All India Aakash Test Series for JEE (Advanced)-2020

TEST - 3A (Paper-1) - Code-A

Test Date : 06/10/2019

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(B, C)	21.	(B, D)	41.	(A, D)
2.	(A, D)	22.	(C)	42.	(B, C)
3.	(A, C, D)	23.	(A, B, D)	43.	(A, B, D)
4.	(A, B)	24.	(A, B, D)	44.	(A, D)
5.	(A, D)	25.	(B, C)	45.	(A, B, C)
6.	(D)	26.	(B)	46.	(B)
7.	(C)	27.	(C)	47.	(B)
8.	(B)	28.	(D)	48.	(C)
9.	(B)	29.	(D)	49.	(A)
10.	(A)	30.	(A)	50.	(D)
11.	(D)	31.	(B)	51.	(D)
12.	(B)	32.	(B)	52.	(B)
13.	(A)	33.	(A)	53.	(C)
14.	(C)	34.	(C)	54.	(C)
15.	(D)	35.	(B)	55.	(C)
16.	A $\rightarrow$ (Q, R, T) B $\rightarrow$ (P, R) C $\rightarrow$ (S, T) D $\rightarrow$ (R, S, T)	36.	A $\rightarrow$ (P, R, T) B $\rightarrow$ (P, R) C $\rightarrow$ (S, T) D $\rightarrow$ (Q, R)	56.	A $\rightarrow$ (P, Q, R) B $\rightarrow$ (P, Q, R) C $\rightarrow$ (S, T) D $\rightarrow$ (P, Q, R, S, T)
17.	A $\rightarrow$ (Q, S) B $\rightarrow$ (P, S) C $\rightarrow$ (P, Q, R, S, T) D $\rightarrow$ (P, Q, R, S, T)	37.	A $\rightarrow$ (R, T) B $\rightarrow$ (P, S, T) C $\rightarrow$ (P, Q, S) D $\rightarrow$ (P, Q, S)	57.	A $\rightarrow$ (Q, R, S, T) B $\rightarrow$ (Q, R, S, T) C $\rightarrow$ (T) D $\rightarrow$ (P, Q, R, S)
18.	(01)	38.	(08)	58.	(02)
19.	(08)	39.	(06)	59.	(07)
20.	(03)	40.	(08)	60.	(00)

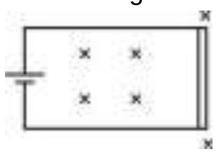
HINTS & SOLUTIONS

PART - I (PHYSICS)

1. Answer (B, C)

Hint : $\int F dt = \int iB\ell \cdot dt$

Solution : Let i be the current at any time during short changes then



$F(t) = i\ell B$ and impulse $\int F(t)\, dt = \int \ell\, B\, i\, dt$

$\Rightarrow\ mv = \ell\, B\, \Delta q \ \Rightarrow\ \Delta q = \dfrac{mv}{B\ell}$

Work done by battery $w_b = \dfrac{\varepsilon mv}{B\ell}$

$= \Delta w_{\text{loss}} = \dfrac{\varepsilon mv}{B\ell} - \dfrac{1}{2}mv^2$

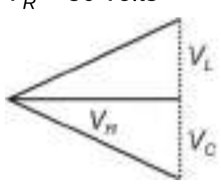
2. Answer (A, D)

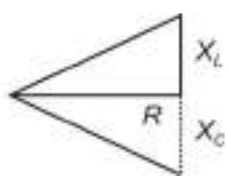
Hint : At resonance $|\omega L| = \dfrac{1}{|\omega C|}$

Solution : At resonance $|\omega L| = \dfrac{1}{|\omega C|}$

V_L = 60 volts

V_R = 80 volts





Clearly, $\dfrac{V_L}{V_R} = \dfrac{\omega L}{R} \quad \Rightarrow \quad \dfrac{60}{80} = \dfrac{\omega L}{240}$

$\Rightarrow\ \omega L = 180\ \Omega \quad \Rightarrow \quad L = \dfrac{180}{90} = 2\text{H}$

Similarly $\dfrac{1}{\omega C} = 180 \ \Rightarrow\ C = \dfrac{1}{90 \times 180} = \dfrac{1}{16200}$ F

For current to lag by 45° $|R| = \left|\omega L - \dfrac{1}{\omega C}\right|$

$\Rightarrow\ \omega^2 LC - 1 = 240\, \omega C$

$\Rightarrow\ \omega^2 - 120\omega - 8100 = 0$

$\therefore\ \ \omega = 60 + 10\sqrt{117}$ rad/s

3. Answer (A, C, D)

Hint :

If there is no dissipating force then the system will oscillate.

Solution : If there is no dissipating force/s then the system will oscillate.

But if there is some small resistance then

$$mg - \frac{vB^2\ell^2}{r} = \frac{mdv}{dt}$$

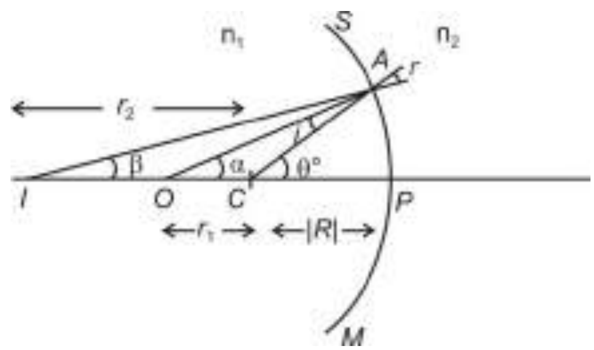
Before it attains the steady constant speed, $\dfrac{dv}{dt}$ is positive. That means part of the work done by gravity is appearing as increase in kinetic energy and rest are dissipating as heat energy.

Finally $\dfrac{dv}{dt} = 0$. *i.e.* $v_0 = \dfrac{mgr}{B^2\ell^2}$

4. Answer (A, B)

Hint : $n_1 \sin i = n_2 \sin r$.

Solution :



Let SPM be the spherical surface separating two media with refractive index n_1 and n_2 respectively.

So for refraction at point A

$n_1 \sin i = n_2 \sin r \quad \Rightarrow \quad \dfrac{\sin i}{\sin r} = \dfrac{n_2}{n_1}$ …(i)

Take $|CP| = R$, $|OC| = r_1$ and $|IC| = r_2$

$\angle AIC = \beta$, $\angle AOC = \alpha$ then

In ΔAOC, $\dfrac{\sin i}{\sin \alpha} = \dfrac{r_1}{|R|}$ …(ii)

Also in ΔAIC $\dfrac{\sin r}{\sin \beta} = \dfrac{r_2}{|R|}$ …(iii)

For a very particular case when $\angle\alpha = \angle r$ and $\angle\beta = \angle i$ then equation (ii) will become.

$\dfrac{\sin i}{\sin \alpha} = \dfrac{r_1}{|R|} = \dfrac{n_2}{n_1}$

$\Rightarrow\ r_1 = \dfrac{n_2}{n_1}|R|$

and equation (iii) will become $\dfrac{\sin r}{\sin B} = \dfrac{r_2}{|R|} = \dfrac{n_1}{n_2}$

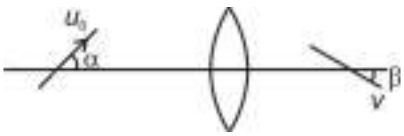
$\Rightarrow \quad |r| = \dfrac{n_1}{n_2}|R|$

Irrespective of the point A being paraxial or marginal. Hence all ray emanating from O seems to be emanating from i and vice-versa.

5. Answer (A, D)

Hint : $|m| = \dfrac{|y|}{|x|}$

Solution :



Let u_0 be the speed of object and v be the speed of image and x and y are the respective distance of object and image, then

Magnification $|m| = \dfrac{|y|}{|x|}$

Also, $v \sin\beta = |m|\, u_0 \sin\alpha \quad \ldots$(i)

$v \cos\beta = |m|^2\, u_0 \cos\alpha \quad \ldots$(ii)

$\therefore \quad \tan\beta = \dfrac{|x|}{|y|}\tan\alpha$

Now $\dfrac{1}{y} + \dfrac{1}{x} = \dfrac{1}{f}$

$\Rightarrow \quad \dfrac{\tan\beta}{x\tan\alpha} + \dfrac{1}{x} = \dfrac{1}{f}$

$\Rightarrow \quad x = f\dfrac{(\tan\alpha + \tan\beta)}{\tan\alpha}$

Also, $\dfrac{1}{y} + \dfrac{\tan\alpha}{y\tan\beta} = \dfrac{1}{f}$

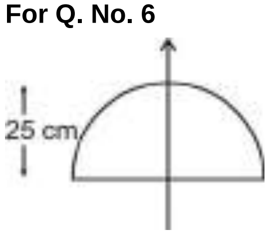
$\Rightarrow \quad y = \dfrac{f(\tan\alpha + \tan\beta)}{\tan\beta}$

6. Answer (D)
7. Answer (C)
8. Answer (B)

Hint : $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$

Solution for Q. Nos. 6 to 8

For Q. No. 6



$\dfrac{1}{V_1} + \dfrac{\mu}{25} = \dfrac{(1-\mu)}{-50}$

$\Rightarrow \quad \dfrac{1}{V_1} + \dfrac{3}{2\times 25} = \dfrac{1}{100}$

$\Rightarrow \quad \dfrac{1}{V_1} = \dfrac{1}{100} - \dfrac{6}{100}$

$\Rightarrow \quad \dfrac{1}{V_1} = \dfrac{-5}{100} \quad \therefore \quad V_1 = -20\text{ cm}$

$\therefore$ Shifting in 1st case is 25 – 20 = 5 cm

For Q. No. 7

$\dfrac{1}{V_2} + \dfrac{\mu}{75} = \dfrac{(1-\mu)}{-50}$

$\Rightarrow \quad \dfrac{1}{V_2} = \dfrac{1}{100} - \dfrac{3}{2\times 75}$

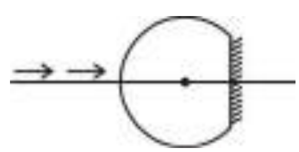
$\Rightarrow \quad \dfrac{1}{V_2} = \dfrac{1}{100} - \dfrac{1}{50}$

$\Rightarrow \quad \dfrac{1}{V_2} = \dfrac{-1}{100}$

$\therefore \quad V_2 = -100$

Shifting $\Rightarrow$ 100 – 75 = 25 cm

For Q. No. 8



$\dfrac{\mu}{V_3} - \dfrac{1}{\infty} = \dfrac{(\mu - 1)}{50}$

$\therefore \quad \dfrac{3}{2V_3} = \dfrac{1}{2\times 50}$

$\Rightarrow \quad V_3$ = 150 cm

So for mirror (at plane surface) object distance is +75 cm.

