All India Aakash Test Series for JEE (Advanced)-2020

TEST - 1A (Paper-2) - Code-E

Test Date : 13/10/2019

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(A, D)	19.	(A, B, C)	37.	(A, C)
2.	(A, B, C, D)	20.	(A, B)	38.	(A, B, C, D)
3.	(B, D)	21.	(A, B)	39.	(A, C)
4.	(A, D)	22.	(A, C)	40.	(A, B, D)
5.	(A, C, D)	23.	(A, B, C)	41.	(A, C, D)
6.	(A, B, C)	24.	(B, C, D)	42.	(A, B, C, D)
7.	(12)	25.	(32)	43.	(16)
8.	(65)	26.	(80)	44.	(41)
9.	(10)	27.	(03)	45.	(98)
10.	(72)	28.	(23)	46.	(17)
11.	(18)	29.	(90)	47.	(21)
12.	(17)	30.	(35)	48.	(64)
13.	(03)	31.	(25)	49.	(24)
14.	(27)	32.	(33)	50.	(01)
15.	(D)	33.	(A)	51.	(C)
16.	(C)	34.	(D)	52.	(B)
17.	(D)	35.	(D)	53.	(A)
18.	(B)	36.	(B)	54.	(B)

HINTS & SOLUTIONS

PART - I (PHYSICS)

1. Answer (A, D)

Hint : At F = 10 N cone is just about to topple.

Solution :

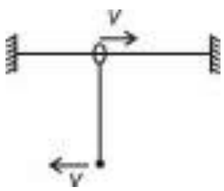
$\Rightarrow$ $N_1 = 0$

$N_2 = 10$ N

2. Answer (A, B, C, D)

Hint : Momentum in horizontal direction and mechanical energy will be conserved.

Solution :



$$mgl = 2\times\frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{gl}$$

$$T_{ms} = (mg) + (m)\left(\frac{4gl}{l}\right)$$

$$= 5mg$$

3. Answer (B, D)

Hint : $\vec{F} = \frac{\overrightarrow{dp}}{dt}$

Solution :

$$t = 2\sqrt{\frac{l}{g}}, \quad F = \left(2\sqrt{gl}\right)\left(\frac{2m}{l}\sqrt{gl}\right) + mg$$

$$F = \left(\frac{m}{l}\right)(4gl) + mg = 5mg$$

$$t = \sqrt{\frac{2\left(\frac{3l}{2}\right)}{g}} = \sqrt{\frac{3l}{g}}, \quad F = \frac{Mg}{2} + \left(\sqrt{2g\frac{3l}{2}}\right)\frac{M}{l}\sqrt{3gl}$$

$$F = \frac{Mg}{2} + \left(\frac{M}{l}\right)(3gl)$$

$$= \left(\frac{7Mg}{2}\right)$$

4. Answer (A, D)

Hint : F_{ST} (Vertical component) = weight of liquid.

Solution :

By geometry

$$\frac{r}{R} = \cos\left(\theta + \frac{\alpha}{2}\right) \quad \Rightarrow \quad R = \frac{r}{\cos 60^\circ} = 2r$$

Now $P_0 - \frac{2S}{R} + h\rho g = P_0$

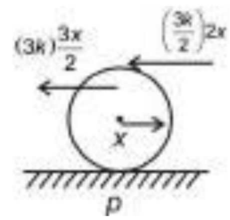
$$\Rightarrow \quad h = \frac{2S}{R\rho g} = \frac{2\times10^{-2}}{2\times10^{-3}\times10^{3}\times10}$$

$$H = 10^{-3}\text{ m} = \left(\frac{1}{10}\text{cm}\right)$$

5. Answer (A, C, D)

Hint : Sphere will perform SHM.

Solution :



$$\tau_p = I_p\alpha$$

$$\left(\frac{9kx}{2}\right)\frac{3R}{2} + (3kx)2R = \left(\frac{7}{5}mp^2\right)\frac{9}{R}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{255k}{28M}} = 1\text{ rad/s}, \; T = 2\pi \text{ s}$$

6. Answer (A, B, C)

Hint : Frictional force has tendency to stop sliding.

Solution :

If $\alpha > 45^\circ$

$\Rightarrow$ $F_1 \sin\alpha > F_2 \cos\alpha$

$\Rightarrow$ Friction will be along Q

7. Answer (12)

Hint : $X = A\cos\left(\frac{2\pi}{T}t\right)$

Solution :

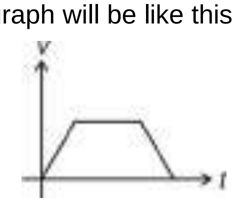
$$X = A\cos\left(\frac{2\pi}{T}t\right)$$

At $t = 2$

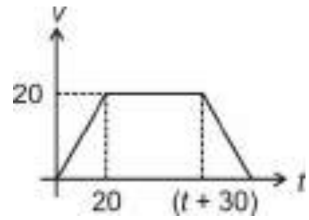
$$X = \frac{A\sqrt{3}}{2}$$

8. Answer (65)

Hint : v - t graph will be like this



Solution :



$A = 1000$

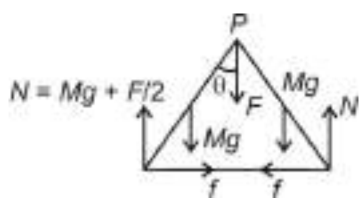
$\Rightarrow$ 200 + 100 + 20 (t) = 1000

$\Rightarrow$ t = 65 s

9. Answer (10)

Hint : $\sum \vec{F} = 0$ and $\vec{\tau} = 0$

Solution :



$\tau_p = 0$

$$\left(Mg + \frac{F}{2}\right)L\,\sin\theta = Mg\,\frac{L}{2}\sin\theta + fL\cos\theta$$

$\Rightarrow$ F = 10 N

10. Answer (72)

Hint : $I = I_1 + I_2$

Solution :

$$I = \left(\frac{1}{6}\right)(2M)64 + \frac{1}{6}(M)16$$

$= 72$

11. Answer (18)

Hint :

$12t - 2a_A - a_0 = 0$

Solution :

$$\Rightarrow \quad a_A = \left(6t - \frac{a_0}{2}\right)$$

$$\int_0^v dv = \int_0^t \left(6t - \frac{a_0}{2}\right) dt$$

$$\Rightarrow \quad v = 3t^2 - \frac{a_0}{2}t$$

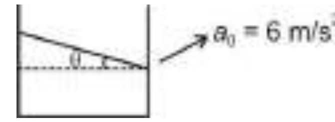
$$\Rightarrow \quad v = 0, \quad \Rightarrow \quad 3t = \frac{a_0}{2}$$

$\Rightarrow$ $a_0 = 6t$

12. Answer (17)

Hint : Net force on free surface is always perpendicular to surface.

