All India Aakash Test Series for JEE (Advanced)-2020

TEST - 1A (Paper-1) - Code-C

Test Date : 13/10/2019

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(B, C)	19.	(A, C, D)	37.	(C, D)
2.	(A, D)	20.	(B, C, D)	38.	(B, C)
3.	(A, C)	21.	(B, C)	39.	(A, D)
4.	(A, D)	22.	(B, C, D)	40.	(A)
5.	(A, B, C)	23.	(B, C, D)	41.	(A, D)
6.	(A, B)	24.	(B, C)	42.	(A, B, C)
7.	(00)	25.	(30)	43.	(35)
8.	(02)	26.	(09)	44.	(63)
9.	(20)	27.	(70)	45.	(41)
10.	(05)	28.	(50)	46.	(26)
11.	(08)	29.	(10)	47.	(10)
12.	(05)	30.	(30)	48.	(61)
13.	(16)	31.	(19)	49.	(06)
14.	(20)	32.	(65)	50.	(27)
15.	(C)	33.	(D)	51.	(C)
16.	(B)	34.	(D)	52.	(B)
17.	(C)	35.	(B)	53.	(A)
18.	(A)	36.	(B)	54.	(C)

HINTS & SOLUTIONS

PART - I (PHYSICS)

1. Answer (B, C)

Hint : $U_2 = \frac{1}{2}k\ X_{max}^2$

Solution :

$k_{eq} = 2k, \quad \Delta X_{max} = \frac{mg}{k}$

2. Answer (A, D)

Hint : $k = \frac{1}{2}I\omega^2$

Solution :

$k_T = \frac{1}{2}\left(2mR^2\right)\left(\frac{v_0^2}{R^2}\right)$

$k_T = mv_0^2$

$k_1 = \frac{1}{2}I_0^{ADB}\left(\frac{v_0}{R}\right)^2$

$k_2 = \frac{1}{2}I_0^{AOB}\left(\frac{v_0}{R}\right)^2$

$\Rightarrow k_2 = \frac{mv_0^2}{2} - \frac{mv_0^2}{\pi}$

$k_1 = mv_0^2 - \left[\frac{mv_0^2}{2} - \frac{mv_0^2}{\pi}\right]$

$= \frac{mv_0^2}{2} + \frac{mv_0^2}{\pi}$

3. Answer (A, C)

Hint: $t_{O\to R} = \frac{T}{12}$

Solution: $t_{R\to Q} = \frac{T}{4} - \frac{T}{12} = \frac{T}{6}$

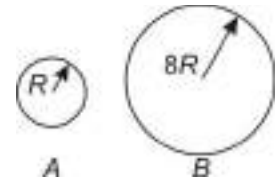
$t_{RP} = \frac{T}{4} + \frac{T}{12} = \frac{T}{3}$

4. Answer (A, D)

Hint : $v_e = \sqrt{\frac{2GM}{R}}$

Solution :

$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G}{R}(e)\frac{4}{3}\pi R^3}$



$V_e = \sqrt{CeR^2}$

$V_A = \sqrt{(C)16e_0R^2} = 4\sqrt{Ce_0R^2}$

$V_B = \sqrt{(C)e\ 8^2 \times R^2} = 8\sqrt{Ce_0R^2}$

5. Answer (A, B, C)

Hint : For equilibrium $\rho_{solid} = \rho_{liquid}$

Solution :

$\rho = \rho_0 + \frac{2\rho_0 x}{H}$

If $2\rho_0 = \rho_0 + \frac{2\rho_0 x}{H}$

$\Rightarrow \rho_0 = \frac{2\rho_0 x}{H} \Rightarrow x = \frac{H}{2}$

$a_{max} = \frac{(\rho_0)vg}{(2\rho_0)vg} = \frac{g}{2}$

6. Answer (A, B)

Hint : $F = \left(\frac{YA}{\ell}\right)\Delta\ell$

Solution :

$\Delta\ell_1 = \frac{\ell Mg}{(Y)2A} = \frac{Mg\ell}{(Y)2A}$

$\Delta\ell_2 = \frac{\ell Mg}{YA} = \frac{Mg\ell}{YA}$

7. Answer (00)

Hint : $\varepsilon\bar{f}_2 = 0 \Rightarrow \bar{a}_2 = 0$

Solution :

No force on plate (2)

$\Rightarrow \Delta X_2 = 0$

8. Answer (02)

Hint : Find IA_0R

Solution :

$\omega = \frac{v_Q}{4}$

$v_{cm} = \left(\frac{5}{2}\right)\omega$

$1.25 = (2.5)\frac{v_Q}{4}$

9. Answer (20)

Hint : $\tau = I\alpha$

Solution :

$\tau = (mg\ell) = (m\ell^2 + gm\ell^2)\alpha$

$\Rightarrow \alpha = \left(\frac{g}{10\ell}\right)$

$a_{cm} = r\alpha$

10. Answer (05)

Hint : $f = \eta A\frac{du}{dx}$

Solution :

$f = \eta A\frac{du}{dx}$

$15 = (\eta)(\ell)(\ell - x)\frac{u}{h}$

$\Rightarrow u = \frac{15h}{\eta\ell(\ell - x)}$

$\frac{dx}{dt} = \frac{15h}{\eta\ell(\ell - x)}$

Solve for t

11. Answer (08)

Hint : $v = \sqrt{\frac{2GM}{R}}$

Solution :

$\frac{v_p}{v_e} = \sqrt{\frac{M_p}{M_e} \times \frac{R_e}{R_p}}$

$v_p = \sqrt{2}\, v_e$

$\Rightarrow N = 08$

12. Answer (05)

Hint : $\Delta\ell \propto F$

Solution :

$\Delta\ell = \left(\frac{F}{YA\ell_2}\right)$

$\Rightarrow \frac{\Delta\ell_1}{\Delta\ell_2} = \frac{F}{5F}$

$\Rightarrow \frac{\Delta\ell_2}{\Delta\ell_1} = 5$

13. Answer (16)

Hint : $F_{ub} = mg$

Solution :

$\frac{\Delta v_s}{v_s} = \frac{e_s}{e_e}$

$\Rightarrow \frac{\Delta v_s}{v_s} = \frac{1}{16}$

14. Answer (20)

Hint: $\Delta p = \rho aL = \rho g\Delta h$

Solution :

$\Delta h = \frac{aL}{g}$

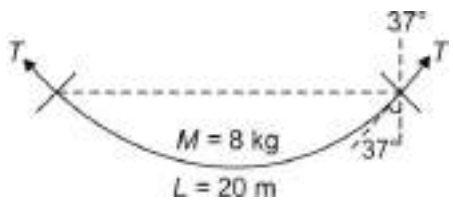
$\Delta h = 20$ cm

15. Answer (C)

16. Answer (B)

Solution for Q. nos. 15 and 16

Hint & Solution : Find tension at mid-point of rope (Net vertical force on string will be zero)



$(2)T \times \frac{4}{5} = Mg$

$\Rightarrow T = \frac{5Mg}{8} = 50$ N

$T_M = (T) \times \frac{3}{5} = (50) \times \frac{3}{5} = 30$ N

Now $(2T_M)\frac{\Delta\theta}{2} = \left(\frac{M}{L}\right)(R)(\Delta\theta)g$

$M = 2$ kg

$$\Rightarrow R = \left(\frac{T_M L}{Mg}\right) = \frac{30 \times 20}{80}$$

$$R = \frac{30}{4} = \frac{15}{2} \text{ m}$$

17. Answer (C)

18. Answer (A)

Solution for Q. nos. 17 and 18

Hint : $v_{s_1 r} = v_0 + \frac{v_0 4}{D}$

Solution :

$$v_{s_1 r} = v_0 + v_e \frac{y}{D}$$

$$v_y = \left(v_0 + v_0 \frac{y}{D}\right)$$

$$\frac{dy}{dt} = \left(v_0 + v_0 \frac{y}{D}\right)$$

$$\int_0^D \frac{dy}{\left(1 + \frac{y}{D}\right)} = \int_0^t v_0 dt$$

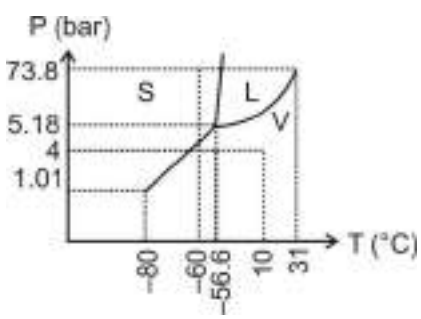
$$D\left[\ln\left(1 + \frac{y}{D}\right)\right]_0^D = v_0 t$$

PART - II (CHEMISTRY)

19. Answer (A, C, D)

Hint : (5.18 bar, –56.6°C) represents the triple point of CO_2. Hence, all three phases exist simultaneously.