$\therefore$ Image would be at 75 cm opposite to silvered surface which falls on the periphery of the sphere.

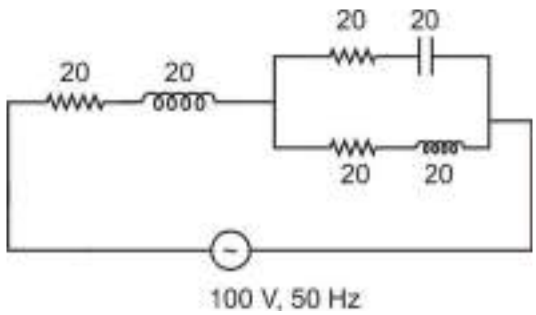
9. Answer (B)

10. Answer (A)

11. Answer (D)

Hint : $I = \frac{V}{z}$ (Here z is impedance)

Solution for Q. Nos. 9 to 11



$$X_L = \omega L = 2\pi \times 50\,\frac{1}{5\pi} = 20\,\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1\,\pi \times 10^6}{2\pi \times 50 \times 500} = 20\,\Omega$$

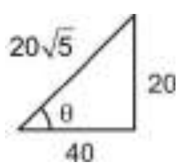
Let $z_1 = 20 + 20j$; $z_2 = 20 - 20j$; $z_3 = 20 + 20j$

$$\therefore\ \frac{1}{z} = \frac{1}{z_2} + \frac{1}{z_3} = \frac{1}{20-20j} + \frac{1}{20+20j}$$

$$= \frac{20j + 20 + 20 - 20j}{400 + 400}$$

$$\Rightarrow\ z' = \frac{800}{40} = 20\,\Omega$$

$\therefore$ $z_{eq} = 20 + 20j + 20 = 40 + 20j$



$$\therefore\ |z| = 20\sqrt{5}\cdot \tan\theta = \frac{1}{2} \quad \cos\theta = \frac{2}{\sqrt{5}}$$

$$\therefore\ I = \frac{100\ \angle 0}{20\sqrt{5}\ \angle\theta} = \sqrt{5}\ \Rightarrow I_0 = \sqrt{10}$$

Current amplitude = $\sqrt{10}$ ampere

$\therefore$ Power dissipation = $I_{rms} \cdot V_{rms} \cos\phi$

$\Rightarrow$ $z_2 I_C = z_3 I_L$ $\quad\Rightarrow$ $z_2 I_C = (I - I_C) z_3$

$\Rightarrow$ $(z_2 + z_3) I_C = z_3 I$

$$\therefore\ I_C = \frac{I(20+20j)}{(20+20j+20-20j)} = \frac{I\times 20(1+j)}{20\times 2}$$

$$\Rightarrow\ I_C = \frac{I}{\sqrt{2}}\angle 45^\circ$$

So I_C is in lead by phase of 45° or $\frac{\pi}{4}$ rad w.r.t. to total current I.

12. Answer (B)

Hint : $i = I_0\left(1 - e^{-\frac{tR}{L}}\right)$

Solution :

$$i = \frac{\varepsilon}{R}\left(1 - e^{-\frac{tR}{L}}\right)$$

$$\therefore\ \Delta q = \int dq = \int i\,dt = \frac{\varepsilon}{R}\int_0^{2\tau}\left(1 - e^{-\frac{tR}{L}}\right)dt$$

$$\Rightarrow\ \Delta q = \frac{\varepsilon}{R}\left[t + \frac{L}{R}e^{-\frac{tR}{L}}\right]_0^{2\tau}$$

$$= \frac{\varepsilon}{R}\left[\frac{2L}{R} + \frac{L}{Re^2} - 0 - \frac{L}{R}\right]$$

$$\Rightarrow\ \Delta q = \frac{\varepsilon}{R}\left[\frac{L}{R} + \frac{L}{Re^2}\right]$$

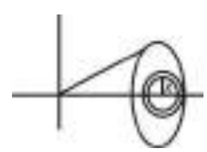
$$\Rightarrow\ \Delta q = \frac{\varepsilon L}{R^2}\left[\frac{e^2+1}{e^2}\right]$$

13. Answer (A)

Hint : $dB = \int B \cdot ds$

Solution :

$$\vec{B} = \frac{B_0\left(x\hat{i} + y\hat{j} + z\hat{k}\right)}{\left(x^2+y^2+z^2\right)^{\frac{3}{2}}}$$



Consider a small ring of radius R and width 'dR'.

Then $R = \sqrt{y^2 + z^2}$

$$d\phi = \frac{B_0}{\left(x^2+y^2+z^2\right)^{\frac{3}{2}}}\ (x\hat{i} + y\hat{j} + z\hat{k})\cdot 2\pi R\,dR\,\hat{i}$$

$$\Rightarrow\ d\phi = \frac{B_0 x \cdot 2\pi R dR}{\left[R^2 + x^2\right]^{\frac{3}{2}}} = B_0 \pi x \int \frac{2RdR}{\left(R^2+x^2\right)^{\frac{3}{2}}}$$

$$\Rightarrow \quad \phi = 2B_0\pi x\left[\frac{-1}{\sqrt{R^2+x^2}}\right]_0^a$$

$$\Rightarrow \quad \phi = 2\pi x\, B_0\left[\frac{1}{x}-\frac{1}{\sqrt{a^2+x^2}}\right]$$

14. Answer (C)

Hint : $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$.

Solution : When viewed from spherical side.

$$\frac{1}{v_1}+\frac{3}{2\times 8}=\frac{1}{2\times 16}$$

$$\Rightarrow \quad \frac{1}{v_1}=\frac{1}{32}-\frac{3}{16}=\frac{-5}{32}$$

Image position is $\frac{32}{5}$ cm inside the hemisphere from its periphery.

And when viewed from plane side then image is at $\frac{16}{3}$ cm. Inside the plane surface.

$$\therefore \quad \Delta x = 16-\frac{16}{3}-\frac{32}{5}=\frac{64}{15}\ \text{cm}$$

15. Answer (D)

Hint : $E\cdot 2\pi R=\pi R^2\frac{dB}{dt}$.

Solution :

$$E\cdot 2\pi R=\pi R^2\times\frac{\Delta B}{\Delta t}$$

$$\Rightarrow \quad E=\frac{R}{2}\frac{B}{\Delta t}$$

$dF = dq\cdot E$ and $d\tau = REdq$

$$\Rightarrow \quad \tau=\int REdq = REq$$

So $\int \tau\cdot dt=\lim\limits_{x\to 0}\frac{R^2q\,B}{2\Delta t}\cdot\Delta t = I\omega$

$$\Rightarrow \quad \frac{R^2qB}{2}=mR^2\cdot\omega$$

$$\Rightarrow \quad \omega=\frac{qB}{2m}$$

16. Answer A(Q, R, T); B(P, R); C (S, T); D (R, S, T)

Hint : If field along the dipole, then torque is zero.

Solution :

For option (A) : Opposite current means they will repel and dipoles are at 180° torque is zero but rotationally unstable.

For option (B) : Decrement of current in one loop will induced the current in same sense in other loop, so attractive force and no torque.

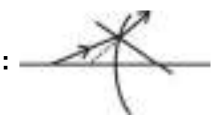
For option (C) : Force/s are zero but torque is not zero.

For option (D) : Force/s are zero, torque is also zero but dipoles are oppositely aligned so rotationally unstable.

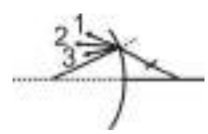
17. Answer A(Q, S); B(P, S); C(P, Q, R, S, T); D(P, Q, R, S, T)

Hint : $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$ and magnification $m=\frac{\mu_1 v}{\mu_2 u}$.

Solution :

For option (A) :  ray seems to diverge always after refraction so it will form virtual and diminished image.

For option (B) :  after refraction ray would bend and meet the axis some what at shorter distance. So, it will form real and diminished image.

For option (C) : 

it will try to bend towards normal so, depending upon the extent of bending the refracted ray may meet really or it may seems to be diverging. So it may form real image or virtual image also. Magnification may also vary on being diminished to magnified.

For option (D) :  Depending upon the extend of bending the refracted ray may meet really or it may seems to be diverging. So, it may form real or virtual image. Also magnification may vary from being diminished to being magnified.

