$$\therefore N\sin\theta = \frac{mv^2}{h} \frac{\sin\theta}{\cos\theta} \Rightarrow N\cos\theta = \frac{mv^2}{h}$$

5. Answer (B, D)

**Hint :** Sudden impulsive force by spring is zero.

**Sol. :** Let the tension in string  $BC$  is  $T$  at equilibrium.

 Then for  $m_1$  to be in equilibrium  $K\Delta x = m_1g$ 

 That means spring will be in extended condition and it will transmit  $T_1 = K\Delta x = m_1g$  force on string attached with spring.

 So, for ( $m_2$ )


$$\Rightarrow T = m_1g - m_2g$$

 When string  $BC$  is burnt suddenly then spring still transmit the same force so acceleration of mass  $m_1$  is zero. And acceleration of mass  $m_2$  is

$$a_2 = \frac{(m_1 - m_2)g}{m_2}$$

6. Answer (A, C)

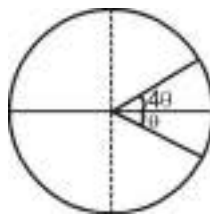
**Hint :**  $\Delta \vec{r} = \vec{r}_f - \vec{r}_i$ 
**Sol. :**

 Displacement  $\vec{r} = \vec{r}_f - \vec{r}_i$ 

$$\Rightarrow \vec{r} = (3\hat{i} + 4\hat{j} + 5\hat{k}) - (2\hat{i} + 3\hat{j} + 5\hat{k}) = \hat{i} + \hat{j}$$

 $\vec{r}$  makes  $45^\circ$  with +ve x-axis in anticlockwise sense.

7. Answer (15)

**Hint :**  $2T\Delta\theta = \Delta m\omega^2 R$ 
**Sol. :**


$$\lambda = \frac{m}{2\pi R}$$

$$\Delta\ell = 2R\Delta\theta$$

$$\therefore 2T \sin \Delta\theta = \Delta m \omega^2 R$$

$$\Rightarrow 2T\Delta\theta = \Delta m \omega^2 R$$

$$\Rightarrow 2T\Delta\theta = \frac{m}{2\pi R} 2R\Delta\theta \cdot \omega^2 R$$

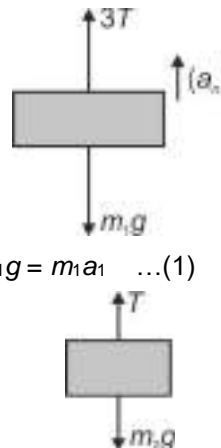
$$\Rightarrow T = \frac{m\omega^2 R}{2\pi} = 6.28 \times 5 \times 5 \times \frac{6}{10 \times 2 \times 3.14}$$

$$\Rightarrow T = 15$$

8. Answer (18)

**Hint :**  $3T - m_1g = m_1a_1$ 

$$m_2g - T = m_2a_2$$

**Sol. :**


$$\Rightarrow 3T - m_1g = m_1a_1 \quad \dots(1)$$

$$\Rightarrow m_2g - T = m_2a_2$$

$$\Rightarrow 3m_2g - 3T = 3m_2a_2$$

$$\therefore 3m_2g - m_1g = m_1a_1 + 3m_2a_2$$

 Also,  $a_2 = 3a_1$ 

$$\therefore (3m_2 - m_1)g = m_1a_1 + 9m_2a_1$$

$$\therefore a_1 = \frac{4g}{20} = 2 \text{ m/s}^2$$

$$\therefore a_2 = 6 \text{ m/s}^2$$

$$\therefore 3a_2 = 18 \text{ m/s}^2$$

9. Answer (16)

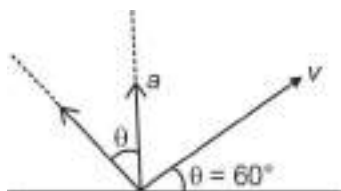
**Hint :**  $a = \frac{dv}{dt}$

**Sol. :**  $v(t) = \frac{8}{\sqrt{3}}\hat{i} + 8\hat{j}$

$\therefore \vec{a} = 8\hat{j}$

At  $t = 1$  s,  $v = \frac{8}{\sqrt{3}}\hat{i} + 8\hat{j}$

$\therefore \tan\theta = \frac{8}{\frac{8}{\sqrt{3}}} \cdot \sqrt{3} \therefore \theta = 60^\circ$



$\therefore a_n = a \cos 60^\circ = 8 \times \frac{1}{2} = 4$

$\therefore 4|\vec{a}_n| = 16$

10. Answer (72)

**Hint :**  $\frac{mg}{120} x \mu = \frac{mg}{120} (120 - x)$

**Sol. :** For state of impending motion, let  $x$  be the length on the table, then

$$\frac{m}{120} x g \mu = \frac{mg}{120} (120 - x)$$

$$\Rightarrow \frac{2}{3}x = 120 - x \Rightarrow \frac{5x}{3} = 120$$

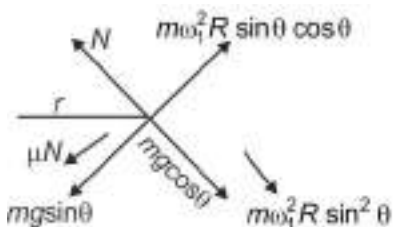
$$\Rightarrow x = \frac{120 \times 3}{5} = 72$$