Solution :



20. Answer (B, C, D)

Hint : $\Delta G° = \Delta H° - T\Delta S°$

and $\Delta G° = -RT \ln k_{eq}$

$$k_{eq} = P_{H_2O}^{3/2}$$

Solution :

$$\Delta H° = \frac{3}{2}(-241.8) + (-1575) - (-2021)$$

= 83.3 kJ/mole

$$\Delta S° = \frac{3}{2}(188{\cdot}6) + 130.5 - 194$$

= 219.4 JK^{-1} mol^{-1}

$\therefore \quad \Delta G° = \Delta H° - T\Delta S°$

= 83.3 – 298 (219.4) × 10^{-3}

= 17.92 $kJmol^{-1}$

$\therefore \quad -RT \ln K_p = 17.92$

$$\ln K_p = \frac{-17.92}{RT}$$

$\therefore \quad \ln K_p < 0$

$\Rightarrow \quad K_p < 1$

$\Rightarrow \quad \left(P_{H_2O}\right)^{3/2} < 1$

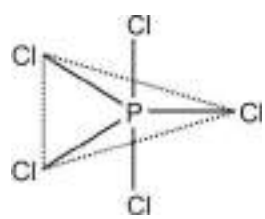
$\Rightarrow \quad P_{H_2O} < 1$

21. Answer (B, C)

Hint : P : Cl = 1 : 5

$\therefore$ Compound is PCl_5

Solution :



Axial P–Cl bond length > Equatorial P–Cl bond length.

22. Answer (B, C, D)

Hint : Mass is lost in the form of α particles.

Solution :

$$Th_{90}^{232} \longrightarrow Pb_{82}^{208}$$

$\alpha = 6, \ \beta = 4$

And emission of one α particle and two β particles lead to formation of an isotope.

23. Answer (B, C, D)

Hint : Closest distance between two atoms = 2R

Solution :

Rank = 2 for BCC

Rank = 4 for FCC

24. Answer (B, C)

Hint : In presence of phenolphthalein, neutralisation of half of the equivalents of carbonate is reported.

Solution :

Let in 100 ml, m moles of NaOH be 'a' and m moles of Na_2CO_3 be 'b'.

So,

$$\begin{array}{lcl} a+b & = & 100 \\ a+2b & = & 160 \\ \hline b & = & 60 \\ a & = & 40 \end{array}$$

$\therefore$ In 200 ml m moles of NaOH = 80

m moles of Na_2CO_3 = 120

Now,

$$CO_2 + 2NaOH \longrightarrow Na_2CO_3 + H_2O$$

$\therefore$ m moles of $CO_2 = 120$

And m moles of NaOH initially = 120 × 2 + 80

= 320

$$\therefore \quad x = \frac{320}{200} = 1.6$$

Now, 120 m mole of C is present in 0.02 mole

$\therefore$ 0.02 (n + 2) = 0.12

n + 2 = 6

n = 4

25. Answer (30)

Hint : Moles of SO_2 = 2 × moles of FeS_2 (Assuming 100% yield)

Solution :

44.8 L at 2 atm and 546 K = 2 moles

$\therefore$ 2 moles of SO_2 are produced with 40% yield.

$$\therefore \quad \left(\frac{40}{100}\right) 2n = 2$$

$$\Rightarrow \quad n = 2.5$$

$$\therefore \quad g = 2.5 \times (120) = 300 \text{ g}$$

$$\therefore \quad \frac{5}{100} \times w = 300 \times 10^{-3} \text{ kg}$$

$$\Rightarrow \quad w = \frac{300 \times 10^{-3} \times 10^2}{5} = 6 \text{ kg}$$

26. Answer (09)

Hint : Diffusion coefficient $\propto$ (mean free path) × (mean speed)

Solution :

Mean free path $\propto \frac{T}{P}$

Mean speed $\propto \sqrt{T}$

$$\therefore \quad \text{Diffusion coefficient} \propto \frac{T^{\frac{3}{2}}}{P}$$

$$\therefore \quad x = \frac{9^{\frac{3}{2}}}{3} = \frac{27}{3} = 9$$

27. Answer (70)

Hint : The given set of quantum number represent 4*f* orbital.

Solution :

So, the last electron enters 4*f* orbital.

So, $X \equiv [Xe]\, 6s^2 4f^{14}$

$\therefore$ Atomic number(Y) = 70

28. Answer (50)

Hint : $\Delta G°$ is the maximum extractable useful work.

Solution :

$386 \times 10^3 = 2(96500) \times 4 \times p$

p = 0.5

$\therefore$ 100 p = 50

29. Answer (10)

Hint : Electrical neutrality

Solution :

$O^{2-} = 4$

$\therefore$ OV = 4

TV = 8

$$2 \times \frac{p}{100} \times 4 + \frac{q}{100} \times 8 \times 3 = 8$$

$$p + 3q = 100$$

$$\therefore \quad 0.1(p + 3q) = 10$$

30. Answer (30)

Hint : Rate = K $[H^+]$ [Ester]

Solution :

$(H^+)_{HA} = 0.001\ [H^+]_{HCl}$.

$$\therefore \quad [H^+]_{HA} = 10^{-3}$$

$$\therefore \quad \underset{(1-x)}{HA} \rightleftharpoons \underset{x}{H^+} + \underset{x}{A^-}, K_a$$

$$\frac{x^2}{1} = K_a$$

$K_a = 10^{-6}$

$$\therefore \underset{(1-h)}{A^-} + \underset{-}{H_2O} \rightleftharpoons \underset{h}{HA} + \underset{h}{OH^-}, K_h = 10^{-8}$$

$$\frac{h^2}{1} = 10^{-8} \Rightarrow h = 10^{-4}$$

$$\therefore \text{pOH} = 4 \Rightarrow \text{pH} = 10$$

31. Answer (19)

Hint :

$$Ag^+ + e^- \longrightarrow Ag \quad \ldots(i)$$

$$AgI \rightleftharpoons Ag^+ + I^- \quad \ldots(ii)$$

$$AgI + e^- \rightleftharpoons Ag + I^- \quad \ldots(iii)$$

Solution : (iii) = (i) + (ii)

$$\therefore -nf(-0.28) = -f(0.8) - 2.303\,RT \log K_{sp}$$

$$\Rightarrow -0.28 = 0.8 + 0.06 \log K_{sp}$$

$$\Rightarrow -1.08 = 0.06 \log K_{sp}$$

$$\Rightarrow \log K_{sp} = -18$$

$K_{sp} = 1 \times 10^{-18}$

32. Answer (65)

Hint :

$$3\,Fe^{2+} \longrightarrow Fe^{3+} + 3e^-$$

$$6C^{2+} \longrightarrow 6C^{+4} + 12e^-$$

$$6N^{3-} \longrightarrow 6N^{+5} + 48e^-$$

$$\therefore \text{n factor} = 63$$

Solution :