Solution :



$$g\sin\theta = a_0\cos(\theta + 37^\circ)$$

$$\tan\theta = \frac{6}{17}$$

13. Answer (03)

Hint : $P_B A = F_{\text{centre}}$

Solution :

$$\int_0^{P_B} (dp)A = \int_{\frac{l}{2}}^{l} \rho A\omega^2 x\,dx$$

$$P = \frac{\rho\omega^2}{2}\left(\frac{3l^2}{4}\right)$$

$$= \frac{1000 \times 100}{2} \times \frac{3}{4}\left(\sqrt{8}\right)$$

$= 3 \times 10^5$ N/m

14. Answer (27)

Hint : $T = \dfrac{F}{2\sin\theta}$

Solution :

$$\frac{T}{A} = \frac{F}{2A\sin\theta}$$

$$\frac{\Delta l}{l} = \frac{\sqrt{d^2 \times l^2} - l}{l}$$

$$\left(1 \times \frac{d^2}{l^2}\right)^{\frac{1}{2}} - 1 = \frac{d^2}{2l^2}$$

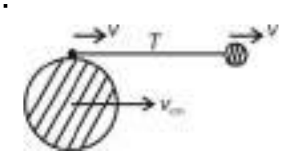
$$\Rightarrow \quad d = l\left(\frac{F}{\pi r^2 Y}\right)^{\frac{1}{3}}$$

$$d = 8100\left(\frac{10^{-2}}{3}\right) = 27 \text{ cm}$$

15. Answer (D)

Hint : Apply conversation of momentum.

Solution :



$Mv_0 = Mv_{cm} + Mv_0$

$\Rightarrow \; v_{cm} + v = v_0$ ...(i)

$|J| = M(v_0 - v)$

$$M(v_0 - v)R = \left(\frac{MR^2}{2}\right)\left(\frac{v - v_{cm}}{R}\right) \quad \text{...(ii)}$$

$\Rightarrow \; 2v_0 - 2v = v - v_{cm}$

$\Rightarrow \; 3v - v_{cm} = 2v_0$ ...(iii)

From (i) and (ii)

$$v = \frac{3v_0}{4} = 3 \text{ m/s}$$

$$v_{cm} = \frac{v_0}{4} = 1 \text{ m/s}$$

$$\omega = \frac{v_0}{2R} = 2 \text{ rad/s}$$

$$J = M\left(v_0 - \frac{3v_0}{4}\right) = \left(\frac{Mv_0}{4}\right)$$

$$= \frac{4 \times 4}{4} = 4$$

16. Answer (C)

Hint : An ideal liquid is incompressible as well as non-viscous.

Solution :

v remains constant in case of incompressible liquid

$\Rightarrow \; \rho = \frac{m}{v} = \text{constant}$

17. Answer (D)

Hint : Use formula of gravitational potential.

Solution :

(A) $V_P = \frac{-2GM}{3R} - \frac{GM}{2R} = -\frac{7}{6}\frac{GM}{R}$

(B) $V = -\frac{GM}{2R^3}\left(3R^2 - \frac{R^2}{4}\right) = -\frac{11}{8}\frac{GM}{R}$

(C) $V = -\frac{GM}{5R}$

18. Answer (B)

Hint : $0 \le f_s \le \mu_s N$ $\quad F = 2$ N

Solution :

$f_1 = f_2 = 2$, $a_1 = a_2 = 0$

at $F = 6$ N

f_1 (required) = 5 N

$\therefore \; a_1 = a_2 = 1 \text{ m/s}^2$

for $F > 6$ N

$a_1 \neq a_2$

PART - II (CHEMISTRY)

19. Answer (A, B, C)

Hint : q + w = ΔU

Both q and w are path functions but ΔU is state function.

Solution :

Free expansion is simultaneously adiabatic as well as isothermal.

20. Answer (A, B)

Hint : The given graph represents negative deviation from Raoults law.

Solution :

When $X_Q = 0$, $X_P = 1$ So $Z = P_p^{\circ}$

Relative volatility can't be predicted on the basis of the given data.

21. Answer (A, B)

Hint : Frequency factor (A) = pZ

where Z is the number of collisions per unit volume per unit time, and p is the probability or steric factor.

Solution :

For Arrhenius theory, p < 1.

Experimentally, p may be less than, greater than or equal to 1.

22. Answer (A, C)

Hint : For endothermic reaction, K increases with temperature.

Solution :

For exothermic reaction, K decreases with temperature.

23. Answer (A, B, C)

Hint : $\left(P + \frac{a}{V^2}\right)(V - b) = RT$

Solution :

When a = 0,

PV + Pb = RT Straight line

When b = 0,

$$\left(P + \frac{a}{V^2}\right)(V) = RT$$

$\Rightarrow \; PV + \frac{a}{V} = RT$ Straight line

When a = b = 0

PV = RT

So PV versus P is a straight line parallel to pressure axis.

24. Answer (B, C, D)

Hint : Conservation of mass and charge.

Solution :

R is proton

S is neutron

T is positron

25. Answer (32)

Hint : Degeneracy of n^{th} shell in H atom is equal to n^2. So, Y = 16

Solution :

In 4^{th} shell $\rightarrow$ 4*s*, 4*p*, 4*d* and 4*f*

$\therefore$ Number of e^- with $m_s = +\frac{1}{2}$ = 16

26. Answer (80)

Hint : $V_{solvent} = V_{solution}$

Solution :

Let 1 L solution be taken

So, moles of solute = 3.2

and mass of solvent = 1000d

$\therefore \quad m = \frac{3.2 \times 1000}{1000d} = 4$

$\Rightarrow \quad \frac{3.2}{d} = 4$

$\Rightarrow \quad d = 0.8$

$\therefore \quad 100d = 80$

27. Answer (03)

Hint :

(1) $\rightarrow$ H_2

(2) $\rightarrow$ He

(3) $\rightarrow$ N_2

(4) $\rightarrow$ CO_2

Solution :

x = 1

y = 3

xy = 3

28. Answer (23)

Hint : Density = $\frac{\text{Mass}}{\text{Volume}}$

Solution :

$12 = \frac{2 \times (x)}{N_0 64 \times 10^{-24}}$

where *x* = atomic mass

$\Rightarrow \quad x = \frac{12 \times N_0 \times 64 \times 10^{-24}}{2}$

= 230.4 gm

So, $\frac{x}{10} = 23.04$

29. Answer (90)

Hint : $\frac{dE}{dT} = \frac{\Delta S}{nF}$

$\Rightarrow \quad \Delta S = -2 \times 10^{-6} \times 2 \times 96500$

= –0.386 J/mol K

Solution :

$\Delta G = \Delta H - T\Delta S$

–2(96500) (1.4) = ΔH – 300 (–0.386)