11. Answer (12)

**Hint :**  $N = mg \cos \theta + m\omega_1^2 R \sin^2 \theta$

$$mg \sin \theta + \mu N = m\omega_1^2 R \sin \theta \cdot \cos \theta$$

**Sol. :** Let  $\omega_1$  be the maximum angular speed and  $\omega_2$  be the minimum angular speed, then



$$N = mg \cos \theta + m\omega_1^2 R \sin^2 \theta$$

$$\text{And } mg \sin \theta + \mu N = m\omega_1^2 R \sin \theta \cdot \cos \theta$$

$$\Rightarrow mg \sin \theta + \mu mg \cos \theta + \mu m\omega_1^2 R \sin^2 \theta = m\omega_1^2 R \sin \theta \cos \theta$$

$$\Rightarrow \omega_1^2 R \sin \theta (\cos \theta - \mu \sin \theta) = g(\sin \theta + \mu \cos \theta)$$

$$\therefore \omega_1^2 = \frac{g(\sin \theta + \mu \cos \theta)}{R \sin \theta (\cos \theta - \mu \sin \theta)}$$

$$\text{Similarly, } \omega_2^2 = \frac{g(\sin \theta - \mu \cos \theta)}{R \sin \theta (\cos \theta + \mu \sin \theta)}$$

$$\therefore \frac{\omega_1^2}{\omega_2^2} = \frac{\left(\frac{1}{\sqrt{2}} + \frac{1}{2\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}} + \frac{2}{2\sqrt{2}}\right)}{\left(\frac{1}{\sqrt{2}} - \frac{1}{2\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}} - \frac{1}{2\sqrt{2}}\right)}$$

$$\Rightarrow \frac{\omega_1^2}{\omega_2^2} = \frac{\left(\frac{3}{2\sqrt{2}}\right)^2}{\left(\frac{1}{2\sqrt{2}}\right)^2} = 9$$

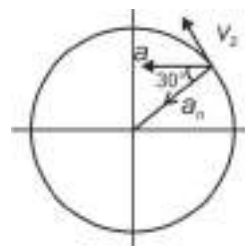
$$\therefore X = \frac{\omega_1}{\omega_2} = 3 \quad \therefore 4X = 12$$

12. Answer (17)

**Hint :**  $\tan \theta = \frac{|a_n|}{|a_t|}$

**Sol. :** Angle with velocity vector is  $30^\circ$ .

$$\therefore \tan 30^\circ = \frac{|a_n|}{|a_t|} = \frac{v^2}{Ra}$$



$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{a^2 t^2}{Ra}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{a^2 t^2}{Ra} \Rightarrow \frac{at^2}{R} = \frac{1}{\sqrt{3}}$$

$$\therefore t^2 = \frac{51}{\sqrt{3} \times \sqrt{3}} = 17$$

13. Answer (24)

**Hint :**  $v = \frac{dr}{dt}$ ,  $a = \frac{d^2r}{dt^2}$

**Sol. :**  $\vec{r} = 2t\hat{i} + 4t^2\hat{j} + \hat{k}$

$\therefore \vec{v} = \frac{dr}{dt} = 2\hat{i} + 8t\hat{j}$

And  $\vec{a} = \frac{d^2r}{dt^2} = 8\hat{j}$

Acceleration is always along y direction.

So, velocity in y direction at  $t = 3$  s is

$|\vec{v}_y| = |8t| \Rightarrow (8 \times 3) = 24$

14. Answer (33)

**Hint :** For vertical upward motion,  $y = u_y t - \frac{1}{2}gt^2$

**Sol. :** Let  $v_0$  was the velocity of dropping of 1<sup>st</sup> stone, then



$\vec{y}_1 = v_0 2 - \frac{1}{2} \times 10 \times 4 = 2v_0 - 20$

$\Rightarrow |\vec{y}_1| = (20 - 2v_0)$  is the distance from dropping point.

After 1 sec balloon shall have velocity  $v_2 = (v_0 + 1)$

And it must have travelled  $|y| = \left(v_0 + \frac{1}{2}\right)$ .