40 × 0.63 × 5 = (m moles) × 63

m moles = 2

$$\therefore M_0 = 3 \times 56 + 6 \times (26) = 324$$

$$\therefore x = 2 \times 324 \times 10^{-3} = 0.648$$

$$\therefore 100x = 64.8 \simeq 65$$

33. Answer (D)

Hint : RLVP = X_{Solute}

Solution :

$$\frac{760 - 670}{760} = \frac{0.1}{0.1 + x}$$

$$\Rightarrow \frac{9}{76} = \frac{0.1}{0.1 + x} \Rightarrow \frac{9}{67} = \frac{0.1}{x}$$

$$\Rightarrow x = \frac{6.7}{9} = 0.744$$

34. Answer (D)

Hint :

$$K_f = \left(\frac{M}{1000}\right) \frac{R\,T_f^2}{(\Delta H_{fusion})}$$

Solution :

$$K_f = \left(\frac{78}{1000}\right) \frac{8.314 \times 77.56}{10.67} = 4.714 \text{ K kgmol}^{-1}$$

35. Answer (B)

Hint : In presence of external H^+,

$$[S^{2-}] = \frac{K_{a_1} K_{a_2} [H_2S]}{[H^+]^2}$$

Solution :

$$[S^{2-}] = \frac{10^{-22}}{(0.1)^2} = 10^{-20}$$

$$\therefore [Ni^{2+}] = \frac{2 \times 10^{-21}}{10^{-20}} = 0.2\,M$$

36. Answer (B)

Hint :

$$H_2S \rightleftharpoons 2H^+ + S^{2-}$$

$$Ni^{2+} + S^{2-} \rightleftharpoons NiS$$

Solution :

Sulphide ion concentration decrease while $[H^+]$ increases. So pH decreases.

PART - III (MATHEMATICS)

37. Answer (C, D)

Hint : $\prod_{k=0}^{n-1} \alpha_{k+1} = -1 \;\; \forall\; n = 2m, m \in N$

$$\prod_{k=0}^{n-1} \alpha_{k+1} = 1 \;\; \forall\; n = 2m+1, m \in N$$

Solution :

$$\prod_{k=0}^{n-1} \alpha_{k+1} = -1 \;\; \forall\; n = 2m, m \in N$$

$$\prod_{k=0}^{n-1} \alpha_{k+1} = 1 \;\; \forall\; n = 2m+1, m \in N$$

$1 - \alpha_2 = 1 - e^{i\frac{2\pi}{n}}$

$= 1 - \cos\frac{2\pi}{n} - i\sin\frac{2\pi}{n}$

$= 1 - 1 + 2\sin^2\frac{\pi}{n} - i\cdot 2\sin\frac{\pi}{n}\cdot\cos\frac{\pi}{n}$

$= 2\sin\frac{\pi}{n}\left(\sin\frac{\pi}{n} - i\cos\frac{\pi}{n}\right)$

$= 2\sin\frac{\pi}{n}\left(\cos\left(\frac{\pi}{n} - \frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{n} - \frac{\pi}{2}\right)\right)$

$1 - \alpha_2 = 2\sin\left(\frac{\pi}{n}\right)e^{i\left(\frac{\pi}{n} - \frac{\pi}{2}\right)}$

38. Answer (B, C)

Hint : $S_n = \frac{1}{3}\sum_{n=1}^{n}\left(\frac{(n+3)-n}{n(n+1)(n+2)(n+3)}\right)$

Solution :

$T_n = \frac{1}{n(n+1)(n+2)(n+3)}$

$S_n = \frac{1}{3}\sum_{n=1}^{n}\left(\frac{(n+3)-n}{n(n+1)(n+2)(n+3)}\right)$

$= \frac{1}{3}\left(\frac{1}{6} - \frac{1}{(n+1)(n+2)(n+3)}\right)$

$S_7 = \frac{119}{2160}$

39. Answer (A, D)

Hint : $f(8) \le f(10) \le f(12)$

Solution :

$f(8) \le f(10) \le f(12)$

Case 1 : $f(8) < f(10) < f(12)$

${}^9C_3$ functions

Case 2 : $f(8) = f(10) < f(12)$

${}^9C_2$ functions

Case 3 : $f(8) < f(10) = f(12)$

${}^9C_2$ functions

Case 4 : $f(8) = f(10) = f(12)$

${}^9C_1$ functions

For $f(i) \le f(j)$, $2\cdot{}^9C_2 + {}^9C_3 + {}^9C_1 = 165$

$f(i) < f(j)$, ${}^9C_3 = 84$

40. Answer (A)

Hint : $T_{r+1} = {}^{10}C_r(x\sin^{-1}p)^{10-r}\left(\frac{\cos^{-1}p}{x}\right)^r$

Solution :

$T_{r+1} = {}^{10}C_r(x\sin^{-1}p)^{10-r}\left(\frac{\cos^{-1}p}{x}\right)^r$

For independent of x, $r = 5$

Independent term = ${}^{10}C_5(\sin^{-1}p)^5(\cos^{-1}p)^5$

So for given interval $p \in [-1, 1]$

Minimum value of $(\sin^{-1}p\cos^{-1}p)^5$ at $p = -1$,

$\left(\frac{-\pi}{2}\times\pi\right)^5 = \frac{-\pi^{10}}{2^5}$

Maximum value of $(\sin^{-1}p\cos^{-1}p)^5$ at $p = \frac{1}{\sqrt{2}}$,

$\left(\frac{\pi}{4}\times\frac{\pi}{4}\right)^5 = \frac{\pi^{10}}{2^{20}}$

41. Answer (A, D)

Hint : Substitute $A = \pi - (B + C)$

Solution :

$\sin\left(\frac{\pi}{2} - \left(B + \frac{C}{2}\right)\right) + \sin\left(\frac{\pi}{2} - \left(C + \frac{B}{2}\right)\right) + \sin\left(\frac{3A}{2}\right) = \frac{3}{2}$

$\Rightarrow \cos\left(\frac{2B+C}{2}\right) + \cos\left(\frac{2C+B}{2}\right) - \cos\left(\frac{3(B+C)}{2}\right) = \frac{3}{2}$ ...(i)

$\Rightarrow 2\cos\left(\frac{3}{4}(B+C)\right)\cos\left(\frac{B-C}{4}\right) - 2\cos^2\left(\frac{3}{4}(B+C)\right) + 1 = \frac{3}{2}$ ...(ii)

For real values of $\cos\left(\frac{3}{4}(B+C)\right)$

$\left(2\cos\left(\frac{B-C}{4}\right)\right)^2 - 4(-2)\left(\frac{-1}{2}\right) \ge 0$

$\Rightarrow \cos^2\left(\frac{B-C}{4}\right) \ge 1$

$\Rightarrow \frac{B-C}{4} = 0 \Rightarrow B = C$ ...(iii)

By equation (ii), we get

$\left(2\cos\left(\frac{3}{4}(B+C)\right) - 1\right)^2 = 0$

$\Rightarrow \cos\frac{3B}{2} = \frac{1}{2}$

$\Rightarrow \frac{3B}{2} = 60^\circ \Rightarrow B = 40^\circ = C$ and $A = 100^\circ$

42. Answer (A, B, C)

Hint : $\therefore$ A is symmetric matrix.

Solution :

The matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

$\therefore$ A is symmetric matrix.

trace (A) = 6

$\therefore$ det(A) = 2(4 – 1) + 1(–2 – 0) = 4

43. Answer (35)

Hint : Simplify using properties of determinants.

Solution :

$$abc\begin{vmatrix} \frac{a^3+1}{a} & b^2 & c^2 \\ a^2 & \frac{b^3+1}{b} & c^2 \\ a^2 & b^2 & \frac{c^3+1}{c} \end{vmatrix} = 154$$

$$\Rightarrow \begin{vmatrix} a^3+1 & b^3 & c^3 \\ a^3 & b^3+1 & c^3 \\ a^3 & b^3 & c^3+1 \end{vmatrix} = 154$$

$\Rightarrow$ $a^3 + b^3 + c^3 = 153$

$\therefore$ a, b, c all must be odd.