18. Answer (01)

Hint : $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \left(\dfrac{\mu_2 - \mu_1}{R}\right)$ use this.

Solution : Let 'x' be the distance of object from S_1 so, for first refraction let v_1 is the image distance then,

$$\frac{3}{2v_1} + \frac{4}{3x} = \frac{-1}{6} \Rightarrow \frac{3}{2v_1} = \frac{-1}{6} - \frac{4}{3x}$$

$$\Rightarrow v_1 = \frac{-9x}{8+x} \quad i.e. \quad \frac{9x}{8+x} \text{ from } S_1$$

Now for 2nd refraction from S_2 object distance would be $u_2 = \dfrac{9x}{8+x} + 1 = \dfrac{8+10x}{8+x}$

For refraction from 2nd surface S_2

$$\frac{1}{v_2} - \frac{3}{2u_2} = \frac{\left(1 - \frac{3}{2}\right)}{-2}$$

$$\Rightarrow v_2 = -(x+1)$$

So, $\dfrac{-1}{x+1} + \dfrac{3(8+x)}{2(8+10x)} = \dfrac{1}{4}$

$$\Rightarrow \frac{-16 - 20x + 24x + 24 + 3x^2 + 3x}{4(x+1)(4+5x)} = \frac{1}{4}$$

$\Rightarrow 3x^2 + 7x + 8 = 5x^2 + 9x + 4$

$\Rightarrow 2x^2 + 2x - 4 = 0$

$\Rightarrow x^2 + x - 2 = 0$

$\Rightarrow x = 1, -2$

$\therefore$ Object distance from S_1 = 1 m

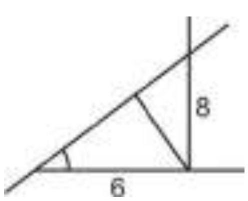
19. Answer (08)

Hint : $E_{\text{Ind}} = \left|\dfrac{d\phi}{dt}\right|$

Solution : $E \cdot 2\pi r = \pi r^2 \cdot \dfrac{dB}{dt}$

$$\Rightarrow \vec{E} = \frac{r}{2}\frac{dB}{dt}$$

$$r = 6\sin 53^\circ = 6 \times \frac{4}{5} = \frac{24}{5}$$



$\therefore \displaystyle\int E dl = \frac{24}{5}\,\frac{1}{2} \times \frac{1}{3} \times 10$

$\left(\int E dl\right) = 8$

20. Answer (03)

Hint : $I = \dfrac{V}{Z}$ (Here Z is impedance)

Solution : In capacitive circuit $I_C = \dfrac{50 \sin \omega t}{2R \angle -60^\circ}$

$$\Rightarrow I_C = \frac{25}{R}\sin(\omega t + 60^\circ)$$

$$\therefore V_C = \frac{25}{R}\sin(\omega t + 60^\circ) \times \sqrt{3}R \angle -90^\circ$$

$$= 25\sqrt{3}\sin(\omega t - 30^\circ)$$

$\therefore \omega t - 30^\circ = 90^\circ$

Now in inductive circuit $I_L = \dfrac{50\sin(\omega t)}{2R \angle 30^\circ}$

$$\therefore v_{R_1} = 3\sqrt{R} \angle 0^\circ \frac{50 \sin(\omega t)}{2R \angle 30^\circ}$$

$$= 25\sqrt{3}\sin(\omega t - 30^\circ)$$

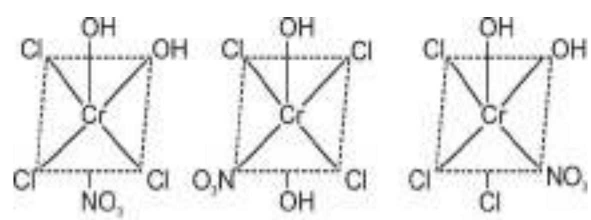
$\therefore v_{R_1} = 25\sqrt{3}$ volts

$\therefore N = 3$

PART - II (CHEMISTRY)

21. Answer (B, D)

Hint : The given complex ion has three geometrical isomers and each of them is optically inactive



(I) (II) (III)

Solution : The central metal ion (Cr^{3+}) has 3 unpaired electrons. So, magnetic moment of complex is 3.87 BM

22. Answer (C)

Hint : Fact based.

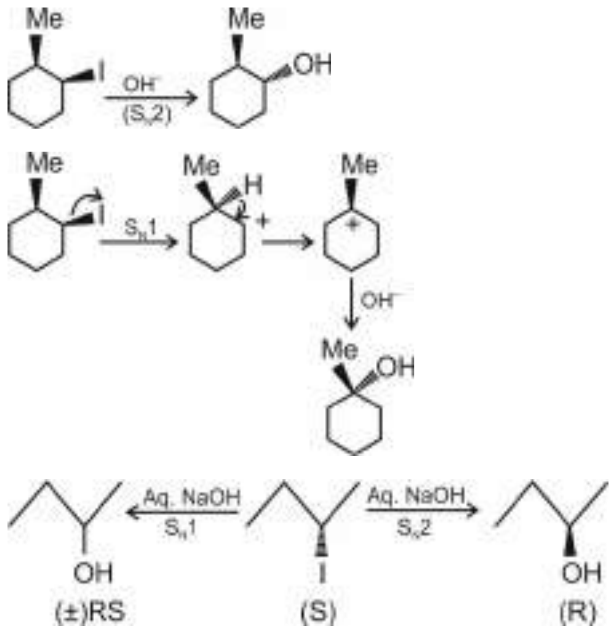
Solution : The following reactions take place in the reaction mixture

$$MnCl_2 + 2NaOH + H_2O_2 \rightarrow MnO(OH)_2 + 2NaCl + H_2O$$

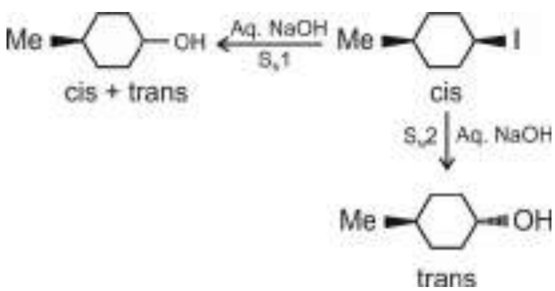
$$2CrCl_3 + 10NaOH + 3H_2O_2 \rightarrow 2Na_2CrO_4 + 6NaCl + 8H_2O$$

23. Answer (A, B, D)

Hint :



Solution :



24. Answer (A, B, D)

Hint : Fact based and evident from diagram.

Solution : Copper (II) acetate is dimeric and hydrated as $\left[Cu\left(OCOCH_3\right)_2 \cdot H_2O\right]_2$.

25. Answer (B, C)

Hint :

$$FeSO_4 + 2NaOH \longrightarrow \underset{\text{Dirty green ppt.}}{Fe(OH)_2} + Na_2SO_4$$

Solution :

$$CuSO_4 + 2NaOH \longrightarrow \underset{\text{Bluish white ppt.}}{Cu(OH)_2} + Na_2SO_4$$

26. Answer (B)

Hint : The solution (A) contains I⁻ ions, as seen from the following reaction

$$\underset{(A)}{2I^-} + Hg(NO_3)_2 \longrightarrow \underset{\substack{\text{(Scarlet ppt.)} \\ (B)}}{HgI_2} + 2NO_3^-$$

Solution : $HgI_2 + 2KI \longrightarrow \underset{(C)}{K_2\left[HgI_4\right]}$

The hybridisation of $Hg^{2+}\left(5d^{10}6s^0\right)$ in (C) is sp^3. Therefore, shape of complex anion in (C) is tetrahedral.

27. Answer (C)

Hint : The reagent (X) must be $Bi(NO_3)_3$ which reacts with I⁻ ions to give black ppt. of BiI_3

Solution :

$$3I^- + Bi\left(NO_3\right)_3 \longrightarrow \underset{\text{Black ppt.}}{BiI_3} + 3NO_3^-$$

28. Answer (D)

Hint : The black ppt. of BiI_3 when heated in presence of water turns orange due to the formation of BiOI

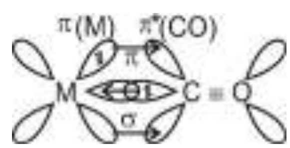
Solution :

$$BiI_3 + H_2O \longrightarrow \underset{\text{Orange ppt.}}{BiOI} + 2HI$$

29. Answer (D)

Hint : CO is π^* acceptor and σ donor.

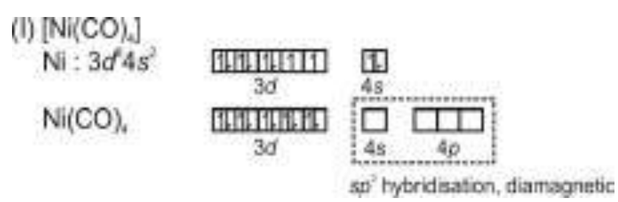
Solution : In metal carbonyl, the C-atom of CO ligand uses its filled orbital and metal uses its vacant orbital to form σ type of co-ordinate covalent bond. Simultaneously, the metal uses its filled orbital and CO uses its π-type of antibonding molecular orbital to form π -type of co-ordinate bond.