Then 1 sec after 2<sup>nd</sup> particle will be at

$y_2 = (v_0 + 1) - \frac{1}{2} \times 10 \times 1$

$\Rightarrow |\vec{y}_2| = 5 - v_0 - 1 = 4 - v_0$

Distance from dropping point

$\therefore \text{Separation} \Rightarrow s = |\vec{y}_1| + |\vec{y}| - |\vec{y}_2|$

$\Rightarrow s = 20 - 2v_0 + v_0 + \frac{1}{2} - 4 + v_0$

$\Rightarrow s = 16 + \frac{1}{2} = \frac{33}{2}$

$\therefore 2s = 33$

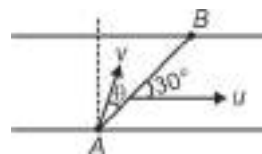
15. Answer (C)

16. Answer (B)

**Hint for Q.Nos. 15 and 16 :**

For forward and backward, both motion, person must maintain the same angle with line AB.

**Solution for Q.Nos. 15 and 16 :**



Clearly  $\sin 30^\circ = \frac{d}{AB}$

$\Rightarrow AB = 2d$

For forward and backward, both motion, person must maintain the same angle with line AB.

$\therefore v \sin \theta = u \sin 30^\circ \Rightarrow \sin \theta = \frac{u}{4\sqrt{3}}$

From A to B  $\Rightarrow v \cos \theta + \frac{u\sqrt{3}}{2} = \frac{2d}{T_1}$

And from B to A  $\Rightarrow v \cos \theta - \frac{u\sqrt{3}}{2} = \frac{2d}{T_2}$

$\therefore u\sqrt{3} = 2d \left( \frac{1}{T_1} - \frac{1}{T_2} \right)$

$\Rightarrow u\sqrt{3} = 2d \left( \frac{T_2 - T_1}{T_1 T_2} \right)$

$\therefore u = \frac{2d}{\sqrt{3}} \left[ \frac{T_2 - T_1}{T_1 T_2} \right]$

And  $\sin \theta = \frac{1}{4\sqrt{3}} \cdot \frac{2d}{\sqrt{3}} \left( \frac{T_2 - T_1}{T_1 T_2} \right)$

$\therefore \theta = \sin^{-1} \left( \frac{d}{6} \left( \frac{T_2 - T_1}{T_1 T_2} \right) \right)$

17. Answer (C)

18. Answer (A)

**Hint for Q.Nos. 17 and 18 :**

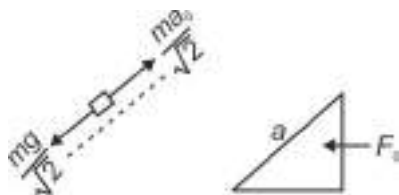
Motion is accelerated reference frame.

**Solution for Q.Nos. 17 and 18 :**

Let the force be  $F_0$  when small block does not slide with respect to wedge.



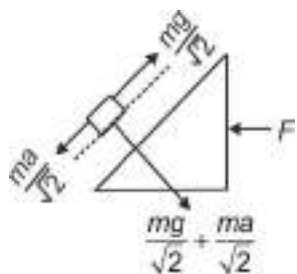
Then



$$\frac{mg}{\sqrt{2}} = \frac{ma_0}{\sqrt{2}} \Rightarrow a_0 = g$$

$$\text{And } F_0 = 2ma_0 = 2mg$$

$$\text{As } F = \frac{3}{2}F_0 \therefore \frac{3}{2} \cdot 2mg \Rightarrow F = 3mg$$



Now as  $F = 3mg$ , let the acceleration of block be  $a_1$  and wedge be  $a$ , then

$$\frac{ma}{\sqrt{2}} - \frac{mg}{\sqrt{2}} = ma_1$$

$$\therefore a_1 = \frac{1}{\sqrt{2}} - \frac{g}{\sqrt{2}}$$

$$\text{And for wedge } 3mg - \left(\frac{mg}{2} + \frac{ma}{2}\right) = ma$$

$$\Rightarrow 6mg - mg - ma = 2ma$$

$$\Rightarrow 5g = 3a \Rightarrow a = \frac{5g}{3}$$

$$\therefore a_1 = \frac{1}{\sqrt{2}} \left(\frac{5}{3} - 1\right)g = \frac{2g}{3\sqrt{2}} = \frac{\sqrt{2}g}{3}$$

$$\text{So, } \frac{L}{2} = \frac{1}{2} \cdot \frac{\sqrt{2}g}{3} \cdot t^2$$

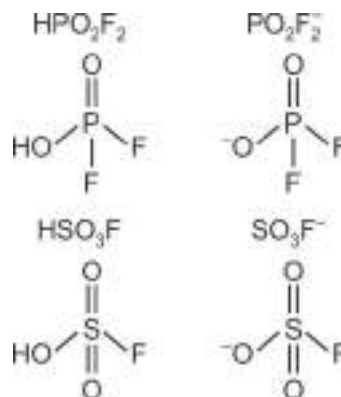
$$\Rightarrow t = \left[ \frac{3L}{\sqrt{2}g} \right]^{\frac{1}{2}}$$