Hence, $1^3 + 3^3 + 5^3$

$\therefore$ (a, b, c) has unordered pair triplets satisfying (1, 3, 5).

$\Rightarrow$ $a^2 + b^2 + c^2 = 1^2 + 3^2 + 5^2 = 1 + 9 + 25 = 35$

44. Answer (63)

Hint : Convert entire problem in cosine function.

Solution :

2 sec$z$ = cos(x + y) + cos(x – y)

$$\Rightarrow \frac{2}{\cos z} = 2\cos x \cos y$$

$\Rightarrow$ cos$x$ cos$y$ cos$z$ = 1

Hence, (1, –1, –1), (–1, –1, 1), (1, 1, 1) and (–1, 1, –1) for (cos$x$, cos$y$, cos$z$) are possible.

Total number of triplets = 27 + 18 + 18 = 63

45. Answer (41)

Hint : $T_r = \dfrac{r-\frac{1}{2}}{r(r-1)+\frac{1}{2}} - \dfrac{(r+1)-\frac{1}{2}}{(r+1)r+\frac{1}{2}}$

Solution :

$$T_r = \frac{r^2-\frac{1}{2}}{r^4+\frac{1}{4}} = \frac{r^2-\frac{1}{2}}{\left(r^2+r+\frac{1}{2}\right)\left(r^2-r+\frac{1}{2}\right)}$$

$$T_r = \frac{r-\frac{1}{2}}{r^2-r+\frac{1}{2}} - \frac{r+\frac{1}{2}}{r^2+r+\frac{1}{2}}$$

$$T_r = \frac{r-\frac{1}{2}}{r(r-1)+\frac{1}{2}} - \frac{(r+1)-\frac{1}{2}}{(r+1)r+\frac{1}{2}}$$

$$\sum_{r=1}^{20} T_r = \frac{\frac{1}{2}}{\frac{1}{2}} - \frac{21-\frac{1}{2}}{420+\frac{1}{2}}$$

$$= 1 - \frac{41}{841} = \frac{800}{841}$$

46. Answer (26)

Hint : Make a quadratic in a.

Solution :

$a^2 - 11ax + (10x^2 + 9x^3 - x^4) = 0$

$$\Rightarrow a = \frac{11x \pm \sqrt{121x^2 + 4(x^4 - 9x^3 - 10x^2)}}{2}$$

$\Rightarrow$ $x^2 + x - a = 0$ or $x^2 - 10x + a = 0$

$\because$ All four roots are real, then

$1 + 4a \geq 0$ and $100 - 4a \geq 0$

$$\Rightarrow a \in \left[-\frac{1}{4}, 25\right]$$

Number of integral values of a = 26

47. Answer (10)

Hint : P is orthocentre of ΔABC.

Solution :

$\because$ DEF is pedal triangle of triangle ABC.

So, P is orthocentre of ΔABC.

$AP = 2R\cos A$, $BP = 2R\cos B$ and $CP = 2R\cos C$

$\because$ $8R^3 \cos A \cos B \cos C = 3$

and $4R^2[\cos^2 A + \cos^2 B + \cos^2 C] = 7$

$\Rightarrow$ $4R^2[1 - 2\cos A \cos B \cos C] = 7$

Now, $DP = 2R\cos B\cos C$, $EP = 2R\cos A\cos C$ and $FP = 2R\cos A\cos B$

$DP \cdot EP \cdot FP = 8R^3[\cos A \cos B \cos C]^2$

$$= 8\left(\frac{3}{2}\right)^3\left[\frac{1}{9}\right]^2$$

$$= \frac{27}{81} = \frac{1}{3}$$

48. Answer (61)

Hint : Such numbers = $5^3 - 4^3$

Solution :

Such numbers = $5^3 - 4^3 = 61$

49. Answer (06)

Hint :

$$(1+3x-2x^2)^5 = \sum \frac{5!}{p!q!r!}(1)^p(3x)^q(-2x^2)^r$$

Solution :

$$(1+3x-2x^2)^5 = \sum \frac{5!}{p!q!r!}(1)^p(3x)^q(-2x^2)^r$$

$p + q + r = 5$ $\quad$ $q + 2r = 3$

	p	q	r
Case 1 :	2	3	0
Case 2 :	3	1	1

Coefficient of x^3 = $\frac{5!}{2!3!}(3)^3(-2)^0 + \frac{5!}{3!1!1!}(3)^1(-2)^1$

$= 10 \times 27 + 20 \times 3 \times (-2)$

$= 3 \times 10(9 - 4) = 150$

Sum of digits of coefficients of $x^3 = 6$

50. Answer (27)

Hint : $\left(\frac{1}{3}\right)S = 3\left(\frac{1}{3}\right) + 5\left(\frac{1}{3}\right)^2 + 9\left(\frac{1}{3}\right)^3 + \ldots\infty$

Solution :

$$S = 3 + 5\left(\frac{1}{3}\right) + 9\left(\frac{1}{3}\right)^2 + 15\left(\frac{1}{3}\right)^3 + \ldots\infty$$

$$\left(\frac{1}{3}\right)S = 3\left(\frac{1}{3}\right) + 5\left(\frac{1}{3}\right)^2 + 9\left(\frac{1}{3}\right)^3 + \ldots\infty$$

$$S\left(\frac{2}{3}\right) = 3 + 2\left(\frac{1}{3}\right) + 4\left(\frac{1}{3}\right)^2 + 6\left(\frac{1}{3}\right)^3 + \ldots\infty$$

Let $S_1 = 2\left(\frac{1}{3}\right) + 4\left(\frac{1}{3}\right)^2 + 6\left(\frac{1}{3}\right)^3 + \ldots\infty$

$$\frac{1}{3}S_1 = \quad 2\left(\frac{1}{3}\right)^2 + 4\left(\frac{1}{3}\right)^3 + \ldots\infty$$

$$\frac{2}{3}S_1 = 2\left(\frac{1}{3}\right) + 2\left(\frac{1}{3}\right)^2 + 2\left(\frac{1}{3}\right)^3 + \ldots$$

$$\frac{2}{3}S_1 = 2\left[\frac{\frac{1}{3}}{1-\frac{1}{3}}\right] = 2 \times \frac{1}{2} = 1$$

$$S_1 = \frac{3}{2}$$

So, $S\left(\frac{2}{3}\right) = 3 + \frac{3}{2} = \frac{9}{2}$

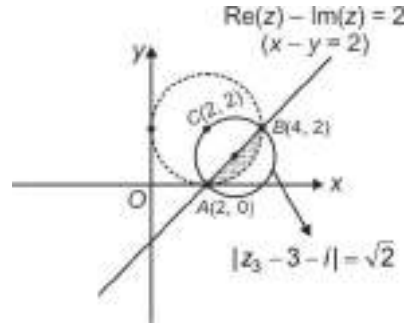
$$S = \frac{27}{4}$$

51. Answer (C)

52. Answer (B)

Hint : Draw graph of A, B, C.

Solution for Q.Nos. 51 and 52 :



$\Rightarrow$ Number of elements in $A \cap B \cap C$ is two.

Area = $\frac{1}{4}\pi \times 4 - \frac{1}{2} \times 2\sqrt{2} \times \sqrt{2} = \pi - 2$

$0 \le \arg(z_4) \le \cot^{-1}2$

53. Answer (A)

Hint : Total number of matrices be 9! and values of determinants are of opposite sign.

Solution :

All the determinants can be grouped into two groups each containing $\frac{9!}{2}$ determinants such that corresponding to each determinant in a group there is another determinant in the other group which is obtained by interchanging the two consecutive rows of the determinant in the first group.

$\therefore$ Sum of the values of the determinants is 0.

54. Answer (C)

Hint : First 9 prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23.