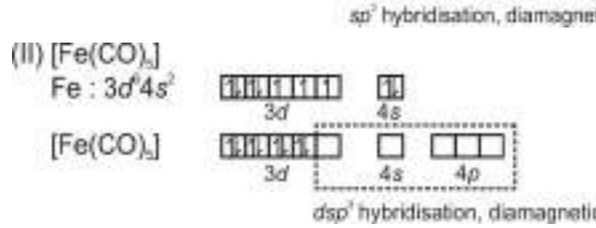


30. Answer (A)

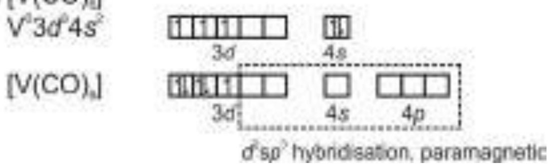
Hint : CO form strong field ligand with metals.

Solution :

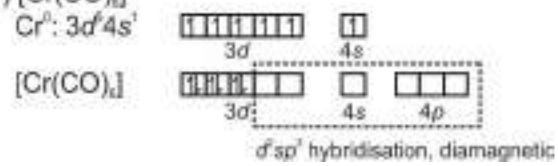




(III) $[V(CO)_6]$



(IV) $[Cr(CO)_6]$



So, (II) and (IV) are inner orbital complexes and diamagnetic.

31. Answer (B)

Hint : Greater the electron density on metal, lesser will be the C — O bond strength.

Solution :

In the given metal carbonyls i.e. $[Ni(CO)_4]$, $[Fe(CO)_5]$ and $[Cr(CO)_6]$ the number of electron pairs available are 5, 4 and 3 respectively. So, the share of $d\pi$ electrons from Ni, Fe and Mn for back donating to each CO molecule in these complexes will be 5/4, 4/5 and 3/6. Hence, C — O bond length in these complexes will be in the order of (I) > (II) > (III).

32. Answer (B)

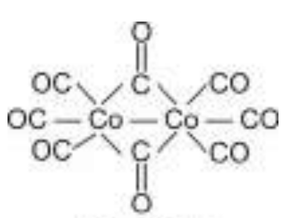
Hint : Co-ordination compound (X) must be cis isomer having permanent dipole moment since it dissolves in polar solvents.

Solution : Co-ordination compound (Y) must be trans isomer which is non-polar since it dissolves in non-polar solvents.

33. Answer (A)

Hint : Fact based.

Solution :



$[Co_2(CO)_8]$

34. Answer (C)

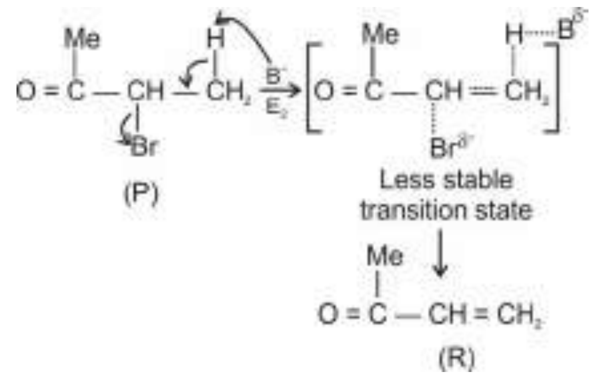
Hint : Hexachloridoplatinic acid is formed when platinum reacts with aqua regia

Solution : $3Pt + 16H^+ + 4NO_3^- + 18Cl^- \longrightarrow$

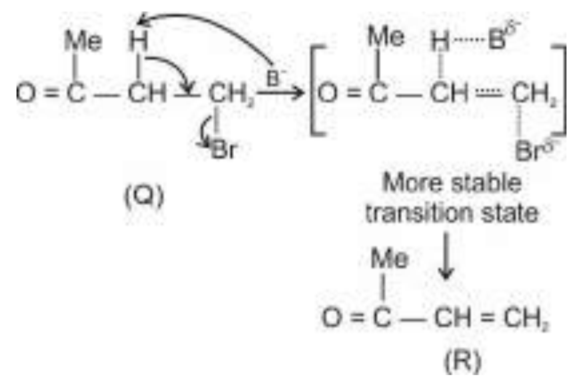
$$3\left[PtCl_6\right]^{2-} + 4NO + 8H_2O$$

35. Answer (B)

Hint :



Solution :



The H-atom attached to β C-atom with respect to Br-atom is more acidic in (Q) than in (P). Therefore, the transition state formed in elimination of HBr from (Q) is more stable than that from (P). So, (Q) reacts faster than (P) in the given elimination reaction.

36. Answer A(P, R, T); B(P, R); C(S, T); D(Q, R)

Hint :

$$CoCl_2 + 7KNO_2 + 2CH_3COOH \rightarrow \underset{\text{Yellow ppt.}}{K_3\left[Co(NO_2)_6\right]}$$

$$+2KCl + 2CH_3COOK + \underset{\text{Neutral gas}}{NO} + H_2O$$

$$H_2S + 2HNO_2 \rightarrow S + \underset{\text{Neutral gas}}{2NO} + 2H_2O$$

Solution :

$$4FeCl_3 + 3K_4[Fe(CN)_6] \longrightarrow Fe_4[Fe(CN)_6]_3 + 12KCl$$

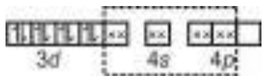
$$Na_2S_2O_3 + 2HCl \rightarrow 2NaCl + \underset{\substack{\text{Acidic}\\\text{gas}}}{SO_2} + \underset{\substack{\text{Yellow}\\\text{ppt.}}}{S} + H_2O$$

37. Answer A(R, T); B(P, S, T); C(P, Q, S); D(P, Q, S)

Hint : CN^- is a strong field ligand and F^- is weak field ligand.

Solution :

(A) $[Ni(CN)_4]^{2-}$

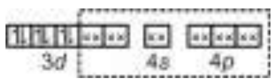


Hybridisation : dsp^2

Shape : Square planar

Magnetic behaviour : Diamagnetic

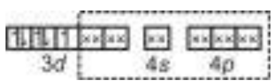
(B) $[NiF_6]^{2-}$



d^2sp^3 hybridisation; Octahedral; Diamagnetic

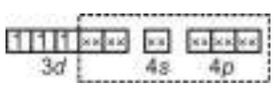
[**Note :** As oxidation state of nickel ion increases, the splitting energy increases. Thus F^- ions cause pairing of unpaired electron]

(C) $[Fe(CN)_6]^{3-}$



d^2sp^3 hybridisation; Octahedral; Paramagnetic

(D) $[Cr(H_2O)_6]^{3+}$

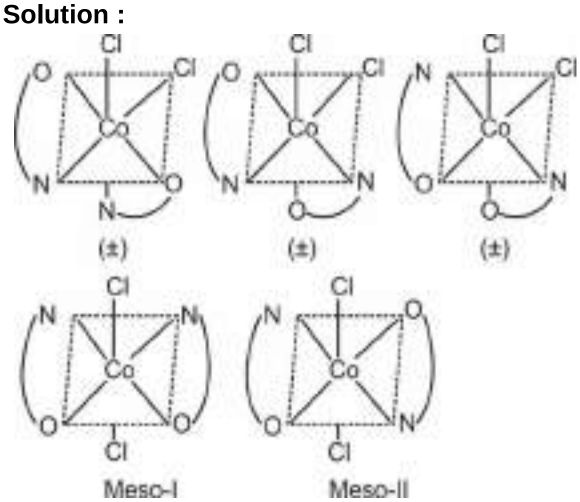


d^2sp^3 hybridisation; Octahedral; Paramagnetic

38. Answer (08)

Hint : The possible stereoisomers of the given complex ion are 8.

Solution :



39. Answer (06)

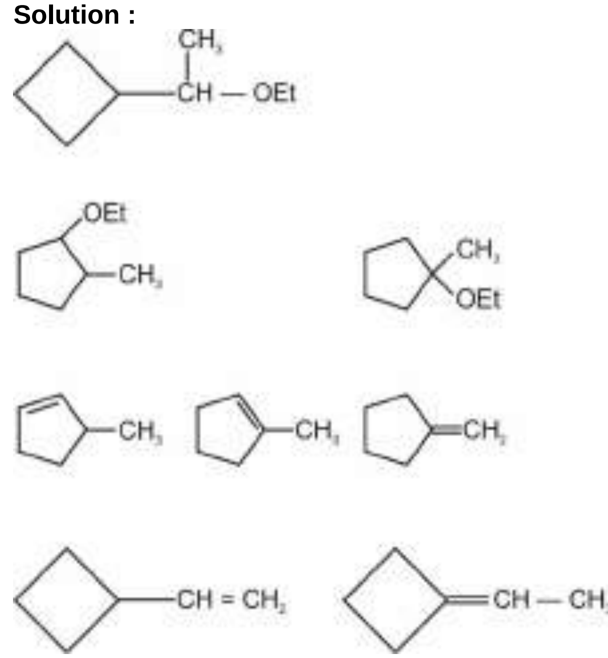
Hint : Test for G-II cations.

Solution : Only Ag^+, Pb^{2+}, Bi^{3+}, Sn^{2+}, As^{3+} and Cd^{2+} ions will get precipitated on passing H_2S gas in presence of dil HCl.

40. Answer (08)

Hint : The total number of substitution (by S_N1) and elimination (by E_1) products formed in the given reaction are 8.