## PART - II (CHEMISTRY)

19. Answer (A, B, C)

**Hint :** Hybridisation of central atom in all 4 molecules is same.

**Sol. :**



20. Answer (B, C)

**Hint :** The probability of finding electron,  $\psi^2$  is zero at radial nodes in an orbital.

**Sol. :** The radial wave function for a Bohr atom is given as

$$\psi(\text{radial}) = \frac{1}{16\sqrt{4}} \left[ \frac{Z}{a_0} \right]^{\frac{3}{2}} [(\sigma-1)(\sigma^2-8\sigma+12)] e^{-\frac{\sigma}{2}}$$

$$\text{where } \sigma = \frac{2Zr}{a_0}$$

At radial nodes,  $\psi^2 = 0$

$$\text{or } (\sigma-1) = 0; \sigma = 1 \Rightarrow r = \frac{a_0}{2Z}$$

$$\text{or } \sigma^2 - 8\sigma + 12 = 0; (\sigma-6)(\sigma-2) = 0$$

$$\Rightarrow \sigma = 6 \text{ or } 2; r = \frac{3a_0}{Z} \text{ or } \frac{a_0}{Z}$$

$$\therefore \text{Minimum position of radial node, } r = \frac{a_0}{2Z}$$

$$\text{Maximum position of radial node, } r = \frac{3a_0}{Z}$$

21. Answer (A, B, C, D)

**Hint :** All statements are correct.

$$\text{Sol. : At constant } V, P = \left( \frac{R}{V-b} \right) T$$

$$\text{At constant } P, V = b + \left( \frac{R}{P} \right) T$$

$$Z = \frac{PV}{RT} = 1 + \frac{Pb}{RT}$$

Since  $Z > 1$ , the repulsive forces dominate over attractive forces.

22. Answer (B)

$$\text{Hint : } P_{\text{real}} = \frac{nRT}{V-nb} - a \left( \frac{n}{V} \right)^2$$

$$P_{\text{ideal}} = \frac{nRT}{V}$$

**Sol. :** When cylinder is full

$$P_{\text{real}} = \frac{60 \times 0.08 \times 300}{15 - (60 \times 0.05)} - 0.25 \left( \frac{60}{15} \right)^2$$

$$= 116 \text{ atm}$$

$$P_{\text{ideal}} = \frac{60 \times 0.08 \times 300}{15} = 96 \text{ atm}$$

After prolonged used,

$$P_{\text{real}} = \frac{0.60 \times 0.08 \times 300}{15 - (0.6 \times 0.05)} - (0.25) \left( \frac{0.6}{15} \right)^2$$

$$\approx 0.96 \text{ atm}$$

$$P_{\text{ideal}} = \frac{0.6 \times 0.08 \times 300}{15} = 0.96 \text{ atm}$$

23. Answer (A, B, D)

**Hint :** Particles in the right zone have greater kinetic energy in distribution curve.

**Sol. :** Greater the kinetic energy, greater would be the tendency to get evaporate  $T_2 > T_1$ .

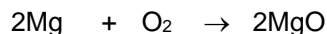
At higher temperature, vapour phase would exist.

24. Answer (B, C, D)

**Hint :** Oxygen is the limiting reagent.

**Sol. :** Number of moles of Mg =  $\frac{1}{24}$

Number of moles of  $O_2 = \frac{1}{64}$



Initial moles  $\frac{1}{24} \quad \frac{1}{64}$

Moles at the end  $\left( \frac{1}{24} - \frac{1}{32} \right) \quad 0 \quad \frac{1}{32}$

of reaction

Mass of Mg left unreacted

$$= \left[ \frac{1}{24} - \frac{1}{32} \right] \times 24 = 0.25 \text{ g}$$

$O_2$  gas is consumed completely.

Mass of MgO formed =  $\frac{1}{32} \times 40 = 1.25 \text{ g}$

25. Answer (50)

**Hint :** Number of moles of  $C_2H_5Br$

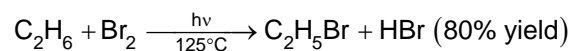
=  $0.80 \times$  Number of moles of  $C_2H_6$  consumed

Number of moles of n-butane

$$= \frac{0.56}{2} \times \text{Number of moles of } C_2H_5Br \text{ consumed}$$

**Sol. :** Let the volume of  $C_2H_6$  required at STP be x L.

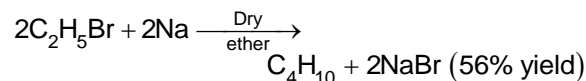
$$\text{Number of moles of } C_2H_6 \text{ required} = \frac{x}{22.4}$$