Solution :

The maximum value of trace of matrix = 23 + 19 + 17 = 59

The minimum value of trace of matrix = 10

$\therefore$ The required value = 59 – 10 = 49

□ □ □

All India Aakash Test Series for JEE (Advanced)-2020

TEST - 1A (Paper-1) - Code-D

Test Date : 13/10/2019

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(A, B)	19.	(B, C)	37.	(A, B, C)
2.	(A, B, C)	20.	(B, C, D)	38.	(A, D)
3.	(A, D)	21.	(B, C, D)	39.	(A)
4.	(A, C)	22.	(B, C)	40.	(A, D)
5.	(A, D)	23.	(B, C, D)	41.	(B, C)
6.	(B, C)	24.	(A, C, D)	42.	(C, D)
7.	(20)	25.	(65)	43.	(27)
8.	(16)	26.	(19)	44.	(06)
9.	(05)	27.	(30)	45.	(61)
10.	(08)	28.	(10)	46.	(10)
11.	(05)	29.	(50)	47.	(26)
12.	(20)	30.	(70)	48.	(41)
13.	(02)	31.	(09)	49.	(63)
14.	(00)	32.	(30)	50.	(35)
15.	(C)	33.	(D)	51.	(C)
16.	(B)	34.	(D)	52.	(B)
17.	(C)	35.	(B)	53.	(A)
18.	(A)	36.	(B)	54.	(C)

HINTS & SOLUTIONS

PART - I (PHYSICS)

1. Answer (A, B)

Hint : $F = \left(\frac{YA}{\ell}\right)\Delta\ell$

Solution :

$$\Delta\ell_1 = \frac{\ell Mg}{(Y)2A} = \frac{Mg\ell}{(Y)2A}$$

$$\Delta\ell_2 = \frac{\ell Mg}{YA} = \frac{Mg\ell}{YA}$$

2. Answer (A, B, C)

Hint : For equilibrium $\rho_{solid} = \rho_{liquid}$

Solution :

$$\rho = \rho_0 + \frac{2\rho_0 x}{H}$$

If $2\rho_0 = \rho_0 + \frac{2\rho_0 x}{H}$

$$\Rightarrow \rho_0 = \frac{2\rho_0 x}{H} \Rightarrow x = \frac{H}{2}$$

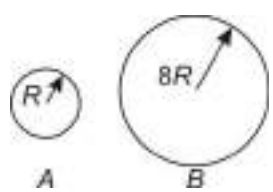
$$a_{max} = \frac{(\rho_0)vg}{(2\rho_0)vg} = \frac{g}{2}$$

3. Answer (A, D)

Hint : $v_e = \sqrt{\frac{2GM}{R}}$

Solution :

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G}{R}(e)\frac{4}{3}\pi R^3}$$



$$V_e = \sqrt{CeR^2}$$

$$V_A = \sqrt{(C)16e_0R^2} = 4\sqrt{Ce_0R^2}$$

$$V_B = \sqrt{(C)e\,8^2 \times R^2} = 8\sqrt{Ce_0R^2}$$

4. Answer (A, C)

Hint: $t_{O\to R} = \frac{T}{12}$

Solution: $t_{R\to Q} = \frac{T}{4} - \frac{T}{12} = \frac{T}{6}$

$$t_{RP} = \frac{T}{4} + \frac{T}{12} = \frac{T}{3}$$

5. Answer (A, D)

Hint : $k = \frac{1}{2}I\omega^2$

Solution :

$$k_T = \frac{1}{2}\left(2mR^2\right)\left(\frac{v_0^2}{R^2}\right)$$

$k_T = mv_0^2$

$$k_1 = \frac{1}{2}I_0^{ADB}\left(\frac{v_0}{R}\right)^2$$

$$k_2 = \frac{1}{2}I_0^{AOB}\left(\frac{v_0}{R}\right)^2$$

$$\Rightarrow k_2 = \frac{mv_0^2}{2} - \frac{mv_0^2}{\pi}$$

$$k_1 = mv_0^2 - \left[\frac{mv_0^2}{2} - \frac{mv_0^2}{\pi}\right]$$

$$= \frac{mv_0^2}{2} + \frac{mv_0^2}{\pi}$$

6. Answer (B, C)

Hint : $U_2 = \frac{1}{2}k\ X_{max}^2$

Solution :

$$k_{eq} = 2k, \quad \Delta X_{max} = \frac{mg}{k}$$

7. Answer (20)

Hint: $\Delta p = \rho aL = \rho g\Delta h$

Solution :

$$\Delta h = \frac{aL}{g}$$

Δh = 20 cm

8. Answer (16)

Hint : $F_{ub} = mg$

Solution :

$$\frac{\Delta v_s}{v_s} = \frac{e_s}{e_e}$$

$$\Rightarrow \frac{\Delta v_s}{v_s} = \frac{1}{16}$$

9. Answer (05)

Hint : $\Delta\ell \propto F$

Solution :

$$\Delta\ell = \left(\frac{F}{YA\ell_2}\right)$$

$$\Rightarrow \frac{\Delta\ell_1}{\Delta\ell_2} = \frac{F}{5F}$$

$$\Rightarrow \frac{\Delta\ell_2}{\Delta\ell_1} = 5$$

10. Answer (08)

Hint : $v = \sqrt{\frac{2GM}{R}}$

Solution :

$$\frac{v_p}{v_e} = \sqrt{\frac{M_p}{M_e} \times \frac{R_e}{R_p}}$$

$$v_p = \sqrt{2}\, v_e$$

$\Rightarrow N = 08$

11. Answer (05)

Hint : $f = \eta A \frac{du}{dx}$

Solution :

$$f = \eta A \frac{du}{dx}$$

$$15 = (\eta)(\ell)(\ell - x)\frac{u}{h}$$

$$\Rightarrow u = \frac{15h}{\eta\ell(\ell - x)}$$

$$\frac{dx}{dt} = \frac{15h}{\eta\ell(\ell - x)}$$

Solve for t

12. Answer (20)

Hint : $\tau = I\alpha$

Solution :

$$\tau = (mg\ell) = (m\ell^2 + gm\ell^2)\alpha$$

$$\Rightarrow \alpha = \left(\frac{g}{10\ell}\right)$$

$a_{cm} = r\alpha$

13. Answer (02)

Hint : Find IA_0R

Solution :

$$\omega = \frac{v_Q}{4}$$

$$v_{cm} = \left(\frac{5}{2}\right)\omega$$

$$1.25 = (2.5)\frac{v_Q}{4}$$

14. Answer (00)

Hint : $\varepsilon\bar{f}_2 = 0 \Rightarrow \bar{a}_2 = 0$

Solution :

No force on plate (2)

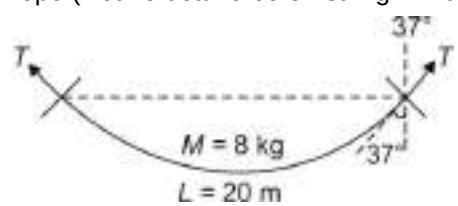
$\Rightarrow \Delta X_2 = 0$

15. Answer (C)

16. Answer (B)

Solution for Q. nos. 15 and 16

Hint & Solution : Find tension at mid-point of rope (Net vertical force on string will be zero)



$$(2)T \times \frac{4}{5} = Mg$$

$$\Rightarrow T = \frac{5Mg}{8} = 50\text{ N}$$

$$T_M = (T) \times \frac{3}{5} = (50) \times \frac{3}{5} = 30\text{ N}$$

Now $(2T_M)\frac{\Delta\theta}{2} = \left(\frac{M}{L}\right)(R)(\Delta\theta)g$

$M = 2$ kg

$$\Rightarrow R = \left(\frac{T_M L}{Mg}\right) = \frac{30 \times 20}{80}$$

$$R = \frac{30}{4} = \frac{15}{2}\text{ m}$$

17. Answer (C)

18. Answer (A)

Solution for Q. nos. 17 and 18

Hint : $v_{s_1r} = v_0 + \frac{v_0 4}{D}$

Solution :

$$v_{s_1r} = v_0 + v_e \frac{y}{D}$$

$$v_y = \left(v_0 + v_0 \frac{y}{D}\right)$$

$$\frac{dy}{dt} = \left(v_0 + v_0 \frac{y}{D}\right)$$

$$\int_0^D \frac{dy}{\left(1+\frac{y}{D}\right)} = \int_0^t v_0 dt$$

$$D\left[\ln\left(1+\frac{y}{D}\right)\right]_0^D = v_0 t$$

PART - II (CHEMISTRY)

19. Answer (B, C)

Hint : In presence of phenolphthalein, neutralisation of half of the equivalents of carbonate is reported.