Solution :



PART - III (MATHEMATICS)

41. Answer (A, D)

Hint : Use property $\int_a^b f(a+b-x)\,dx = \int_a^b f(x)\,dx$

Solution : $I = \int_0^{\pi/2} t(\sin 2x)\sin x\,dx \quad \ldots(i)$

$$I = \int_0^{\pi/2} f\left(\sin(\pi - 2x)\right)\cos x\,dx$$

$$I = \int_0^{\pi/2} t(\sin 2x)\cos x\,dx \quad \ldots(ii)$$

Add (i) and (ii) we get

$$2I = \int_0^{\pi/2} f(\sin 2x)(\sin x + \cos x)\,dx$$

$$= \sqrt{2}\int_0^{\pi/2} f(\sin 2x)\sin\left(x + \frac{\pi}{4}\right)dx$$

Put $x = \frac{\pi}{4} - \theta$

$\Rightarrow \quad dx = -d\theta$

$$= \sqrt{2}\int_{+\pi/4}^{-\pi/4} f\left(\sin\left(2\left(\frac{\pi}{4}-\theta\right)\right)\right)\cos\theta(-d\theta)$$

$$= \sqrt{2}\int_{\pi/4}^{-\pi/4} f(\cos 2\theta)\cos\theta(-d\theta)$$

$$= 2\sqrt{2}\int_{0}^{\pi/4} f(\cos 2\theta)\cos\theta\, d\theta$$

$$I = \sqrt{2}\int_{0}^{\pi/4} f(\cos 2\theta)\cos\theta\, d\theta$$

42. Answer (B, C)

Hint :

Minimize $\int_{x_1}^{x_2}\left((x^2+2x-3)-(kx+1)\right)dx$ where x_1 and x_2 are intersection points of given curves.

Solution : $kx + 1 = x^2 + 2x - 3$

$\Rightarrow \quad x^2 + (2 - k)x - 4 = 0$

$$\Rightarrow \quad x = \frac{k-2\pm\sqrt{(k-2)^2-4(-4)}}{2}$$

$$= \frac{k-2\pm\sqrt{k^2-4k+20}}{2} \begin{matrix} \nearrow x_1 \\ \text{(say)} \\ \searrow x_2 \end{matrix} \quad \ldots\text{(i)}$$

$$A = \left|\int_{x_1}^{x_2}\left((x^2+2x-3)-(kx+1)\right)dx\right|$$

$$= \left|\frac{x^3}{3}+\frac{(2-k)x^2}{2}-4x\right|_{x_1}^{x_2}$$

$$= \left|\frac{x_2^3-x_1^3}{3}+\left(\frac{2-k}{2}\right)(x_2^2-x_1^2)-4(x_2-x_1)\right| \quad \ldots\text{(ii)}$$

From (i) $x_1 + x_2 = k - 2$, $x_1x_2 = -4$ and $|x_1 - x_2| = \sqrt{k^2-4k+20}$ and using these results in (ii) we get

$$= \left|\frac{(x_2-x_1)\left((x_1+x_2)^2-x_1x_2\right)}{3}+\left(\frac{2-k}{2}\right)(x_2-x_1)(x_1+x_2)-4(x_2-x_1)\right|$$

$$= \left|\sqrt{k^2-4k+20}\left(\frac{(k-2)^2+4}{3}+\left(\frac{2-k}{2}\right)(k-2)-4\right)\right|$$

$$= \left|\sqrt{k^2-4k+20}\left(\frac{2(k-2)^2+8-3(k-2)^2-24}{6}\right)\right|$$

$$= \left|\sqrt{k^2-4k+20}\left(\frac{-(k-2)^2}{6}-\frac{8}{3}\right)\right|$$

$\Rightarrow \quad A_{\min}$ at $k = 2$

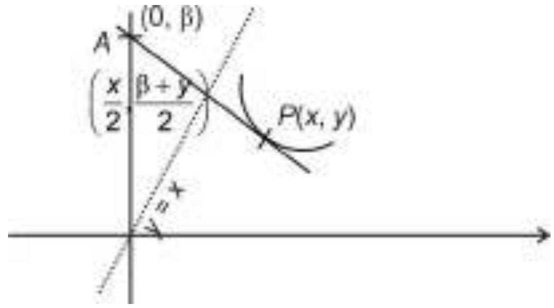
and $A_{\min} = \left|\sqrt{16}\left(\frac{-8}{3}\right)\right| = \frac{32}{3}$ sq. units

43. Answer (A, B, D)

Hint : Form differential equation using the equation of tangent at (x, y) i.e.

$$Y - y = \frac{dy}{dx}(X - x)$$

Solution :



Equation of tangent at $P(x, y)$

$$Y - y = \frac{dy}{dx}(X - x)$$

$$\beta = y - x\frac{dy}{dx}$$

Hence $\quad \dfrac{x}{2} = \dfrac{y - x\frac{dy}{dx} + y}{2}$

$$\Rightarrow \quad x = 2y - x\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} - \frac{2y}{x} = -1$$

$$\Rightarrow \quad d\left(\frac{y}{x^2}\right) = -\frac{1}{x^2}dx$$

$$\Rightarrow \quad \frac{y}{x^2} = \frac{1}{x} + c$$

$\Rightarrow \quad y = x - x^2$

44. Answer (A, D)

Hint : $\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$

Solution : $\int \dfrac{x^4 dx}{1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}}$

$$= 4!\,x - 4!\ln\left(1+\frac{x}{1!}+\frac{x^2}{2!}+\ldots+\frac{x^4}{4!}\right)+c$$

$\Rightarrow \quad k = 4,\ f(1) = \frac{15}{24}$ and $f(0) = 1$

45. Answer (A, B, C)

Hint :

$\int \cos x\, dx = \sin x + c$ and $\int \sin x\, dx = -\cos x + c$

Solution :

$$\left. k^2\frac{\sin 3x}{3} + \frac{k^2}{2}\sin x - \cos x - 2k\sin x\right|_0^{\pi/2} \le 1$$

$\Rightarrow \left(-\frac{k^2}{3}+\frac{k^2}{2}-0-2k\right)-(0+0-1-0)\le 1$

$\Rightarrow + k^2 - 12k + 6 \le 6$

$\Rightarrow k(k - 12) \le 0$

$k \in [0, 12]$

46. Answer (B)
47. Answer (B)
48. Answer (C)

Hint for Q. No. 46 to 48

$$\frac{d}{dx}\int_{f(x)}^{g(x)} h(x) = h(g(x))\cdot g'(x) - h(f(x))\cdot f'(x)$$

Solution for Q. Nos. 46 to 48

Using Leibnitz's rule

$f(x) = x^2 - x^2 f(x)$

$\Rightarrow f(x) = \frac{x^2}{1+x^2}$ ($f(x)$ is even function)

$g(x) = f(x) - 1 \Rightarrow g(x) = \frac{x^2-1-x^2}{1+x^2} = -\frac{1}{1+x^2}$

Range of $g(x) \in [-1, 0)$

$$I = \int_0^2 \left[-\frac{1}{g(x)}\right]^{dx} = \int_0^2 1+\left[x^2\right]dx$$

$$= x\big|_0^2 + \int_0^1 0dx + \int_1^{\sqrt{2}} dx + \int_{\sqrt{2}}^{\sqrt{3}} 2dx + \int_{\sqrt{3}}^2 3dx$$

$$= 2+0+(\sqrt{2}-1)+2(\sqrt{3}-\sqrt{2})+3(2-\sqrt{3})$$

$$= 7-\sqrt{2}-\sqrt{3}$$

$\Rightarrow [I] = 3$

49. Answer (A)
50. Answer (D)
51. Answer (D)

Hint for Q. No. 49 to 51

$\frac{dD}{dt} \propto D \Rightarrow \frac{dD}{dt} = kD$ (where k is constant of proportionality)

Solution for Q. No. 49 to 51 :

Let data at any time 't' is D

$\frac{dD}{dt} \propto D$

$\Rightarrow \frac{dD}{dt} = kD$

$\Rightarrow \int_{500}^{D} \frac{dD}{D} = \int_0^t kdt$

$\Rightarrow \ln D\big|_{500}^{D} = kt$

$\Rightarrow \frac{D}{500} = e^{kt}$

$\Rightarrow D = 500\, e^{kt}$

At $t = 2$, $D = 0.9 \times 500$

$\therefore 0.9 \times 500 = 500\, e^{k2}$

$\Rightarrow 0.9 = e^{2k}$

$\Rightarrow 2k = \ln 0.9$

$\Rightarrow k = \frac{\ln(0.9)}{2}$

$\Rightarrow D = 500e^{\left(\frac{1}{2}\right)(\ln(0.9))t}$ ...(i)

At $t = 4$ h, if $D = \frac{500}{2}$ in (i)

$D = 500e^{2\ln(0.9)}$

$\Rightarrow \frac{1}{2} = e^{\frac{1}{2}\ln(0.9)t}$

$\Rightarrow -\ln 2 = \frac{1}{2}\ln(0.9)t$

$\Rightarrow t = \frac{2\ln\left(\frac{1}{2}\right)}{\ln(0.9)}$

52. Answer (B)

Hint : Put $\sin 2x = 2\sin x\cos x$ and make partial fractions

Solution : $\int \frac{2\sin x(1-\cos x)dx}{2\sin x\cos x(1+\cos x)}$

$\Rightarrow \int \frac{1-\cos x\, dx}{(1+\cos x)\cos x}$

$\Rightarrow \int \left(\frac{1}{\cos x} - \frac{2}{1+\cos x}\right)dx$

$\Rightarrow \int \sec x dx - 2\int \frac{dx}{2\cos^2\frac{x}{2}}$

$\Rightarrow \int \sec x dx - \int \sec^2\frac{x}{2}dx$

$\Rightarrow \ln|\sec x + \tan x| - 2\tan\frac{x}{2} + c$

53. Answer (C)

Hint : $\lim_{n\to\infty}\sum_{r=1}^{kn} f\left(\frac{r}{n}\right)\cdot\frac{1}{n} = \int_0^k f(x)dx$