Number of moles of  $C_2H_5Br$  produced

=  $0.80 \times$  Number of moles of  $C_2H_6$  consumed

$$= \frac{0.80 \times x}{22.4}$$



Number of moles of  $C_4H_{10}$  produced

$$= \frac{0.56}{2} \times \text{number of moles of } C_2H_5Br \text{ consumed}$$

$$= \frac{0.56 \times 0.80 \times x}{2 \times 22.4}$$

$$\text{Mass of } C_4H_{10} \text{ produced} = \frac{0.56 \times 0.80 \times x \times 58}{2 \times 22.4}$$

$$= 29 \text{ g}$$

x = 50 L

26. Answer (20)

**Hint :** Molarity of stock solution  $\times V$  (ml) =  $0.4 \times 460$

**Sol. :** Millimoles of HCl in the final solution

$$= 0.4 \times 460$$

$$= 184$$

Mass of HCl in stock solution = 29.2 gm

Number of moles of HCl in stock solution

$$= \frac{29.2}{36.5} = 0.8$$

Mass of HCl stock solution = 100 gm

Density of stock solution =  $1.15 \text{ g mL}^{-1}$

$$\text{Volume of 100 g stock solution} = \frac{100}{1.15} \text{ mL}$$

$$\text{Molarity of stock solution} = \frac{0.8 \times 1.15 \times 1000}{100}$$

$$= 9.2 \text{ M}$$

Let V ml of stock solution is required

$$9.2 \times V = 184$$

$$V = \frac{184}{9.2} = 20 \text{ ml}$$

27. Answer (18)

**Hint :** Angular momentum of electron in 3<sup>rd</sup> orbit of He<sup>+</sup> ion

$$mv_3 r_3 = 3 \left( \frac{h}{2\pi} \right)$$

Radius of electron in 3<sup>rd</sup> orbit of He<sup>+</sup> ion

$$r_3 = \frac{(3)^2 a_0}{2}$$

KE of electron in 3<sup>rd</sup> orbit of He<sup>+</sup> ion =  $\frac{(mv_3)^2}{2m}$ **Sol. :** Angular momentum of an electron in n<sup>th</sup> orbit of a Bohr atom is given by

$$mvr = n \frac{h}{2\pi}$$

For an electron in 3<sup>rd</sup> orbit of He<sup>+</sup> ion,

$$mv_3 r_3 = 3 \frac{h}{2\pi}$$

$$mv_3 = \frac{3h}{2\pi r_3}$$

$$r_3 = \frac{(3)^2 a_0}{2} = \frac{9a_0}{2}$$

$$\therefore mv_3 = \frac{3h \times 2}{2\pi \times 9a_0} = \frac{h}{3\pi a_0}$$

$$KE = \frac{(mv_3)^2}{2m} = \frac{h^2}{2m \times 9\pi^2 a_0^2} = \left( \frac{h^2}{\pi^2 m a_0^2} \right) \left( \frac{1}{18} \right)$$

$$\therefore x = 18$$

28. Answer (12)

**Hint :** For  $n = 4$ ,  $l = 0, 1, 2$ , and  $3$ For  $|m_l| = 1$ ,  $m_l = \pm 1$  and

$$\text{For } |m_s| = \frac{1}{2}, m_s = \pm \frac{1}{2}$$

**Sol. :** For principal quantum number,  $n = 4$ , the possible values of azimuthal quantum number and magnetic quantum number are

$$l = 0 \quad m_l = 0$$

$$l = 1 \quad m_l = 0, \pm 1$$

$$l = 2 \quad m_l = 0, \pm 1, \pm 2$$

$$l = 3 \quad m_l = 0, \pm 1, \pm 2, \pm 3$$

Given values of magnetic and spin quantum numbers are

$$|m_l| = 1; \Rightarrow m_l = \pm 1$$

$$|m_s| = \frac{1}{2}; \Rightarrow m_s = \pm \frac{1}{2}$$

There are 6 orbitals which satisfy the given conditions and can accommodate 12 electrons.

29. Answer (15)

**Hint :** Molality of solution

$$= \frac{\text{Number of moles of solute}}{\text{Mass of solvents in kg}}$$

**Sol. :** Molarity of the given solution = 3.9 M

Volume of solvent in 1 L solution = 1 L

Density of solvent = 0.26 g mL<sup>-1</sup>

Mass of 1 L solvent = 260 gm

$$\text{Molality of solution} = \frac{3.9 \times 1000}{260} = 15 \text{ mol kg}^{-1}$$

30. Answer (68)

**Hint :** Number of photons

$$= \frac{\text{Total energy absorbed}}{\text{Photon energy}}$$

**Sol. :** Energy of a photon of wavelength,

$$\lambda = 612 \text{ nm}$$

$$E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{612 \times 10^{-9}} = \frac{6.6 \times 10^{-17}}{204}$$

Minimum energy needed to see an object

$$= 2.2 \times 10^{-17} \text{ J}$$

Number of photons required to see an object

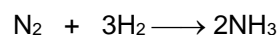
$$= \frac{2.2 \times 10^{-17} \times 204}{6.6 \times 10^{-17}} = 68$$

31. Answer (51)

**Hint :** N<sub>2</sub> is the limiting reagent, that decides the maximum mass of NH<sub>3</sub>.

$$\text{Sol. : Number of moles of } N_2 = \frac{42}{28} = 1.5$$

$$\text{Number of moles of } H_2 = \frac{12}{2} = 6.0$$



Initial moles    1.5        6.0

Final moles     0        1.5        3.0

Maximum mass of NH<sub>3</sub> gas formed = 3.0 × 17  
= 51 g

32. Answer (10)

**Hint :** Average atomic mass =  $\sum X_i M_i$ , where  $X_i$  is the mole fraction of an isotope and  $M_i$  is its atomic mass.