–270200 = ΔH + 115.8

ΔH = –270315.8 J/mol

ΔH = –270.32 kJ/mol

$\therefore \quad$ X = 270.32

30. Answer (35)

Hint : PV^4 = Constant

TV^3 = Constant

Solution :

$C = C_v - \frac{R}{n-1}$

$= \frac{3R}{2} - \frac{R}{3}$

$= \frac{7R}{6}$

$\therefore \quad X = \frac{7}{6}$

30X = 35

31. Answer (25)

Hint :

$$\underset{1-x-z}{P(g)} \rightleftharpoons \underset{x-y}{Q(g)} + \underset{x+y+z}{R(g)}$$

$$\underset{(x-y)}{Q(g)} + \underset{(1-y+z)}{S(g)} \rightleftharpoons \underset{(x+y+z)}{R(g)} + \underset{y-z}{T(g)}$$

$$\underset{(1-x-z)}{P(g)} + \underset{(y-z)}{T(g)} \rightleftharpoons \underset{(x+y+z)}{R(g)} + \underset{(1-y+z)}{S(g)}$$

Solution :

At equilibrium,

$x + y + z = 1$ …(i)

$x - y = y - z$ …(ii)

$1 - y + z = \frac{5}{6}$ …(iii)

From (iii), $y - z = \frac{1}{6} \Rightarrow y = z + \frac{1}{6}$

From (ii), $x = 2y - z \Rightarrow x = z + \frac{1}{3}$

From (i), $z + \frac{1}{3} + z + \frac{1}{6} + z = 1$

$\Rightarrow \quad z + \frac{1}{2} = 1 \Rightarrow z = \frac{1}{6}$

$\therefore \quad x = z + \frac{1}{3} = \frac{1}{6} + \frac{1}{3} = \frac{1}{2}$

$\therefore \quad y = \frac{1}{3}$

$$\therefore \left.\begin{matrix}[S] = 1 - \frac{1}{3} + \frac{1}{6} = \frac{5}{6} \\ [P] = 1 - \frac{1}{2} - \frac{1}{6} = \frac{1}{3}\end{matrix}\right\} [S] = \frac{5}{2}[P]$$

32. Answer (33)

Hint : P : PF_5

Q : SF_6

Solution :

a = 15

b = 18

33. Answer (A)

Hint : $HCN \rightleftharpoons H^+ + CN^-$

Adding NH_3 shifts equilibrium to the right

Solution :

$$\begin{array}{ccccc} ZnS & \rightleftharpoons & Zn^{2+} & + & S^{2-} \\ & & \downarrow OH^- & & \downarrow H^+ \\ & & Zn(OH)_2 & & H_2S \end{array}$$

On adding H^+, S^{2-} gets consumed

On adding OH^-, Zn^{2+} gets consumed

$NH_3 + H_2O \rightleftharpoons NH_4^+ + OH^-$

$NaH_2PO_4 \rightleftharpoons Na_2HPO_4^- + H^+$

Increasing pH shifts equilibrium to the right

$H_2O + Na_2CO_3 \rightleftharpoons NaHCO_3 + NaOH$

Decreasing pH shifts equilibrium to the right.

34. Answer (D)

Hint : Equivalent mass $= \frac{\text{Molar mass}}{\text{n-factor}}$

Solution :

(P) M = 376, x = 6 (R) M = 142, x = 2

(Q) M = 144, x = 3 (S) M = 112, x = 8

35. Answer (D)

Hint :

N_2 : HOMO is $\sigma 2p_z$, LUMO is $\pi^* 2p_x = \pi^* 2p_y$

O_2^- : HOMO is $\pi^* 2p_x = \pi^* 2p_y$, LUMO is $\sigma^* 2p_z$

Solution :

C_2 : HOMO is $\pi 2p_x = \pi 2p_y$, LUMO is $\sigma 2p_z$

Be_2^+ : HOMO is σ^*_{2s}, LUMO is $\pi 2p_x = \pi 2p_y$

36. Answer (B)

Hint : CH_3COOH + NaOH : Addition of WA to SB. Conductivity decreases and then does not change much.

NaOH + CH_3COOH : Addition of SB to WA. Conductivity increases till neutralisation and then increases at a much faster rate.

Solution :

CH_3NH_2 + CH_3COOH : Addition of WB to WA. Conductivity increases due to neutralisation and then does not change much.

PART - III (MATHEMATICS)

37. Answer (A, C)

Hint : Think $f(x)$.

Solution :

$\because \quad f(x) = 1 + x + x^2 + \ldots.. x^n$

$\therefore \quad f'(x) \cdot g(x) = (1 + 2x + 3x^2 + \ldots. + nx^{n-1})$

$\left(1 - \frac{2}{x} + \frac{3}{x^2} \ldots.. + (-1)^n \frac{n+1}{x^n}\right)$

$\therefore$ The constant term

$= 1 - 2^2 + 3^2 - 4^2 + \ldots. (-1)^{n-1} n^2$

$\therefore$ Option (A) and (C) are correct according $n \in$ odd and even

38. Answer (A, B, C, D)

Hint : Assume 3 numbers in G.P.

Solution :

$\because \quad z + 3x > 4y \quad \Rightarrow nr^2 + 3x > 4rx$

$\Rightarrow \quad r^2 - 4r + 3 > 0 \quad \Rightarrow r \in (-\infty, 1) - \{0\} \cup (3, \infty)$

39. Answer (A, C)

Hint : Sum of coefficient zero.

Solution :

$\alpha = 1$ and $\beta = \frac{a-b}{b-c}$ all roots of first equation

$y = 1$ and $\delta = \frac{c(a-b)}{a(b-c)}$ are roots of second equation.

If both roots are common then $\beta = \delta$

$\Rightarrow$ $a = c$ not possible

$\therefore$ (A) and (C) are only correct options

40. Answer (A, B, D)

Hint : Property of modulus.

Solution :

$$|2z_1 + z_2| \le |2z_1| + |z_2| \le 2 + 2 \le 4$$

$|z_1 + z_2|$ is least when O, z_1, z_2 are collinear.

$\therefore$ $|z_1 + z_2| = 1$

$$\left|z_2 + \frac{1}{z_1}\right| \le |z_2| + \frac{1}{|z_1|} \le 2 + 1 \le 3$$

41. Answer (A, C, D)

Hint : Fundamental principle of multiplication.

Solution :

$\because$ All digits are distinct

$\therefore$ Number of matrices formed = $\underline{|9}$

$\because$ Corresponding to a value of determinant the another determinant will have a negative value

$\therefore$ $\sum_{i=2}^{K} \det(\Delta_i) = 0$

42. Answer (A, B, C, D)

Hint : Reducible to quadratic.