Solution : Let $L = \lim_{n\to\infty}\sum_{r=1}^{4n}\frac{1}{r+\sqrt{rn}}$

$$= \lim_{n\to\infty}\sum_{r=1}^{4n}\frac{1}{n}\left(\frac{1}{\left(\frac{r}{n}\right)+\sqrt{\frac{r}{n}}}\right)$$

$$= \int_0^4 \frac{dx}{\left(x+\sqrt{x}\right)}$$

Put $x = t^2$

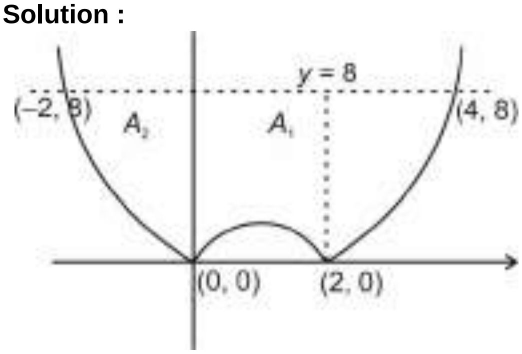
$\Rightarrow$ $dx = 2t\,dt$

$\Rightarrow$ $L = \int_0^2 \frac{2t\,dt}{t(1+t)} = 2\ln(1+t)\Big|_0^2$ = 2ln3 = ln9

54. Answer (C)

Hint : Area = $\int_\alpha^\beta (f(x)-g(x))dx$, where α and β are intersection points of given curves

Solution :



$$A_1 = \int_0^2 8-\left(2x-x^2\right)dx + \int_2^4 8-\left(x^2-2x\right)dx$$

$$= 8x - x^2 + \frac{x^3}{3}\Big|_0^2 + 8x - \frac{x^3}{3} + x^2\Big|_2^4$$

$$= \left(16-4+\frac{8}{3}\right)+\left(32-\frac{64}{3}+16-16+\frac{8}{3}-4\right)$$

$$= \frac{44}{3}+\left(28-\frac{56}{3}\right) = \frac{44}{3}+\frac{28}{3} = \frac{72}{3} = 24 \text{ sq.units}$$

$$A_2 = \int_{-2}^0 8-\left(x^2-2x\right)dx = 8x - \frac{x^3}{3}+x^2\Big|_{-2}^0$$

$$= 0-\left(-16+\frac{8}{3}+4\right) = 12-\frac{8}{3} = \frac{28}{3} \text{ sq. units}$$

$k = \frac{24}{28}\cdot 3 \quad \Rightarrow \quad [k] = 2$

55. Answer (C)

Hint :

$f(x)g'(y)dy + f'(x)g(y)dx = \int d\left(f(x)g(y)\right)$

Solution : $x dy + \cos y \ln x dy = 6x^2 dx - \sin y\, dx$

$\Rightarrow$ $dy + \cos y \ln x dy = 6x dx - \frac{\sin y}{x}dx$

$\Rightarrow$ $dy + \cos y \ln x dy + \frac{\sin y dx}{x} = 6x dx$

$\Rightarrow$ $\int dy + \int d\left(\sin y \ln x\right) = 6\int x dx$

$\Rightarrow$ $y + \sin y \ln x = 3x^2 + c$

For $y(1) = 3 \Rightarrow 3 + 0 = 3 + c \Rightarrow c = 0$

$\Rightarrow$ $y + \sin y \ln x = 3x^2$

Put $x = e$ we get

$y + \sin y = 3e^2$

56. Answer A(P, Q, R); B(P, Q, R); C(S, T); D(P, Q, R, S, T)

Hint : Apply estimation of integrals

Solution : $I_1 = \int_0^1 e^{x^2}dx \Rightarrow 1 < I < e-1$

$$I_2 = \int_0^{\pi/2}\frac{\sin x}{x}dx \Rightarrow \frac{2}{\pi} < I < \frac{\pi}{2}$$

$$I_3 = \left|\int_0^{\pi/2}(\sin x)dx\right| = \frac{\pi}{2}\ln 2$$

$$= \frac{\pi}{2}\times(2.303)\times(0.301) \cong 0.9$$

$$I_4 = 0 < \int_0^1 \frac{x^7}{\left(1+x^8\right)^{1/7}}dx < x^7$$

$\Rightarrow$ $0 < I_4 < \frac{1}{8}$

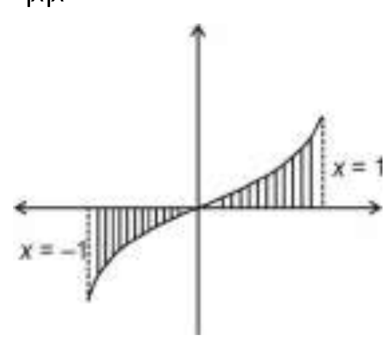
57. Answer A(Q, R, S, T); B(Q, R, S, T); C(T); D(P, Q, R, S)

Hint : Area between two curve $f(x)$ and $g(x)$

$= \int_\alpha^\beta (f(x)-g(x))dx$, where α, β are the point of intersection.

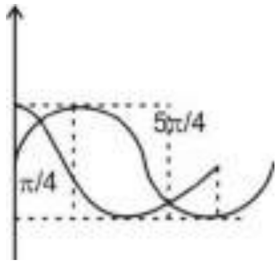
Solution :

(A) $y = |x|x$



$$A = \int_{-1}^0 -x^2dx + \int_0^1 x^2dx = \frac{1}{3}+\frac{1}{3} = \frac{2}{3} \text{ sq. units}$$

(B)

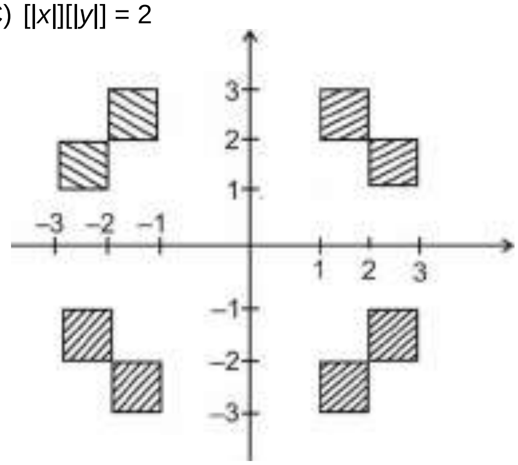


$$A = \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)\,dx = (-\cos x - \sin x)\Big|_{\pi/4}^{5\pi/4}$$

$$= \left(+\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)$$

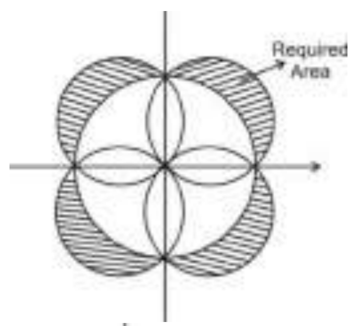
$$= 2\sqrt{2} \text{ sq. units}$$

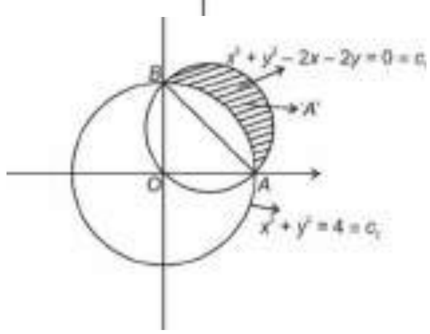
(C) $[|x|][|y|] = 2$



$A = 8$ sq. units

(D)





Required Area = $4A$ = 4(Area of semicircle of C_1 – (Area of quarter circle of C_2) + Area of ΔAOB)

$$= 4\left(\frac{\pi(\sqrt{2})^2}{2} - \frac{\pi(2)^2}{4} + \frac{1}{2}\cdot 2\cdot 2\right)$$

$= 4(\pi - \pi + 2) = 8$ sq. units

58. Answer (02)

Hint : If $f(x) = t$

$\Rightarrow\ f'(x)dx = dt$

Solution : $\displaystyle\int \frac{e^{2x}}{(e^{2x}+1)^2}dx$

Put $e^{2x} + 1 = t$

$2.e^{2x}\,dx = dt$

$$\Rightarrow\ \frac{1}{2}\int \frac{dt}{t^2} = -\frac{1}{2}\frac{1}{t} + c = -\frac{1}{2(e^{2x}+1)} + c \Rightarrow k = 2$$

59. Answer (07)

Hint : Put $y = vx$ and solve as homogeneous differential equation.