**Sol. :** Let the mole % of  $^{25}\text{Mg}$  be  $x$ . Therefore, mole % of  $^{26}\text{Mg}$  is  $(20 - x)\%$ .

$$\therefore 0.80 \times 24 + 0.01x \times 25 + 0.01(20 - x) \times 26 = 24.3$$

$$\therefore x = 10\%$$

33. Answer (A)

**Hint :** Rate =  $\frac{\text{Volume diffused}}{\text{Time}}$

$$\text{Sol. : } \frac{r_X}{r_{O_2}} = \frac{V \times 5.65}{4 \times V} = \sqrt{\frac{32}{M_X}}; M_X = 16$$

34. Answer (B)

**Hint :** Rate =  $\frac{\text{Moles diffused}}{\text{Time}}$

$$\text{Sol. : } \frac{r_{H_2}}{r_{O_2}} = \frac{x \times 32 \times 30}{2 \times 60 \times 1} = \sqrt{\frac{32}{2}}; x = 0.50 \text{ g}$$

35. Answer (A)

**Hint & Sol. :** Correct order of dipole moment



36. Answer (C)

**Hint & Sol. :** Compounds (I) and (II) have non-zero dipole moment because the resultant of all the bond dipole moments do not get cancelled.

### PART - III (MATHEMATICS)

37. Answer (B, D)

**Hint :** Use condition for common root.

$$\begin{aligned} \text{Sol. : } (a_1 b_2 - a_2 b_1)(b_1 c_2 - b_2 c_1) &= (a_1 c_2 - a_2 c_1)^2 \\ \Rightarrow 3(-2\lambda) &= (-\lambda)^2 \\ \Rightarrow \lambda &= 0, -6 \end{aligned}$$

38. Answer (B, C)

**Hint :** Use properties.

$$\text{Sol. : } \because 1 \notin A \cup (B \cap \{1, 2, 3\})$$

$$\Rightarrow 1 \notin A \text{ and } 1 \notin B \cap \{1, 2, 3\}$$

$$\Rightarrow 1 \notin A \text{ and } 1 \notin B$$

$$\Rightarrow 1 \notin A \cup B$$

$$\Rightarrow 1 \in (A \cup B)'$$

$$\therefore 4 \notin B \cap \{1, 2, 3\} \text{ and } 5 \notin B \cap \{1, 2, 3\}$$

So, the smallest possible set  $A = \{4, 5\}$

Also, smallest possible set  $B = \phi$  (when  $A = \{2, 3, 4, 5\}$ )

39. Answer (A, D)

**Hint :** Range of  $f(x)$  is  $\left[-\frac{1}{5}, \frac{1}{3}\right]$ .

**Sol. :** Domain of  $f(x)$  is  $R$  as  $x^2 + x + 4 \neq 0$ .

$$\text{Let } y = \frac{x+1}{x^2+x+4} = yx^2 + x(y-1) + (4y-1) = 0$$

$$\therefore x \in R, (y-1)^2 - 4y(4y-1) \geq 0$$

$$\Rightarrow 15y^2 - 2y - 1 \leq 0$$

$$y \in \left[-\frac{1}{5}, \frac{1}{3}\right]$$

40. Answer (B, D)

**Hint :** Put  $z = x + iy$  and solve for  $x$  and  $y$ .

**Sol. :** Let  $z = x + iy$

$$x + iy + 1 + i = \sqrt{x^2 + y^2}$$

$$\Rightarrow (x+1) + i(y+1) = \sqrt{x^2 + y^2}$$

$$\Rightarrow y+1 = 0 \text{ and } x+1 = \sqrt{x^2 + y^2}$$

$$\Rightarrow y = -1 \text{ and } x = 0$$

$$\text{So, } z = -i$$

41. Answer (B, C, D)

**Hint :** Put  $x - 2 = t$

**Sol. :** Let  $x - 2 = t$

$$\Rightarrow (t+1)^4 + (t-1)^4 = k$$

$$\Rightarrow t^4 + 6t^2 + 1 = \frac{k}{2}$$

$$\Rightarrow (t^2 + 3)^2 = 8 + \frac{k}{2}$$

$$\Rightarrow t^2 = -3 \pm \sqrt{8 + \frac{k}{2}} \quad \dots(i)$$

When  $t^2 > 0 \Rightarrow$  Two distinct real values of  $x$

$t^2 < 0 \Rightarrow$  Two imaginary values of  $x$ .