Solution :

$$\left(\frac{2x^2}{x^2-1}\right)^2 - \frac{2x^2}{x^2-1} - m(m-1) = 0$$

Let $t = \dfrac{2x^2}{x^2-1}$

$\Rightarrow$ $t^2 - t - m(m-1) = 0$

$\Rightarrow$ $t = m$, $1 - m$ are two roots

$\Rightarrow$ $\dfrac{2x^2}{x^2-1} = m$ or $\dfrac{2x^2}{x^2-1} = 1 - m$

$2x^2 = mx^2 - m$ or $2x^2 = x^2 - mx^2 - 1 + m$

$\Rightarrow$ $m = x^2(m-2)$ $(m+1)x^2 = m - 1$

$\Rightarrow$ $x = \pm\sqrt{\dfrac{m}{m-2}}$ $\Rightarrow$ $x = \pm\sqrt{\dfrac{m-1}{m+1}}$

Now verify all the options.

43. Answer (16)

Hint : Apply property of determinants.

Solution :

By $R_2 \to R_2 - 2R_1$

and $R_3 \to R_3 - 3R_1$

We get, $u^3 = 64$ $\Rightarrow$ $u = 4$

$\therefore$ $\boxed{u^2 = 16}$

44. Answer (41)

Hint : Take submission out of integration.

Solution :

Let $\boxed{n = 40}$

$$\sum (-1)^{r-1}\, {}^nC_r \int_0^1 \left(1 + x + x^2 + \ldots + x^{r-1}\right) dx$$

$$= \sum (-1)^{r-1}\, {}^nC_r \int_0^1 \left(\frac{1-x^r}{1-x}\right) dx$$

$$= \int_0^1 \frac{\sum_{r=1}^{n}\left((-1)^{r-1}\, {}^nC_r - (-1)^{r-1}\, {}^nC_r\, x^r\right) dx}{(1-x)}$$

$$= \int_0^1 \frac{\left(-1 + 1 + (1-x)^n\right)}{1-x} dx$$

$$= \int_0^1 (1-x)^{n-1} dx = \frac{1}{n}$$

$\therefore$ $\boxed{\dfrac{1}{n} = \dfrac{1}{40}}$

45. Answer (98)

Hint : Use mathematical induction approach.

Solution :

If $A^2 = O$ the 'n' must be multiple of 3.

$\therefore$ largest value of 'n' is $\boxed{98}$

46. Answer (17)

Hint : Use characteristic equation.

Solution :

$\because$ $A^3 + mA^2 + nA - 6I = 0$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & -11 & 19 \\ 0 & -38 & 46 \end{bmatrix} + m\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 5 \\ 0 & -10 & 14 \end{bmatrix} +$$

$$n\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix} - 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$\therefore$ $1 + m + n - 6 = 0$ and $-11 - m + n - 6 = 0$

$\Rightarrow$ $n - m = 17$

47. Answer (21)

Hint : Product of two matrices *A* and *B*.

Solution :

$$AB = \begin{bmatrix} 3um^2 & 3m^2v & 3wm^2 \\ u & v & w \\ 6mu & 6mv & 6mw \end{bmatrix}$$

$\therefore$ $3mu^2 + v + 6wm = (m + 2)^2 + 2m + 5m^2$

$\Rightarrow$ $3um^2 + 6wm + v = 6m^2 + 6m + 4$

$\Rightarrow$ $u = 2; v = 4; w = 1$

$\Rightarrow$ $u^2 + v^2 + w^2 = 4 + 16 + 1 = \boxed{21}$

48. Answer (64)

Hint : Property of determinants.

Solution :

$$|B| = \begin{vmatrix} 2^2\, a_{11} & 2^3\, a_{12} & 2^4\, a_{13} \\ 2^3\, a_{21} & 2^4\, a_{22} & 2^5\, a_{23} \\ 2^4\, a_{31} & 2^5\, a_{32} & 2^6\, a_{33} \end{vmatrix}$$

$$= 2^2 \cdot 2^3 \cdot 2^4 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 2a_{21} & 2a_{22} & 2a_{23} \\ 2^2a_{31} & 2^2a_{32} & 2^2a_{33} \end{vmatrix}$$

$|B| = 2^{12}\,|A|$ $\quad\Rightarrow\quad$ $|B| = 64$

49. Answer (24)

Hint : Definition of idempotent matrix.

Solution :

$\because$ $A^2 = A$ $\quad\Rightarrow A = I$ ($\because$ A is non singular)

$\therefore$ $a = \pm 2; b = \pm 3; c = \pm 4; p = q = r = 0$

$\Rightarrow$ $N = |0 - (\pm 24)|$

$N = 24$

50. Answer (01)

Hint : Use binomial inside binomial.

Solution :

Coefficient of x^{100} in

$$\left({}^{100}C_0\,(1+x)^{200} - {}^{100}C_1\,(1+x)^{199} + ...\right)$$

Coefficient of x^{100} in $(1 + x)^{100} \cdot x^{100} = 1$

51. Answer (C)

Hint : Gap method.

Solution :

$n_1 = {}^7C_5 \times \lfloor 6 \times \lfloor 5 \quad n_2 = (\lfloor 5)^2 \times {}^6C_1$

$n_3 = \lfloor 6 \times \lfloor 5$; $\quad \boxed{n_4 = 0}$

52. Answer (B)

Hint : A.M $\geq$ G.M.

Solution :

$\because$ All the roots are positive

$$\therefore \quad \frac{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6}{6} = z = \text{A.M.}$$

Also $(\alpha_1\, \alpha_2\, \alpha_3\, \alpha_4\, \alpha_5\, \alpha_6)^{\frac{1}{6}} = z = \text{G.M.}$

$\therefore$ All the roots are equal and are 2

$\therefore$ $f(x) = (x - 2)^6$

Now verify each of the values.

53. Answer (A)

Hint : Theory of equation.

Solution :

$$\frac{1-\tan\theta}{1+\tan\theta} = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$$

$\Rightarrow$ $\tan^4\theta + 4\tan^3\theta - 6\tan^2\theta - 4\tan\theta + 1 = 0$

$\therefore$ $\sum \tan\alpha = -4$

$\sum (\tan\alpha\, \tan\beta) = -6$

$\sum (\tan\alpha\, \tan\beta\, \tan\gamma) = 4$

$\pi(\tan\alpha) = 1$

54. Answer (B)

Hint : Assumption of odd consecutive integer.