Solution :

Let in 100 ml, m moles of NaOH be 'a' and m moles of Na_2CO_3 be 'b'.

So,

$$\begin{array}{lcl} a+b & = & 100 \\ a+2b & = & 160 \\ \hline b & = & 60 \\ a & = & 40 \end{array}$$

∴ In 200 ml m moles of NaOH = 80

m moles of Na_2CO_3 = 120

Now,

$$CO_2 + 2NaOH \longrightarrow Na_2CO_3 + H_2O$$

∴ m moles of $CO_2 = 120$

And m moles of NaOH initially = 120 × 2 + 80

= 320

$$\therefore \quad x = \frac{320}{200} = 1.6$$

Now, 120 m mole of C is present in 0.02 mole

∴ 0.02 (n + 2) = 0.12

n + 2 = 6

n = 4

20. Answer (B, C, D)

Hint : Closest distance between two atoms = 2R

Solution :

Rank = 2 for BCC

Rank = 4 for FCC

21. Answer (B, C, D)

Hint : Mass is lost in the form of α particles.

Solution :

$$Th_{90}^{232} \longrightarrow Pb_{82}^{208}$$

$\alpha = 6,\ \beta = 4$

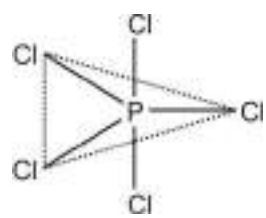
And emission of one α particle and two β particles lead to formation of an isotope.

22. Answer (B, C)

Hint : P : Cl = 1 : 5

∴ Compound is PCl_5

Solution :



Axial P–Cl bond length > Equatorial P–Cl bond length.

23. Answer (B, C, D)

Hint : $\Delta G° = \Delta H° - T\Delta S°$

and $\Delta G° = -RT \ln k_{eq}$

$$k_{eq} = P_{H_2O}^{3/2}$$

Solution :

$$\Delta H° = \frac{3}{2}(-241.8) + (-1575) - (-2021)$$

= 83.3 kJ/mole

$$\Delta S° = \frac{3}{2}(188{\cdot}6) + 130.5 - 194$$

= 219.4 $JK^{-1}\ mol^{-1}$

$$\therefore \quad \Delta G° = \Delta H° - T\Delta S°$$

= 83.3 – 298 (219.4) × 10^{-3}

= 17.92 $kJmol^{-1}$

$\therefore\ -RT\ln K_p = 17.92$

$\ln K_p = \frac{-17.92}{RT}$

$\therefore\ \ln K_p < 0$

$\Rightarrow\ K_p < 1$

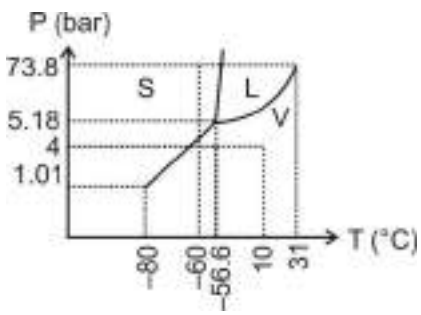
$\Rightarrow\ \left(P_{H_2O}\right)^{3/2} < 1$

$\Rightarrow\ P_{H_2O} < 1$

24. Answer (A, C, D)

Hint : (5.18 bar, –56.6°C) represents the triple point of CO_2. Hence, all three phases exist simultaneously.

Solution :



25. Answer (65)

Hint :

$3\,Fe^{2+} \longrightarrow Fe^{3+} + 3e^-$

$6C^{2+} \longrightarrow 6C^{+4} + 12e^-$

$6N^{3-} \longrightarrow 6N^{+5} + 48e^-$

$\therefore$ n factor = 63

Solution :

40 × 0.63 × 5 = (m moles) × 63

m moles = 2

$\therefore\ M_0 = 3\times 56 + 6\times(26) = 324$

$\therefore\ x = 2\times 324\times 10^{-3} = 0.648$

$\therefore\ 100x = 64.8 \simeq 65$

26. Answer (19)

Hint :

$Ag^+ + e^- \longrightarrow Ag$ …(i)

$AgI \rightleftharpoons Ag^+ + I^-$ …(ii)

$AgI + e^- \rightleftharpoons Ag + I^-$ …(iii)

Solution : (iii) = (i) + (ii)

$\therefore\ -nf(-0.28) = -f(0.8) - 2.303\,RT\log K_{sp}$

$\Rightarrow\ -0.28 = 0.8 + 0.06\log K_{sp}$

$\Rightarrow\ -1.08 = 0.06\log K_{sp}$

$\Rightarrow\ \log K_{sp} = -18$

$K_{sp} = 1 \times 10^{-18}$

27. Answer (30)

Hint : Rate = K [H^+] [Ester]

Solution :

$(H^+)_{HA} = 0.001\,[H^+]_{HCl}$.

$\therefore\ [H^+]_{HA} = 10^{-3}$

$\therefore\ \underset{(1-x)}{HA} \rightleftharpoons \underset{x}{H^+} + \underset{x}{A^-},\ K_a$

$\frac{x^2}{1} = K_a$

$K_a = 10^{-6}$

$\therefore\ \underset{(1-h)}{A^-} + \underset{-}{H_2O} \rightleftharpoons \underset{h}{HA} + \underset{h}{OH^-},\ K_h = 10^{-8}$

$\frac{h^2}{1} = 10^{-8} \Rightarrow h = 10^{-4}$

$\therefore\ pOH = 4 \Rightarrow pH = 10$

28. Answer (10)

Hint : Electrical neutrality

Solution :

$O^{2-} = 4$

$\therefore\ OV = 4$

TV = 8

$2\times\frac{p}{100}\times 4 + \frac{q}{100}\times 8\times 3 = 8$

$p + 3q = 100$

$\therefore\ 0.1(p+3q) = 10$

29. Answer (50)

Hint : ΔG° is the maximum extractable useful work.

Solution :

$386 \times 10^3 = 2(96500) \times 4 \times p$

p = 0.5

$\therefore\ 100\,p = 50$

30. Answer (70)

Hint : The given set of quantum number represent 4*f* orbital.

Solution :

So, the last electron enters 4*f* orbital.

So, $X \equiv [Xe]\, 6s^2 4f^{14}$

$\therefore$ Atomic number$(Y) = 70$

31. Answer (09)

Hint : Diffusion coefficient $\propto$ (mean free path) × (mean speed)

Solution :

Mean free path $\propto \frac{T}{P}$

Mean speed $\propto \sqrt{T}$

$\therefore$ Diffusion coefficient $\propto \frac{T^{\frac{3}{2}}}{P}$

$\therefore \quad x = \frac{9^{\frac{3}{2}}}{3} = \frac{27}{3} = 9$

32. Answer (30)

Hint : Moles of SO_2 = 2 × moles of FeS_2 (Assuming 100% yield)

Solution :

44.8 L at 2 atm and 546 K = 2 moles

$\therefore$ 2 moles of SO_2 are produced with 40% yield.