From (i) at least one value of  $t^2$  is negative, while other value may be positive if  $k > 2$ .

42. Answer (A, C)

**Hint :** Form an equation whose roots are  $\frac{\alpha_i}{1+\alpha_i}$ ,

where  $i = 1, 2, 3, 4$ .

**Sol. :**  $x^4 - 7x + 1 = 0$  has roots  $\alpha_1, \alpha_2, \alpha_3$  and  $\alpha_4$ .

$$\text{Let } y = \frac{x}{1+x} \Rightarrow x = \frac{y}{1-y}$$

$$\left(\frac{y}{1-y}\right)^4 - 7\left(\frac{y}{1-y}\right) + 1 = 0$$

$$\Rightarrow y^4 - 7y(1-y)^3 + (1-y)^4 = 0$$

$$\Rightarrow 9y^4 - 25y^3 + 27y^2 - 11y + 1 = 0 \quad \dots(i)$$

The roots of equation (i) are  $\frac{\alpha_i}{1+\alpha_i}$ ;  $i = 1, 2, 3, 4$

$$\sum_{i=1}^4 \frac{\alpha_i}{1+\alpha_i} = \text{Sum of roots of (i)} = \frac{25}{9}$$

$$\prod_{i=1}^4 \frac{\alpha_i}{1+\alpha_i} = \text{Product of roots of (i)} = \frac{1}{9}$$

43. Answer (07)

**Hint :**  $f(x) = 3 + 2(\tan^2 x + \cot^2 x)$

**Sol. :**  $f(x) = \sin^2 x + \cos^2 x + \tan^2 x + \cot^2 x + \sec^2 x + \csc^2 x$

$$\Rightarrow f(x) = 3 + 2(\tan^2 x + \cot^2 x)$$

$$\Rightarrow f(x) = 7 + 2(\tan x - \cot x)^2$$

$\therefore$  Minimum value of  $f(x) = 7$

44. Answer (01)

**Hint :**  $x \in \left[2n\pi + \frac{\pi}{4}, 2n\pi + \frac{\pi}{2}\right)$

**Sol. :**  $\log_{\frac{\sqrt{3}}{2}}(\sin x) \leq \log_{\frac{\sqrt{3}}{2}}(\cos x)$

$$\Rightarrow \sin x \geq \cos x \text{ also } \sin x > 0 \cap \cos x > 0$$

$$\Rightarrow x \in \left[2n\pi + \frac{\pi}{4}, 2n\pi + \frac{\pi}{2}\right)$$

$$\therefore x \in [0, 12], \text{ then } x \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right) \cup \left[\frac{9\pi}{4}, \frac{5\pi}{2}\right)$$

$\therefore$   $x$  is an integer, then  $x = 1$  only.

45. Answer (04)

**Hint :** Put  $\log_2 3 = a$  to simplify  $X$  and use

$$(\sqrt{3}-1)^2 = 2(2-\sqrt{3}) \text{ to simplify } Y.$$

**Sol. :**

$$X = (4 + \log_2 3)(5 + \log_2 3) - (3 + \log_2 3)(6 + \log_2 3)$$

$$\text{Put } \log_2 3 = a$$

$$\Rightarrow X = (4 + a)(5 + a) - (3 + a)(6 + a) = 2$$

$$Y = \frac{1 + \log_2(2 - \sqrt{3})}{\log_2(\sqrt{3} - 1)} = \frac{\log_2(4 - 2\sqrt{3})}{\log_2(\sqrt{3} - 1)} = 2$$

46. Answer (24)

**Hint :**  $(A \times B \times B) \cap (A \times A \times B) = A \times (A \cap B) \times B$

**Sol. :** If  $(x, y, z) \in (A \times B \times B) \cap (A \times A \times B)$ , then  $x \in A$ ,  $y \in A$  and  $y \in B$ ,  $z \in B$

Possible number of values of  $x = 3$

Possible number of values of  $y = 2$

Possible number of values of  $z = 4$

$$\therefore n((A \times B \times B) \cap (A \times A \times B)) = 24$$

47. Answer (12)

**Hint :**  $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$

**Sol. :**  $\arg(z) = \arg(\sqrt{i}) - \arg(\sqrt{3} + i)$

$$= \frac{1}{2}\arg(i) - \frac{\pi}{6}$$

$$= \frac{\pi}{4} - \frac{\pi}{6}$$

$$= \frac{\pi}{12}$$

48. Answer (11)

**Hint :** Find the range of both trigonometric functions.

**Sol. :**  $\therefore 2k + 1 \in [-\sqrt{193}, \sqrt{193}] \quad \dots(i)$

Also,  $2k = 4\sec^2 y + \csc^2 y$

$$2k = 5 + 4\tan^2 y + \frac{1}{\tan^2 y}$$

$$2k \in [9, \infty] \quad \dots(ii)$$

From (i) and (ii),

$$k = 5 \text{ or } 6$$

49. Answer (63)

**Hint :**  $\tan C = -\tan(A + B) = -\frac{2\tan\left(\frac{A+B}{2}\right)}{1 - \tan^2\left(\frac{A+B}{2}\right)}$

**Sol. :**  $\therefore C = \pi - (A + B)$

$$\Rightarrow \tan C = -\tan(A + B)$$

$$\Rightarrow \tan C = -\frac{2\tan\left(\frac{A+B}{2}\right)}{1 - \tan^2\left(\frac{A+B}{2}\right)}$$