Solution :

Let odd integer are $2m + 1$, $2m + 3$, …. let number be n

$\therefore$ $57^2 - 13^2 = n(2m + n)$

$(m + n)^2 - m^2 = 57^2 - 13^2$

$\Rightarrow$ $m = 13; n + m = 57$

$\therefore$ $n = 44$

$\therefore$ The least integer = 26 + 1 = 27

Largest integer = 113

Number of factors of type $(4n + 1)$ for 113 is 1

Sum of least + largest integer = $140 = 5 \times 7 \times 2^2$

$\therefore$ Number of factors $2 \times 2 \times 3 = 12$

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All India Aakash Test Series for JEE (Advanced)-2020

TEST - 1A (Paper-2) - Code-F

Test Date : 13/10/2019

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(A, B, C)	19.	(B, C, D)	37.	(A, B, C, D)
2.	(A, C, D)	20.	(A, B, C)	38.	(A, C, D)
3.	(A, D)	21.	(A, C)	39.	(A, B, D)
4.	(B, D)	22.	(A, B)	40.	(A, C)
5.	(A, B, C, D)	23.	(A, B)	41.	(A, B, C, D)
6.	(A, D)	24.	(A, B, C)	42.	(A, C)
7.	(27)	25.	(33)	43.	(01)
8.	(03)	26.	(25)	44.	(24)
9.	(17)	27.	(35)	45.	(64)
10.	(18)	28.	(90)	46.	(21)
11.	(72)	29.	(23)	47.	(17)
12.	(10)	30.	(03)	48.	(98)
13.	(65)	31.	(80)	49.	(41)
14.	(12)	32.	(32)	50.	(16)
15.	(B)	33.	(B)	51.	(B)
16.	(D)	34.	(D)	52.	(A)
17.	(C)	35.	(D)	53.	(B)
18.	(D)	36.	(A)	54.	(C)

HINTS & SOLUTIONS

PART - I (PHYSICS)

1. Answer (A, B, C)

Hint : Frictional force has tendency to stop sliding.

Solution :

If $\alpha > 45°$

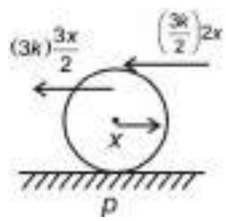
$\Rightarrow$ $F_1 \sin\alpha > F_2 \cos\alpha$

$\Rightarrow$ Friction will be along Q

2. Answer (A, C, D)

Hint : Sphere will perform SHM.

Solution :



$$\tau_p = I_p\alpha$$

$$\left(\frac{9kx}{2}\right)\frac{3R}{2} + (3kx)2R = \left(\frac{7}{5}mp^2\right)\frac{9}{R}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{255k}{28M}} = 1\,\text{rad/s},\ T = 2\pi \text{ s}$$

3. Answer (A, D)

Hint : F_{ST} (Vertical component) = weight of liquid.

Solution :

By geometry

$$\frac{r}{R} = \cos\left(\theta + \frac{\alpha}{2}\right) \quad \Rightarrow \quad R = \frac{r}{\cos 60°} = 2r$$

Now $P_0 - \dfrac{2S}{R} + h\rho g = P_0$

$$\Rightarrow \quad h = \frac{2S}{R\rho g} = \frac{2\times10^{-2}}{2\times10^{-3}\times10^{3}\times10}$$

$$H = 10^{-3}\text{ m} = \left(\frac{1}{10}\text{cm}\right)$$

4. Answer (B, D)

Hint : $\vec{F} = \dfrac{\overrightarrow{dp}}{dt}$

Solution :

$$t = 2\sqrt{\frac{l}{g}}, \quad F = \left(2\sqrt{gl}\right)\left(\frac{2m}{l}\sqrt{gl}\right) + mg$$

$$F = \left(\frac{m}{l}\right)(4gl) + mg = 5mg$$

$$t = \sqrt{\frac{2\left(\frac{3l}{2}\right)}{g}} = \sqrt{\frac{3l}{g}}, \quad F = \frac{Mg}{2} + \left(\sqrt{2g\frac{3l}{2}}\right)\frac{M}{l}\sqrt{3gl}$$

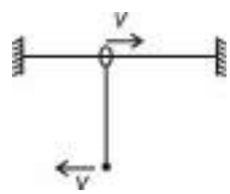
$$F = \frac{Mg}{2} + \left(\frac{M}{l}\right)(3gl)$$

$$= \left(\frac{7Mg}{2}\right)$$

5. Answer (A, B, C, D)

Hint : Momentum in horizontal direction and mechanical energy will be conserved.

Solution :



$$mgl = 2\times\frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{gl}$$

$$T_{ms} = (mg) + (m)\left(\frac{4gl}{l}\right)$$

$$= 5mg$$

6. Answer (A, D)

Hint : At F = 10 N cone is just about to topple.

Solution :

$\Rightarrow$ $N_1 = 0$

$N_2 = 10$ N

7. Answer (27)

Hint : $T = \dfrac{F}{2\sin\theta}$

Solution :

$$\frac{T}{A} = \frac{F}{2A\sin\theta}$$

$$\frac{\Delta l}{l} = \frac{\sqrt{d^2\times l^2} - l}{l}$$

$$\left(1\times\frac{d^2}{l^2}\right)^{\frac{1}{2}} - 1 = \frac{d^2}{2l^2}$$

$$\Rightarrow \quad d = l\left(\frac{F}{\pi r^2 Y}\right)^{\frac{1}{3}}$$

$$d = 8100\left(\frac{10^{-2}}{3}\right) = 27\text{ cm}$$

8. Answer (03)

Hint : $P_B A = F_{\text{centre}}$

Solution :

$$\int_0^{P_B}(dp)A = \int_{\frac{l}{2}}^{l}\rho A\omega^2 x dx$$

$$P = \frac{\rho\omega^2}{2}\left(\frac{3l^2}{4}\right)$$

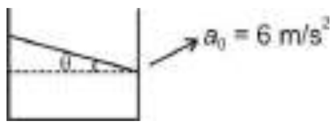
$$= \frac{1000\times 100}{2}\times\frac{3}{4}\left(\sqrt{8}\right)$$

$= 3 \times 10^5$ N/m

9. Answer (17)

Hint : Net force on free surface is always perpendicular to surface.

Solution :



$$g\sin\theta = a_0\cos(\theta + 37°)$$

$$\tan\theta = \frac{6}{17}$$

10. Answer (18)

Hint :

$12t - 2a_A - a_0 = 0$

Solution :

$$\Rightarrow \quad a_A = \left(6t - \frac{a_0}{2}\right)$$

$$\int_0^v dv = \int_0^t\left(6t - \frac{a_0}{2}\right)dt$$

$$\Rightarrow \quad v = 3t^2 - \frac{a_0}{2}t$$

$$\Rightarrow \quad v = 0, \qquad \Rightarrow \quad 3t = \frac{a_0}{2}$$

$\Rightarrow$ $a_0 = 6t$

11. Answer (72)

Hint : $I = I_1 + I_2$

Solution :

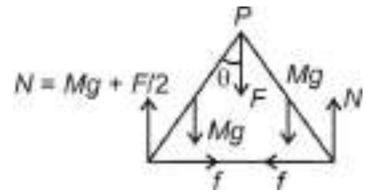
$$I = \left(\frac{1}{6}\right)(2M)64 + \frac{1}{6}(M)16$$