$\therefore \quad \left(\frac{40}{100}\right) 2n = 2$

$\Rightarrow \quad n = 2.5$

$\therefore \quad g = 2.5 \times (120) = 300$ g

$\therefore \quad \frac{5}{100} \times w = 300 \times 10^{-3}$ kg

$\Rightarrow \quad w = \frac{300 \times 10^{-3} \times 10^2}{5} = 6$ kg

33. Answer (D)

Hint : RLVP = X_{Solute}

Solution :

$\frac{760 - 670}{760} = \frac{0.1}{0.1 + x}$

$\Rightarrow \quad \frac{9}{76} = \frac{0.1}{0.1 + x} \Rightarrow \frac{9}{67} = \frac{0.1}{x}$

$\Rightarrow \quad x = \frac{6.7}{9} = 0.744$

34. Answer (D)

Hint :

$$K_f = \left(\frac{M}{1000}\right) \frac{R\, T_f^2}{(\Delta H_{fusion})}$$

Solution :

$$K_f = \left(\frac{78}{1000}\right) \frac{8.314 \times 77.56}{10.67} = 4.714 \text{ K kgmol}^{-1}$$

35. Answer (B)

Hint : In presence of external H^+,

$$[S^{2-}] = \frac{K_{a_1} K_{a_2} [H_2S]}{[H^+]^2}$$

Solution :

$$[S^{2-}] = \frac{10^{-22}}{(0.1)^2} = 10^{-20}$$

$$\therefore \quad [Ni^{2+}] = \frac{2 \times 10^{-21}}{10^{-20}} = 0.2 \text{ M}$$

36. Answer (B)

Hint :

$H_2S \rightleftharpoons 2H^+ + S^{2-}$

$Ni^{2+} + S^{2-} \rightleftharpoons NiS$

Solution :

Sulphide ion concentration decrease while $[H^+]$ increases. So pH decreases.

PART - III (MATHEMATICS)

37. Answer (A, B, C)

Hint : $\therefore$ *A* is symmetric matrix.

Solution :

The matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

$\therefore$ *A* is symmetric matrix.

trace (*A*) = 6

$\therefore$ det(*A*) = 2(4 – 1) + 1(–2 – 0) = 4

38. Answer (A, D)

Hint : Substitute $A = \pi - (B + C)$

Solution :

$$\sin\left(\frac{\pi}{2} - \left(B + \frac{C}{2}\right)\right) + \sin\left(\frac{\pi}{2} - \left(C + \frac{B}{2}\right)\right) + \sin\left(\frac{3A}{2}\right) = \frac{3}{2}$$

$$\Rightarrow \cos\left(\frac{2B+C}{2}\right)+\cos\left(\frac{2C+B}{2}\right)-$$

$$\cos\left(\frac{3(B+C)}{2}\right)=\frac{3}{2} \quad \ldots(i)$$

$$\Rightarrow 2\cos\left(\frac{3}{4}(B+C)\right)\cos\left(\frac{B-C}{4}\right)-$$

$$2\cos^2\left(\frac{3}{4}(B+C)\right)+1=\frac{3}{2} \quad \ldots(ii)$$

For real values of $\cos\left(\frac{3}{4}(B+C)\right)$

$$\left(2\cos\left(\frac{B-C}{4}\right)\right)^2-4(-2)\left(\frac{-1}{2}\right)\geq 0$$

$$\Rightarrow \cos^2\left(\frac{B-C}{4}\right)\geq 1$$

$$\Rightarrow \frac{B-C}{4}=0 \Rightarrow B=C \quad \ldots(iii)$$

By equation (ii), we get

$$\left(2\cos\left(\frac{3}{4}(B+C)\right)-1\right)^2=0$$

$$\Rightarrow \cos\frac{3B}{2}=\frac{1}{2}$$

$$\Rightarrow \frac{3B}{2}=60^\circ \Rightarrow B=40^\circ=C \text{ and } A=100^\circ$$

39. Answer (A)

Hint : $T_{r+1}={}^{10}C_r(x\sin^{-1}p)^{10-r}\left(\frac{\cos^{-1}p}{x}\right)^r$

Solution :

$$T_{r+1}={}^{10}C_r(x\sin^{-1}p)^{10-r}\left(\frac{\cos^{-1}p}{x}\right)^r$$

For independent of x, $r = 5$

Independent term = ${}^{10}C_5(\sin^{-1}p)^5(\cos^{-1}p)^5$

So for given interval $p \in [-1, 1]$

Minimum value of $(\sin^{-1}p\cos^{-1}p)^5$ at $p = -1$,

$$\left(\frac{-\pi}{2}\times\pi\right)^5=\frac{-\pi^{10}}{2^5}$$

Maximum value of $(\sin^{-1}p\cos^{-1}p)^5$ at $p=\frac{1}{\sqrt{2}}$,

$$\left(\frac{\pi}{4}\times\frac{\pi}{4}\right)^5=\frac{\pi^{10}}{2^{20}}$$

40. Answer (A, D)

Hint : $f(8) \leq f(10) \leq f(12)$

Solution :

$f(8) \leq f(10) \leq f(12)$

Case 1 : $f(8) < f(10) < f(12)$

${}^9C_3$ functions

Case 2 : $f(8) = f(10) < f(12)$

${}^9C_2$ functions

Case 3 : $f(8) < f(10) = f(12)$

${}^9C_2$ functions

Case 4 : $f(8) = f(10) = f(12)$

${}^9C_1$ functions

For $f(i) \leq f(j)$, $2\cdot{}^9C_2 + {}^9C_3 + {}^9C_1 = 165$

$f(i) < f(j)$, ${}^9C_3 = 84$

41. Answer (B, C)

Hint : $S_n=\frac{1}{3}\sum_{n=1}^{n}\left(\frac{(n+3)-n}{n(n+1)(n+2)(n+3)}\right)$

Solution :

$$T_n=\frac{1}{n(n+1)(n+2)(n+3)}$$

$$S_n=\frac{1}{3}\sum_{n=1}^{n}\left(\frac{(n+3)-n}{n(n+1)(n+2)(n+3)}\right)$$

$$=\frac{1}{3}\left(\frac{1}{6}-\frac{1}{(n+1)(n+2)(n+3)}\right)$$

$$S_7=\frac{119}{2160}$$

42. Answer (C, D)

Hint : $\prod_{k=0}^{n-1}\alpha_{k+1}=-1 \;\forall\; n=2m, m\in N$

$$\prod_{k=0}^{n-1}\alpha_{k+1}=1 \;\forall\; n=2m+1, m\in N$$

Solution :

$$\prod_{k=0}^{n-1}\alpha_{k+1}=-1 \;\forall\; n=2m, m\in N$$

$$\prod_{k=0}^{n-1}\alpha_{k+1}=1 \;\forall\; n=2m+1, m\in N$$

$$1-\alpha_2 = 1-e^{i\frac{2\pi}{n}}$$

$$= 1-\cos\frac{2\pi}{n}-i\sin\frac{2\pi}{n}$$

$$= 1-1+2\sin^2\frac{\pi}{n}-i\cdot 2\sin\frac{\pi}{n}\cdot\cos\frac{\pi}{n}$$

$$= 2\sin\frac{\pi}{n}\left(\sin\frac{\pi}{n} - i\cos\frac{\pi}{n}\right)$$

$$= 2\sin\frac{\pi}{n}\left(\cos\left(\frac{\pi}{n}-\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{n}-\frac{\pi}{2}\right)\right)$$

$$1 - \alpha_2 = 2\sin\left(\frac{\pi}{n}\right)e^{i\left(\frac{\pi}{n}-\frac{\pi}{2}\right)}$$

43. Answer (27)

Hint : $\left(\frac{1}{3}\right)S = 3\left(\frac{1}{3}\right) + 5\left(\frac{1}{3}\right)^2 + 9\left(\frac{1}{3}\right)^3 + \ldots\infty$

Solution :

$$S = 3 + 5\left(\frac{1}{3}\right) + 9\left(\frac{1}{3}\right)^2 + 15\left(\frac{1}{3}\right)^3 + \ldots\infty$$

$$\left(\frac{1}{3}\right)S = 3\left(\frac{1}{3}\right) + 5\left(\frac{1}{3}\right)^2 + 9\left(\frac{1}{3}\right)^3 + \ldots\infty$$

$$S\left(\frac{2}{3}\right) = 3 + 2\left(\frac{1}{3}\right) + 4\left(\frac{1}{3}\right)^2 + 6\left(\frac{1}{3}\right)^3 + \ldots\infty$$