$$\text{Now, } \tan\left(\frac{A+B}{2}\right) = \frac{\tan\frac{A}{2} + \tan\frac{B}{2}}{1 - \tan\frac{A}{2} \cdot \tan\frac{B}{2}}$$

$$= 8$$

$$\text{So, } \tan C = \frac{16}{63}$$

$$\cos C = \frac{63}{65}$$

50. Answer (15)

**Hint :** Use  $\tan\theta \cdot \tan(60^\circ - \theta) \cdot \tan(60^\circ + \theta) = \tan 3\theta$

**Sol. :**  $\tan 4^\circ \cdot \tan 8^\circ \cdot \tan 12^\circ \dots \tan 88^\circ$

$$= (\tan 4^\circ \cdot \tan 56^\circ \cdot \tan 64^\circ)(\tan 8^\circ \cdot \tan 52^\circ \cdot \tan 68^\circ) \dots$$

$$(\tan 28^\circ \cdot \tan 32^\circ \cdot \tan 88^\circ) \cdot \tan 60^\circ$$

$$= (\tan 12^\circ \cdot \tan 24^\circ \cdot \tan 36^\circ \cdot \tan 48^\circ \cdot \tan 60^\circ \cdot \tan 72^\circ \cdot \tan 84^\circ) \sqrt{3}$$

$$= 3[(\tan 12^\circ \cdot \tan 48^\circ \cdot \tan 72^\circ)(\tan 24^\circ \cdot \tan 36^\circ \cdot \tan 84^\circ)]$$

$$= 3 \tan 36^\circ \cdot \tan 72^\circ$$

$$= 3 \cdot \frac{\sin 36^\circ \cdot \cos 18^\circ}{\cos 36^\circ \cdot \sin 18^\circ}$$

$$= 3 \left[ \frac{\sqrt{10-2\sqrt{5}} \cdot \sqrt{10+2\sqrt{5}}}{(\sqrt{5}+1)(\sqrt{5}-1)} \right]$$

$$= 3 \left[ \frac{\sqrt{100-20}}{4} \right]$$

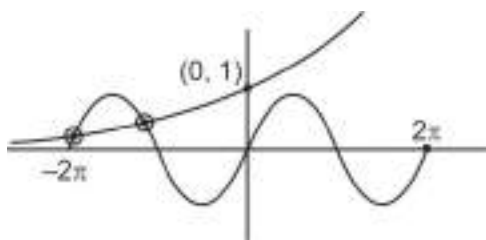
$$= 3\sqrt{5}$$

51. Answer (C)

**Hint :**  $e^x = \sin x$ ; draw the graphs of LHS and RHS

**Sol. :**  $\therefore e^x = \sin x$

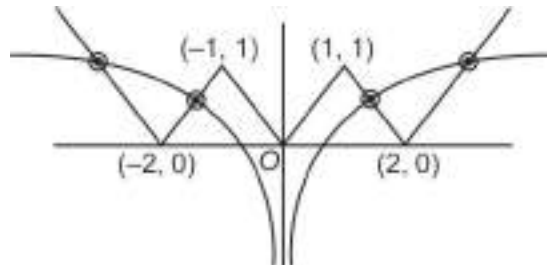
From the graph, there are two points of intersection.



52. Answer (C)

**Hint :** Draw graphs of LHS and RHS.

**Sol. :**  $\log_2 |x| = ||x| - 1| - 1$



From the graph; we get 4 solutions.

53. Answer (A)

**Hint :** Put  $x = \sin\theta \Rightarrow \sin 3\theta = \frac{1}{\sqrt{2}}$

**Sol. :** Let  $x = \sin\theta$

$$\Rightarrow \sin 3\theta = \frac{1}{\sqrt{2}} = \sin 45^\circ$$

So, possible value of  $\theta$  is  $15^\circ$ , then

$$x = \sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

54. Answer (D)

**Hint :** Put  $x = \sin\theta \Rightarrow \sin 2\theta = \frac{\sqrt{10-2\sqrt{5}}}{4}$

**Sol. :** Let  $x = \sin\theta$

$$\Rightarrow \sin 2\theta = \frac{\sqrt{10-2\sqrt{5}}}{4}$$

$$\Rightarrow \sin 2\theta = \sin 36^\circ$$

So, possible value of  $\theta$  is  $18^\circ$ , then

$$x = \sin 18^\circ = \frac{\sqrt{5}-1}{4}$$