$= 72$

12. Answer (10)

Hint : $\sum\vec{F} = 0$ and $\vec{\tau} = 0$

Solution :



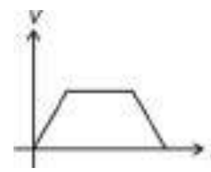
$$\tau_p = 0$$

$$\left(Mg + \frac{F}{2}\right)L\sin\theta = Mg\frac{L}{2}\sin\theta + fL\cos\theta$$

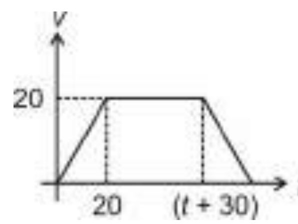
$\Rightarrow$ $F = 10$ N

13. Answer (65)

Hint : v - t graph will be like this



Solution :



$A = 1000$

$\Rightarrow$ $200 + 100 + 20(t) = 1000$

$\Rightarrow$ $t = 65$ s

14. Answer (12)

Hint : $X = A\cos\left(\frac{2\pi}{T}t\right)$

Solution :

$$X = A\cos\left(\frac{2\pi}{T}t\right)$$

At $t = 2$

$$X = \frac{A\sqrt{3}}{2}$$

15. Answer (B)

Hint : $0 \le f_s \le \mu_s N$ $\quad F = 2$ N

Solution :

$f_1 = f_2 = 2$, $a_1 = a_2 = 0$

at $F = 6$ N

f_1 (required) = 5 N

$\therefore$ $a_1 = a_2 = 1$ m/s^2

for $F > 6$ N

$a_1 \ne a_2$

16. Answer (D)

Hint : Use formula of gravitational potential.

Solution :

(A) $V_P = \frac{-2GM}{3R} - \frac{GM}{2R} = -\frac{7}{6}\frac{GM}{R}$

(B) $V = -\frac{GM}{2R^3}\left(3R^2 - \frac{R^2}{4}\right) = -\frac{11}{8}\frac{GM}{R}$

(C) $V = -\frac{GM}{5R}$

17. Answer (C)

Hint : An ideal liquid is incompressible as well as non-viscous.

Solution :

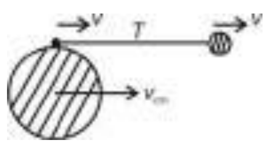
v remains constant in case of incompressible liquid

$\Rightarrow \quad \rho = \frac{m}{v} = \text{constant}$

18. Answer (D)

Hint : Apply conversation of momentum.

Solution :



$Mv_0 = Mv_{cm} + Mv_0$

$\Rightarrow \quad v_{cm} + v = v_0$ …(i)

$|J| = M(v_0 - v)$

$M(v_0 - v)R = \left(\frac{MR^2}{2}\right)\left(\frac{v - v_{cm}}{R}\right)$ …(ii)

$\Rightarrow \quad 2v_0 - 2v = v - v_{cm}$

$\Rightarrow \quad 3v - v_{cm} = 2v_0$ …(iii)

From (i) and (ii)

$v = \frac{3v_0}{4} = 3 \text{ m/s}$

$v_{cm} = \frac{v_0}{4} = 1 \text{ m/s}$

$\omega = \frac{v_0}{2R} = 2 \text{ rad/s}$

$J = M\left(v_0 - \frac{3v_0}{4}\right) = \left(\frac{Mv_0}{4}\right)$

$= \frac{4 \times 4}{4} = 4$

PART - II (CHEMISTRY)

19. Answer (B, C, D)

Hint : Conservation of mass and charge.

Solution :

R is proton

S is neutron

T is positron

20. Answer (A, B, C)

Hint : $\left(P + \frac{a}{V^2}\right)(V - b) = RT$

Solution :

When a = 0,

PV + Pb = RT Straight line

When b = 0,

$\left(P + \frac{a}{V^2}\right)(V) = RT$

$\Rightarrow \quad PV + \frac{a}{V} = RT$ Straight line

When a = b = 0

PV = RT

So PV versus P is a straight line parallel to pressure axis.

21. Answer (A, C)

Hint : For endothermic reaction, K increases with temperature.

Solution :

For exothermic reaction, K decreases with temperature.

22. Answer (A, B)

Hint : Frequency factor (A) = pZ

where Z is the number of collisions per unit volume per unit time, and p is the probability or steric factor.

Solution :

For Arrhenius theory, p < 1.

Experimentally, p may be less than, greater than or equal to 1.

23. Answer (A, B)

Hint : The given graph represents negative deviation from Raoults law.

Solution :

When $X_Q = 0$, $X_P = 1$ So $Z = P_p^{\circ}$

Relative volatility can't be predicted on the basis of the given data.

24. Answer (A, B, C)

Hint : q + w = ΔU

Both q and w are path functions but ΔU is state function.

Solution :

Free expansion is simultaneously adiabatic as well as isothermal.

25. Answer (33)

Hint : P : PF_5

Q : SF_6

Solution :

a = 15

b = 18

26. Answer (25)

Hint :

$$\begin{array}{ccccc} P(g) & \rightleftharpoons & Q(g) & + & R(g) \\ 1-x-z & & x-y & & x+y+z \end{array}$$

$$\begin{array}{ccccccc} Q(g) & + & S(g) & \rightleftharpoons & R(g) & + & T(g) \\ (x-y) & & (1-y+z) & & (x+y+z) & & y-z \end{array}$$

$$\begin{array}{ccccccc} P(g) & + & T(g) & \rightleftharpoons & R(g) & + & S(g) \\ (1-x-z) & & (y-z) & & (x+y+z) & & (1-y+z) \end{array}$$

Solution :

At equilibrium,

$x + y + z = 1$ …(i)

$x - y = y - z$ …(ii)

$1 - y + z = \frac{5}{6}$ …(iii)

From (iii), $y - z = \frac{1}{6} \Rightarrow y = z + \frac{1}{6}$

From (ii), $x = 2y - z \Rightarrow x = z + \frac{1}{3}$

From (i), $z + \frac{1}{3} + z + \frac{1}{6} + z = 1$

$\Rightarrow \quad z + \frac{1}{2} = 1 \Rightarrow z = \frac{1}{6}$

$\therefore \quad x = z + \frac{1}{3} = \frac{1}{6} + \frac{1}{3} = \frac{1}{2}$

$\therefore \quad y = \frac{1}{3}$

$$\therefore \quad \left.\begin{array}{l} [S] = 1 - \frac{1}{3} + \frac{1}{6} = \frac{5}{6} \\ [P] = 1 - \frac{1}{2} - \frac{1}{6} = \frac{1}{3} \end{array}\right\} [S] = \frac{5}{2}[P]$$