Solution : Put $y = vx$

$$u + x\frac{du}{x} = \frac{v}{1+v^2}$$

$$\Rightarrow\ x\frac{du}{dx} = \frac{v - v - v^3}{1+v^2}$$

$$\Rightarrow\ \int \frac{(1+v)^2}{v^3}dv = -\int \frac{dx}{x}$$

$$\Rightarrow\ -\frac{1}{2v^2} + \ln u = -\ln x + c$$

$$\Rightarrow\ -\frac{x^2}{2y^2} + \ln\left(\frac{y}{x}\right) + \ln x = c \Rightarrow -\frac{1}{2}$$

And $\displaystyle\frac{-a^2}{2e^2} + 1 = -\frac{1}{2} \Rightarrow \frac{a^2}{3} = e^2$

60. Answer (00)

Hint : Differentiate expression of area using Leibnitz's Rule

Solution : Given

$$\alpha^3 + 8 = \int_2^{\alpha} \left(x^2 + 2 - f(x)\right)dx$$

Differentiate both sides we get

$3\alpha^2 = \alpha^2 + 2 - f(\alpha)$

$\Rightarrow\ f(\alpha) = -2\alpha^2 + 2$

$\Rightarrow\ f(x) = -2x^2 + 2$

$\Rightarrow\ x_0 = 0$

□ □ □

All India Aakash Test Series for JEE (Advanced)-2020

TEST - 3A (Paper-1) - Code-B

Test Date : 06/10/2019

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(A, D)	21.	(B, C)	41.	(A, B, C)
2.	(A, B)	22.	(A, B, D)	42.	(A, D)
3.	(A, C, D)	23.	(A, B, D)	43.	(A, B, D)
4.	(A, D)	24.	(C)	44.	(B, C)
5.	(B, C)	25.	(B, D)	45.	(A, D)
6.	(D)	26.	(B)	46.	(B)
7.	(C)	27.	(C)	47.	(B)
8.	(B)	28.	(D)	48.	(C)
9.	(B)	29.	(D)	49.	(A)
10.	(A)	30.	(A)	50.	(D)
11.	(D)	31.	(B)	51.	(D)
12.	(D)	32.	(B)	52.	(C)
13.	(C)	33.	(C)	53.	(C)
14.	(A)	34.	(A)	54.	(C)
15.	(B)	35.	(B)	55.	(B)
16.	A $\rightarrow$ (Q, S)	36.	A $\rightarrow$ (R, T)	56.	A $\rightarrow$ (Q, R, S, T)
	B $\rightarrow$ (P, S)		B $\rightarrow$ (P, S, T)		B $\rightarrow$ (Q, R, S, T)
	C $\rightarrow$ (P, Q, R, S, T)		C $\rightarrow$ (P, Q, S)		C $\rightarrow$ (T)
	D $\rightarrow$ (P, Q, R, S, T)		D $\rightarrow$ (P, Q, S)		D $\rightarrow$ (P, Q, R, S)
17.	A $\rightarrow$ (Q, R, T)	37.	A $\rightarrow$ (P, R, T)	57.	A $\rightarrow$ (P, Q, R)
	B $\rightarrow$ (P, R)		B $\rightarrow$ (P, R)		B $\rightarrow$ (P, Q, R)
	C $\rightarrow$ (S, T)		C $\rightarrow$ (S, T)		C $\rightarrow$ (S, T)
	D $\rightarrow$ (R, S, T)		D $\rightarrow$ (Q, R)		D $\rightarrow$ (P, Q, R, S, T)
18.	(03)	38.	(08)	58.	(00)
19.	(08)	39.	(06)	59.	(07)
20.	(01)	40.	(08)	60.	(02)

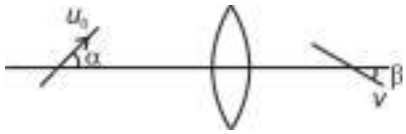
HINTS & SOLUTIONS

PART - I (PHYSICS)

1. Answer (A, D)

Hint : $|m| = \frac{|y|}{|x|}$

Solution :



Let u_0 be the speed of object and v be the speed of image and x and y are the respective distance of object and image, then

Magnification $|m| = \frac{|y|}{|x|}$

Also, $v\sin\beta = |m|\, u_0 \sin\alpha$ …(i)

$v\cos\beta = |m|^2\, u_0 \cos\alpha$ …(ii)

$\therefore \quad \tan\beta = \frac{|x|}{|y|}\tan\alpha$

Now $\frac{1}{y} + \frac{1}{x} = \frac{1}{f}$

$\Rightarrow \quad \frac{\tan\beta}{x\tan\alpha} + \frac{1}{x} = \frac{1}{f}$

$\Rightarrow \quad x = f\frac{(\tan\alpha + \tan\beta)}{\tan\alpha}$

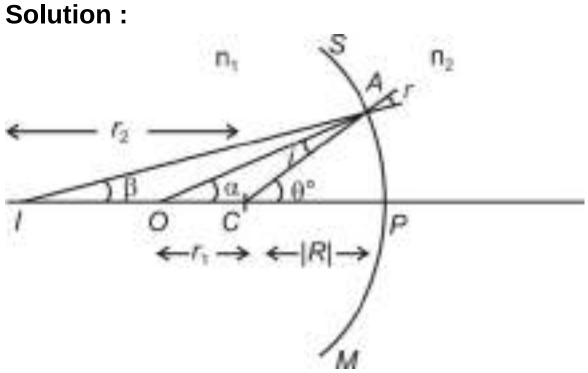
Also, $\frac{1}{y} + \frac{\tan\alpha}{y\tan\beta} = \frac{1}{f}$

$\Rightarrow \quad y = \frac{f(\tan\alpha + \tan\beta)}{\tan\beta}$

2. Answer (A, B)

Hint : $n_1 \sin i = n_2 \sin r$.

Solution :



Let SPM be the spherical surface separating two media with refractive index n_1 and n_2 respectively.

So for refraction at point A

$n_1 \sin i = n_2 \sin r \quad \Rightarrow \quad \frac{\sin i}{\sin r} = \frac{n_2}{n_1}$ …(i)

Take $|CP| = R$, $|OC| = r_1$ and $|IC| = r_2$

$\angle AIC = \beta$, $\angle AOC = \alpha$ then

in ΔAOC, $\frac{\sin i}{\sin\alpha} = \frac{r_1}{|R|}$ …(ii)

Also in ΔAIC $\frac{\sin r}{\sin\beta} = \frac{r_2}{|R|}$ …(iii)

For a very particular case when $\angle\alpha = \angle r$ and $\angle\beta = \angle i$ then equation (ii) will become.

$\frac{\sin i}{\sin\alpha} = \frac{r_1}{|R|} = \frac{n_2}{n_1}$

$\Rightarrow \quad r_1 = \frac{n_2}{n_1}|R|$

and equation (iii) will become $\frac{\sin r}{\sin B} = \frac{r_2}{|R|} = \frac{n_1}{n_2}$

$\Rightarrow \quad |r| = \frac{n_1}{n_2}|R|$

Irrespective of the point A being paraxial or marginal. Hence all ray emanating from O seems to be emanating from i and vice-versa.

3. Answer (A, C, D)

Hint :

If there is no dissipating force then the system will oscillate.

Solution : If there is no dissipating force/s then the system will oscillate.

But if there is some small resistance then

$mg - \frac{vB^2\ell^2}{r} = \frac{mdv}{dt}$

Before it attains the steady constant speed, $\frac{dv}{dt}$ is positive. That means part of the work done by gravity is appearing as increase in kinetic energy and rest are dissipating as heat energy.

Finally $\frac{dv}{dt} = 0$. *i.e.* $v_0 = \frac{mgr}{B^2\ell^2}$

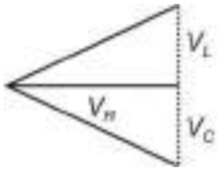
4. Answer (A, D)

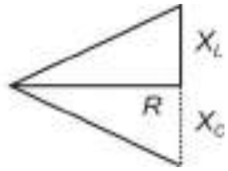
Hint : At resonance $|\omega L| = \frac{1}{|\omega C|}$

Solution : At resonance $|\omega L| = \frac{1}{|\omega C|}$

$V_L = 60$ volts

$V_R = 80$ volts





Clearly, $\frac{V_L}{V_R} = \frac{\omega L}{R} \quad \Rightarrow \quad \frac{60}{80} = \frac{\omega L}{240}$

$\Rightarrow \ \omega L = 180\ \Omega \quad \Rightarrow \quad L = \frac{180}{90} = 2\text{H}$

Similarly $\frac{1}{\omega C} = 180 \ \Rightarrow \ C = \frac{1}{90 \times 180} = \frac{1}{16200}\text{F}$

For current to lag by 45° $|R| = \left|\omega L - \frac{1}{\omega C}\right|$

$\Rightarrow \ \omega^2 LC - 1 = 240\ \omega C$

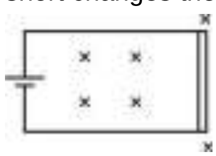
$\Rightarrow \ \omega^2 - 120\omega - 8100 = 0$

$\therefore \quad \omega = 60 + 10\sqrt{117}$ rad/s

5. Answer (B, C)

Hint : $\int F dt = \int iB\ell \cdot dt$

Solution : Let i be the current at any time during short changes then



$F(t) = i\ell B$ and impulse $\int F(t)\, dt = \int \ell\, B\, i\, dt$

$\Rightarrow \ mv = \ell\, B\, \Delta q \quad \Rightarrow \quad \Delta q = \frac{mv}{B\ell}$

Work done by battery $w_b = \frac{\varepsilon mv}{B\ell}$

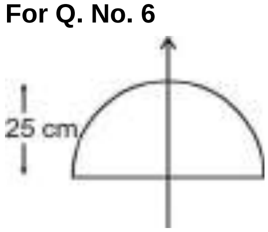
$= \Delta w_{\text{loss}} = \frac{\varepsilon mv}{B\ell} - \frac{1}{2}mv^2$

6. Answer (D)
7. Answer (C)
8. Answer (B)

Hint : $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

Solution for Q. Nos. 6 to 8

For Q. No. 6



$\frac{1}{V_1} + \frac{\mu}{25} = \frac{(1-\mu)}{-50}$

$\Rightarrow \ \frac{1}{V_1} + \frac{3}{2 \times 25} = \frac{1}{100}$

$\Rightarrow \ \frac{1}{V_1} = \frac{1}{100} - \frac{6}{100}$

$\Rightarrow \ \frac{1}{V_1} = \frac{-5}{100} \quad \therefore \ V_1 = -20\text{ cm}$

$\therefore$ Shifting in 1st case is 25 – 20 = 5 cm

For Q. No. 7

$\frac{1}{V_2} + \frac{\mu}{75} = \frac{(1-\mu)}{-50}$

$\Rightarrow \ \frac{1}{V_2} = \frac{1}{100} - \frac{3}{2 \times 75}$

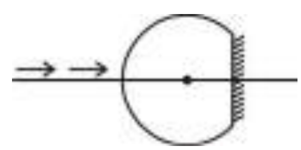
$\Rightarrow \ \frac{1}{V_2} = \frac{1}{100} - \frac{1}{50}$

$\Rightarrow \ \frac{1}{V_2} = \frac{-1}{100}$

$\therefore \ V_2 = -100$

Shifting $\Rightarrow$ 100 – 75 = 25 cm

For Q. No. 8



$\frac{\mu}{V_3} - \frac{1}{\infty} = \frac{(\mu - 1)}{50}$

$\therefore \ \frac{3}{2V_3} = \frac{1}{2 \times 50}$

$\Rightarrow \ V_3 = 150$ cm

So for mirror (at plane surface) object distance is +75 cm.