Let $S_1 = 2\left(\frac{1}{3}\right) + 4\left(\frac{1}{3}\right)^2 + 6\left(\frac{1}{3}\right)^3 + \ldots\infty$

$$\frac{1}{3}S_1 = \qquad 2\left(\frac{1}{3}\right)^2 + 4\left(\frac{1}{3}\right)^3 + \ldots\infty$$

$$\frac{2}{3}S_1 = 2\left(\frac{1}{3}\right) + 2\left(\frac{1}{3}\right)^2 + 2\left(\frac{1}{3}\right)^3 + \ldots$$

$$\frac{2}{3}S_1 = 2\left[\frac{\frac{1}{3}}{1-\frac{1}{3}}\right] = 2\times\frac{1}{2} = 1$$

$$S_1 = \frac{3}{2}$$

So, $S\left(\frac{2}{3}\right) = 3 + \frac{3}{2} = \frac{9}{2}$

$$S = \frac{27}{4}$$

44. Answer (06)

Hint :

$$(1+3x-2x^2)^5 = \sum\frac{5!}{p!q!r!}(1)^p(3x)^q(-2x^2)^r$$

Solution :

$$(1+3x-2x^2)^5 = \sum\frac{5!}{p!q!r!}(1)^p(3x)^q(-2x^2)^r$$

$p + q + r = 5$ $\qquad$ $q + 2r = 3$

	p	q	r
Case 1 :	2	3	0
Case 2 :	3	1	1

Coefficient of x^3 = $\frac{5!}{2!3!}(3)^3(-2)^0 + \frac{5!}{3!1!1!}(3)^1(-2)^1$

$= 10 \times 27 + 20 \times 3 \times (-2)$

$= 3 \times 10(9 - 4) = 150$

Sum of digits of coefficients of $x^3 = 6$

45. Answer (61)

Hint : Such numbers = $5^3 - 4^3$

Solution :

Such numbers = $5^3 - 4^3 = 61$

46. Answer (10)

Hint : P is orthocentre of ΔABC.

Solution :

$\because$ DEF is pedal triangle of triangle ABC.

So, P is orthocentre of ΔABC.

$AP = 2R\cos A$, $BP = 2R\cos B$ and $CP = 2R\cos C$

$\because$ $8R^3\cos A\cos B\cos C = 3$

and $4R^2[\cos^2 A + \cos^2 B + \cos^2 C] = 7$

$\Rightarrow$ $4R^2[1 - 2\cos A\cos B\cos C] = 7$

Now, $DP = 2R\cos B\cos C$, $EP = 2R\cos A\cos C$ and $FP = 2R\cos A\cos B$

$DP\cdot EP\cdot FP = 8R^3[\cos A\cos B\cos C]^2$

$$= 8\left(\frac{3}{2}\right)^3\left[\frac{1}{9}\right]^2$$

$$= \frac{27}{81} = \frac{1}{3}$$

47. Answer (26)

Hint : Make a quadratic in a.

Solution :

$a^2 - 11ax + (10x^2 + 9x^3 - x^4) = 0$

$$\Rightarrow \quad a = \frac{11x \pm \sqrt{121x^2 + 4(x^4 - 9x^3 - 10x^2)}}{2}$$

$\Rightarrow$ $x^2 + x - a = 0$ or $x^2 - 10x + a = 0$

$\because$ All four roots are real, then

$1 + 4a \geq 0$ and $100 - 4a \geq 0$

$$\Rightarrow \quad a \in \left[-\frac{1}{4}, 25\right]$$

Number of integral values of $a = 26$

48. Answer (41)

Hint : $T_r = \dfrac{r-\frac{1}{2}}{r(r-1)+\frac{1}{2}} - \dfrac{(r+1)-\frac{1}{2}}{(r+1)r+\frac{1}{2}}$

Solution :

$$T_r = \frac{r^2 - \frac{1}{2}}{r^4 + \frac{1}{4}} = \frac{r^2 - \frac{1}{2}}{\left(r^2 + r + \frac{1}{2}\right)\left(r^2 - r + \frac{1}{2}\right)}$$

$$T_r = \frac{r - \frac{1}{2}}{r^2 - r + \frac{1}{2}} - \frac{r + \frac{1}{2}}{r^2 + r + \frac{1}{2}}$$

$$T_r = \frac{r - \frac{1}{2}}{r(r-1) + \frac{1}{2}} - \frac{(r+1) - \frac{1}{2}}{(r+1)r + \frac{1}{2}}$$

$$\sum_{r=1}^{20} T_r = \frac{\frac{1}{2}}{\frac{1}{2}} - \frac{21 - \frac{1}{2}}{420 + \frac{1}{2}}$$

$$= 1 - \frac{41}{841} = \frac{800}{841}$$

49. Answer (63)

Hint : Convert entire problem in cosine function.

Solution :

$2\sec z = \cos(x + y) + \cos(x - y)$

$\Rightarrow \quad \frac{2}{\cos z} = 2\cos x \cos y$

$\Rightarrow \quad \cos x \cos y \cos z = 1$

Hence, (1, –1, –1), (–1, –1, 1), (1, 1, 1) and (–1, 1, –1) for ($\cos x$, $\cos y$, $\cos z$) are possible.

Total number of triplets = 27 + 18 + 18 = 63

50. Answer (35)

Hint : Simplify using properties of determinants.

Solution :

$$abc\begin{vmatrix} \frac{a^3+1}{a} & b^2 & c^2 \\ a^2 & \frac{b^3+1}{b} & c^2 \\ a^2 & b^2 & \frac{c^3+1}{c} \end{vmatrix} = 154$$

$$\Rightarrow \begin{vmatrix} a^3+1 & b^3 & c^3 \\ a^3 & b^3+1 & c^3 \\ a^3 & b^3 & c^3+1 \end{vmatrix} = 154$$

$\Rightarrow \quad a^3 + b^3 + c^3 = 153$

$\therefore \quad a, b, c$ all must be odd.

Hence, $1^3 + 3^3 + 5^3$

$\therefore \quad (a, b, c)$ has unordered pair triplets satisfying (1, 3, 5).

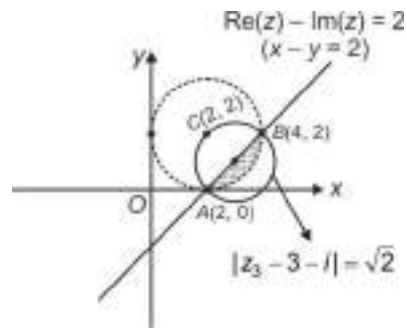
$\Rightarrow \quad a^2 + b^2 + c^2 = 1^2 + 3^2 + 5^2 = 1 + 9 + 25 = 35$

51. Answer (C)

52. Answer (B)

Hint : Draw graph of A, B, C.

Solution for Q.Nos. 51 and 52 :



$\Rightarrow$ Number of elements in $A \cap B \cap C$ is two.

Area $= \frac{1}{4}\pi \times 4 - \frac{1}{2} \times 2\sqrt{2} \times \sqrt{2} = \pi - 2$

$0 \le \arg(z_4) \le \cot^{-1}2$

53. Answer (A)

Hint : Total number of matrices be 9! and values of determinants are of opposite sign.

Solution :

All the determinants can be grouped into two groups each containing $\frac{9!}{2}$ determinants such that corresponding to each determinant in a group there is another determinant in the other group which is obtained by interchanging the two consecutive rows of the determinant in the first group.

$\therefore$ Sum of the values of the determinants is 0.

54. Answer (C)

Hint : First 9 prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23.

Solution :

The maximum value of trace of matrix = 23 + 19 + 17 = 59

The minimum value of trace of matrix = 10

$\therefore$ The required value = 59 – 10 = 49

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