27. Answer (35)

Hint : PV^4 = Constant

TV^3 = Constant

Solution :

$$C = C_v - \frac{R}{n-1}$$

$$= \frac{3R}{2} - \frac{R}{3}$$

$$= \frac{7R}{6}$$

$\therefore \quad X = \frac{7}{6}$

$30X = 35$

28. Answer (90)

Hint : $\frac{dE}{dT} = \frac{\Delta S}{nF}$

$\Rightarrow \quad \Delta S = -2 \times 10^{-6} \times 2 \times 96500$

$= -0.386$ J/mol K

Solution :

$\Delta G = \Delta H - T\Delta S$

$-2(96500)\ (1.4) = \Delta H - 300\ (-0.386)$

$-270200 = \Delta H + 115.8$

$\Delta H = -270315.8$ J/mol

$\Delta H = -270.32$ kJ/mol

$\therefore \quad X = 270.32$

29. Answer (23)

Hint : Density = $\frac{\text{Mass}}{\text{Volume}}$

Solution :

$$12 = \frac{2 \times (x)}{N_0 64 \times 10^{-24}}$$

where x = atomic mass

$$\Rightarrow \quad x = \frac{12 \times N_0 \times 64 \times 10^{-24}}{2}$$

= 230.4 gm

So, $\frac{x}{10} = 23.04$

30. Answer (03)

Hint :

(1) $\rightarrow$ H_2

(2) $\rightarrow$ He

(3) $\rightarrow$ N_2

(4) $\rightarrow$ CO_2

Solution :

$x = 1$

$y = 3$

$xy = 3$

31. Answer (80)

Hint : $V_{solvent} = V_{solution}$

Solution :

Let 1 L solution be taken

So, moles of solute = 3.2

and mass of solvent = 1000d

$$\therefore \quad m = \frac{3.2 \times 1000}{1000d} = 4$$

$$\Rightarrow \quad \frac{3.2}{d} = 4$$

$$\Rightarrow \quad d = 0.8$$

$$\therefore \quad 100d = 80$$

32. Answer (32)

Hint : Degeneracy of n^{th} shell in H atom is equal to n^2. So, Y = 16

Solution :

In 4^{th} shell $\rightarrow$ 4*s*, 4*p*, 4*d* and 4*f*

$\therefore$ Number of e^- with $m_s = +\frac{1}{2}$ = 16

33. Answer (B)

Hint : CH_3COOH + NaOH : Addition of WA to SB. Conductivity decreases and then does not change much.

NaOH + CH_3COOH : Addition of SB to WA. Conductivity increases till neutralisation and then increases at a much faster rate.

Solution :

CH_3NH_2 + CH_3COOH : Addition of WB to WA. Conductivity increases due to neutralisation and then does not change much.

34. Answer (D)

Hint :

N_2 : HOMO is $\sigma 2p_z$, LUMO is $\pi^* 2p_x = \pi^* 2p_y$

O_2^- : HOMO is $\pi^* 2p_x = \pi^* 2p_y$, LUMO is $\sigma^* 2p_z$

Solution :

C_2 : HOMO is $\pi 2p_x = \pi 2p_y$, LUMO is $\sigma 2p_z$

Be_2^+ : HOMO is σ^*_{2s}, LUMO is $\pi 2p_x = \pi 2p_y$

35. Answer (D)

Hint : Equivalent mass $= \frac{\text{Molar mass}}{\text{n-factor}}$

Solution :

(P) M = 376, x = 6 (R) M = 142, x = 2

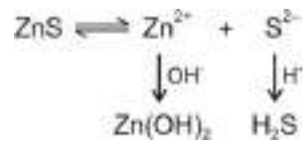
(Q) M = 144, x = 3 (S) M = 112, x = 8

36. Answer (A)

Hint : $HCN \rightleftharpoons H^+ + CN^-$

Adding NH_3 shifts equilibrium to the right

Solution :



On adding H^+, S^{2-} gets consumed

On adding OH^-, Zn^{2+} gets consumed

$$NH_3 + H_2O \rightleftharpoons NH_4^+ + OH^-$$

$$NaH_2PO_4 \rightleftharpoons Na_2HPO_4^- + H^+$$

Increasing pH shifts equilibrium to the right

$$H_2O + Na_2CO_3 \rightleftharpoons NaHCO_3 + NaOH$$

Decreasing pH shifts equilibrium to the right.

PART - III (MATHEMATICS)

37. Answer (A, B, C, D)

Hint : Reducible to quadratic.

Solution :

$$\left(\frac{2x^2}{x^2-1}\right)^2 - \frac{2x^2}{x^2-1} - m(m-1) = 0$$

Let $t = \frac{2x^2}{x^2-1}$

$\Rightarrow$ $t^2 - t - m(m-1) = 0$

$\Rightarrow$ $t = m$, $1 - m$ are two roots

$$\Rightarrow \quad \frac{2x^2}{x^2-1} = m \quad \text{or} \quad \frac{2x^2}{x^2-1} = 1-m$$

$$2x^2 = mx^2 - m \quad \text{or} \quad 2x^2 = x^2 - mx^2 - 1 + m$$

$$\Rightarrow \quad m = x^2(m-2) \qquad (m+1)x^2 = m-1$$

$$\Rightarrow \quad x = \pm\sqrt{\frac{m}{m-2}} \qquad \Rightarrow \quad x = \pm\sqrt{\frac{m-1}{m+1}}$$

Now verify all the options.

38. Answer (A, C, D)

Hint : Fundamental principle of multiplication.

Solution :

$\because$ All digits are distinct

$\therefore$ Number of matrices formed = $\underline{|9}$

$\because$ Corresponding to a value of determinant the another determinant will have a negative value

$\therefore$ $\sum_{i=2}^{K} \det(\Delta_i) = 0$

39. Answer (A, B, D)

Hint : Property of modulus.

Solution :

$|2z_1 + z_2| \le |2z_1| + |z_2| \le 2 + 2 \le 4$

$|z_1 + z_2|$ is least when O, z_1, z_2 are collinear.

$\therefore$ $|z_1 + z_2| = 1$

$$\left|z_2 + \frac{1}{z_1}\right| \le |z_2| + \frac{1}{|z_1|} \le 2 + 1 \le 3$$

40. Answer (A, C)

Hint : Sum of coefficient zero.

Solution :

$\alpha = 1$ and $\beta = \dfrac{a-b}{b-c}$ all roots of first equation

$y = 1$ and $\delta = \dfrac{c(a-b)}{a(b-c)}$ are roots of second equation.

If both roots are common then $\beta = \delta$

$\Rightarrow$ $a = c$ not possible

$\therefore$ (A) and (C) are only correct options

41. Answer (A, B, C, D)

Hint : Assume 3 numbers in G.P.

Solution :

$\because$ $z + 3x > 4y$ $\Rightarrow nr^2 + 3x > 4rx$

$\Rightarrow$ $r^2 - 4r + 3 > 0$ $\Rightarrow r \in (-\infty, 1) - \{0\} \cup (3, \infty)$

42. Answer (A, C)

Hint : Think $f(x)$.

Solution :

$\because$ $f(x) = 1 + x + x^2 + \ldots.. x^n$

$\therefore$ $f'(x) \cdot g(x) = (1 + 2x + 3x^2 + \ldots. + nx^{n-1})$ $\left(1 - \dfrac{2}{x} + \dfrac{3}{x^2} \ldots.. + (-1)^n \dfrac{n+1}{x^n}\right)$

$\therefore$ The constant term

$= 1 - 2^2 + 3^2 - 4^2 + \ldots. (-1)^{n-1} n^2$

$\therefore$ Option (A) and (C) are correct according $n \in$ odd and even

43. Answer (01)

Hint : Use binomial inside binomial.

Solution :

Coefficient of x^{100} in

$$\left({}^{100}C_0 (1+x)^{200} - {}^{100}C_1 (1+x)^{199} + \ldots..\right)$$

Coefficient of x^{100} in $(1 + x)^{100} \cdot x^{100} = 1$

44. Answer (24)

Hint : Definition of idempotent matrix.

Solution :

$\because$ $A^2 = A$ $\Rightarrow A = I$ ($\because$ A is non singular)

$\therefore$ $a = \pm 2$; $b = \pm 3$; $c = \pm 4$; $p = q = r = 0$

$\Rightarrow$ $N = |0 - (\pm 24)|$

$N = 24$

45. Answer (64)

Hint : Property of determinants.

Solution :

$$|B| = \begin{vmatrix} 2^2 a_{11} & 2^3 a_{12} & 2^4 a_{13} \\ 2^3 a_{21} & 2^4 a_{22} & 2^5 a_{23} \\ 2^4 a_{31} & 2^5 a_{32} & 2^6 a_{33} \end{vmatrix}$$

$$= 2^2 \cdot 2^3 \cdot 2^4 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 2a_{21} & 2a_{22} & 2a_{23} \\ 2^2 a_{31} & 2^2 a_{32} & 2^2 a_{33} \end{vmatrix}$$

$|B| = 2^{12} |A|$ $\Rightarrow$ $|B| = 64$

46. Answer (21)

Hint : Product of two matrices A and B.

Solution :

$$AB = \begin{bmatrix} 3um^2 & 3m^2v & 3wm^2 \\ u & v & w \\ 6mu & 6mv & 6mw \end{bmatrix}$$

$\therefore$ $3mu^2 + v + 6wm = (m + 2)^2 + 2m + 5m^2$

$\Rightarrow$ $3um^2 + 6wm + v = 6m^2 + 6m + 4$

$\Rightarrow$ $u = 2$; $v = 4$; $w = 1$

$\Rightarrow$ $u^2 + v^2 + w^2 = 4 + 16 + 1 = \boxed{21}$

47. Answer (17)

Hint : Use characteristic equation.

Solution :

$\because \quad A^3 + mA^2 + nA - 6I = 0$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & -11 & 19 \\ 0 & -38 & 46 \end{bmatrix} + m\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 5 \\ 0 & -10 & 14 \end{bmatrix} +$$

$$n\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix} - 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$\therefore \quad 1 + m + n - 6 = 0$ and $-11 - m + n - 6 = 0$

$\Rightarrow \quad n - m = 17$

48. Answer (98)

Hint : Use mathematical induction approach.

Solution :

If $A^2 = O$ the 'n' must be multiple of 3.

$\therefore$ largest value of 'n' is $\boxed{98}$

49. Answer (41)

Hint : Take submission out of integration.

Solution :

Let $\boxed{n = 40}$

$$\sum (-1)^{r-1}\, {}^nC_r \int_0^1 \left(1 + x + x^2 + ... + x^{r-1}\right) dx$$

$$= \sum (-1)^{r-1}\, {}^nC_r \int_0^1 \left(\frac{1 - x^r}{1 - x}\right) dx$$

$$= \int_0^1 \frac{\sum_{r=1}^{n} \left((-1)^{r-1}\, {}^nC_r - (-1)^{r-1}\, {}^nC_r\, x^r\right) dx}{(1 - x)}$$

$$= \int_0^1 \frac{\left(-1 + 1 + (1 - x)^n\right)}{1 - x} dx$$

$$= \int_0^1 (1 - x)^{n-1} dx = \frac{1}{n}$$

$$\therefore \quad \boxed{\frac{1}{n} = \frac{1}{40}}$$

50. Answer (16)

Hint : Apply property of determinants.

Solution :

By $R_2 \to R_2 - 2R_1$

and $R_3 \to R_3 - 3R_1$

We get, $u_3 = 64 \quad \Rightarrow \quad u = 4$

$\therefore \quad \boxed{u^2 = 16}$

51. Answer (B)

Hint : Assumption of odd consecutive integer.

Solution :

Let odd integer are $2m + 1$, $2m + 3$, …. let number be n

$\therefore \quad 57^2 - 13^2 = n(2m + n)$

$(m + n)^2 - m^2 = 57^2 - 13^2$

$\Rightarrow \quad m = 13;\ n + m = 57$

$\therefore \quad n = 44$

$\therefore$ The least integer = 26 + 1 = 27

Largest integer = 113

Number of factors of type $(4n + 1)$ for 113 is 1

Sum of least + largest integer = $140 = 5 \times 7 \times 2^2$

$\therefore$ Number of factors $2 \times 2 \times 3 = 12$

52. Answer (A)

Hint : Theory of equation.

Solution :

$$\frac{1 - \tan\theta}{1 + \tan\theta} = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

$\Rightarrow \quad \tan^4\theta + 4\tan^3\theta - 6\tan^2\theta - 4\tan\theta + 1 = 0$

$\therefore \quad \sum \tan\alpha = -4$

$\sum (\tan\alpha \tan\beta) = -6$

$\sum (\tan\alpha\ \tan\beta \tan\gamma) = 4$

$\pi(\tan\alpha) = 1$

53. Answer (B)

Hint : A.M $\geq$ G.M.

Solution :

$\because$ All the roots are positive

$$\therefore \quad \frac{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6}{6} = z = \text{A.M.}$$

Also $\left(\alpha_1\, \alpha_2\, \alpha_3\, \alpha_4\, \alpha_5\, \alpha_6\right)^{\frac{1}{6}} = z = \text{G.M.}$

$\therefore$ All the roots are equal and are 2

$\therefore \quad f(x) = (x - 2)^6$

Now verify each of the values.

54. Answer (C)

Hint : Gap method.

Solution :

$n_1 = {}^7C_5 \times \lfloor 6 \times \lfloor 5 \quad n_2 = (\lfloor 5)^2 \times {}^6C_1$

$n_3 = \lfloor 6 \times \lfloor 5\,; \quad \boxed{n_4 = 0}$

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