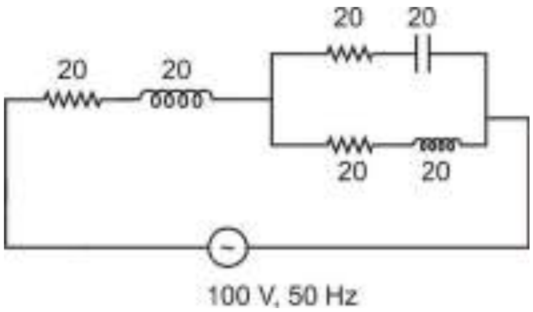
$\therefore$ Image would be at 75 cm opposite to silvered surface which falls on the periphery of the sphere.

9. Answer (B)
10. Answer (A)

11. Answer (D)

Hint : $I = \frac{V}{z}$ (Here z is impedance)

Solution for Q. Nos. 9 to 11



$$X_L = \omega L = 2\pi \times 50 \frac{1}{5\pi} = 20\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1\ \pi \times 10^6}{2\pi \times 50 \times 500} = 20\ \Omega$$

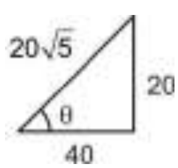
Let $z_1 = 20 + 20j$; $z_2 = 20 - 20j$; $z_3 = 20 + 20j$

$$\therefore \quad \frac{1}{z} = \frac{1}{z_2} + \frac{1}{z_3} = \frac{1}{20-20j} + \frac{1}{20+20j}$$

$$= \frac{20j + 20 + 20 - 20j}{400 + 400}$$

$$\Rightarrow \quad z' = \frac{800}{40} = 20\ \Omega$$

$\therefore \quad z_{eq} = 20 + 20j + 20 = 40 + 20j$



$$\therefore \quad |z| = 20\sqrt{5} \cdot \tan\theta = \frac{1}{2} \quad \cos\theta = \frac{2}{\sqrt{5}}$$

$$\therefore \quad I = \frac{100\ \angle 0}{20\sqrt{5}\ \angle\theta} = \sqrt{5} \ \Rightarrow I_0 = \sqrt{10}$$

Current amplitude = $\sqrt{10}$ ampere

$\therefore$ Power dissipation = $I_{rms} \cdot V_{rms} \cos\phi$

$\Rightarrow \quad z_2 I_C = z_3 I_L \qquad \Rightarrow \quad z_2 I_C = (I - I_C) z_3$

$\Rightarrow \quad (z_2 + z_3) I_C = z_3 I$

$$\therefore \quad I_C = \frac{I(20+20j)}{(20+20j+20-20j)} = \frac{I \times 20(1+j)}{20 \times 2}$$

$$\Rightarrow \quad I_C = \frac{I}{\sqrt{2}} \angle 45^\circ$$

So I_C is in lead by phase of 45° or $\frac{\pi}{4}$ rad w.r.t. to total current I.

12. Answer (D)

Hint : $E \cdot 2\pi R = \pi R^2 \frac{dB}{dt}$.

Solution :

$$E \cdot 2\pi R = \pi R^2 \times \frac{\Delta B}{\Delta t}$$

$$\Rightarrow \quad E = \frac{R}{2} \frac{B}{\Delta t}$$

$dF = dq \cdot E$ and $d\tau = REdq$

$$\Rightarrow \quad \tau = \int REdq = REq$$

$$\text{So} \int \tau \cdot dt = \lim_{x \to 0} \frac{R^2 q\, B}{2\Delta t} \cdot \Delta t = I\omega$$

$$\Rightarrow \quad \frac{R^2 q B}{2} = mR^2 \cdot \omega$$

$$\Rightarrow \quad \omega = \frac{qB}{2m}$$

13. Answer (C)

Hint : $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$.

Solution : When viewed from spherical side.

$$\frac{1}{v_1} + \frac{3}{2 \times 8} = \frac{1}{2 \times 16}$$

$$\Rightarrow \quad \frac{1}{v_1} = \frac{1}{32} - \frac{3}{16} = \frac{-5}{32}$$

Image position is $\frac{32}{5}$ cm inside the hemisphere from its periphery.

And when viewed from plane side then image is at $\frac{16}{3}$ cm. Inside the plane surface.

$$\therefore \quad \Delta x = 16 - \frac{16}{3} - \frac{32}{5} = \frac{64}{15} \text{ cm}$$

14. Answer (A)

Hint : $dB = \int B \cdot ds$

Solution :

$$\vec{B} = \frac{B_0\left(x\hat{i} + y\hat{j} + z\hat{k}\right)}{\left(x^2 + y^2 + z^2\right)^{\frac{3}{2}}}$$

Consider a small ring of radius R and width 'dR'.

Then $R = \sqrt{y^2 + z^2}$

$$d\phi = \frac{B_0}{\left(x^2+y^2+z^2\right)^{\frac{3}{2}}}\ (x\hat{i} + y\hat{j} + z\hat{k}) \cdot 2\pi R\ dR\ \hat{i}$$

$$\Rightarrow\ d\phi = \frac{B_0 x \cdot 2\pi R dR}{\left[R^2+x^2\right]^{\frac{3}{2}}} = B_0 \pi x \int \frac{2RdR}{\left(R^2+x^2\right)^{\frac{3}{2}}}$$

$$\Rightarrow\ \phi = 2B_0 \pi x \left[\frac{-1}{\sqrt{R^2+x^2}}\right]_0^a$$

$$\Rightarrow\ \phi = 2\pi x\ B_0 \left[\frac{1}{x} - \frac{1}{\sqrt{a^2+x^2}}\right]$$

15. Answer (B)

Hint : $i = I_0\left(1 - e^{-\frac{tR}{L}}\right)$

Solution :

$$i = \frac{\varepsilon}{R}\left(1 - e^{-\frac{tR}{L}}\right)$$

$$\therefore\ \Delta q = \int dq = \int i\,dt = \frac{\varepsilon}{R}\int_0^{2\tau}\left(1 - e^{-\frac{tR}{L}}\right)dt$$

$$\Rightarrow\ \Delta q = \frac{\varepsilon}{R}\left[t + \frac{L}{R}e^{-\frac{tR}{L}}\right]_0^{2\tau}$$

$$= \frac{\varepsilon}{R}\left[\frac{2L}{R} + \frac{L}{Re^2} - 0 - \frac{L}{R}\right]$$

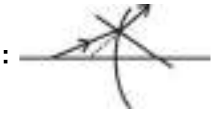
$$\Rightarrow\ \Delta q = \frac{\varepsilon}{R}\left[\frac{L}{R} + \frac{L}{Re^2}\right]$$

$$\Rightarrow\ \Delta q = \frac{\varepsilon L}{R^2}\left[\frac{e^2+1}{e^2}\right]$$

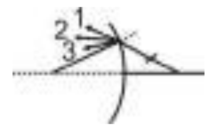
16. Answer A(Q, S); B(P, S); C(P, Q, R, S, T); D(P, Q, R, S, T)

Hint : $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ and magnification $m = \frac{\mu_1 v}{\mu_2 u}$.

Solution :

For option (A) :  ray seems to diverge always after refraction so it will form virtual and diminished image.

For option (B) :  after refraction ray would bend and meet the axis some what at shorter distance. So, it will form real and diminished image.

For option (C) : 

it will try to bend towards normal so, depending upon the extent of bending the refracted ray may meet really or it may seems to be diverging. So it may form real image or virtual image also. Magnification may also vary on being diminished to magnified.

For option (D) :  Depending upon the extend of bending the refracted ray may meet really or it may seems to be diverging. So, it may form real or virtual image. Also magnification may vary from being diminished to being magnified.

17. Answer A(Q, R, T); B(P, R); C (S, T); D (R, S, T)

Hint : If field along the dipole, then torque is zero.

Solution :

For option (A) : Opposite current means they will repel and dipoles are at 180° torque is zero but rotationally unstable.

For option (B) : Decrement of current in one loop will induced the current in same sense in other loop, so attractive force and no torque.

For option (C) : Force/s are zero but torque is not zero.

For option (D) : Force/s are zero, torque is also zero but dipoles are oppositely aligned so rotationally unstable.

18. Answer (03)

Hint : $I = \frac{V}{Z}$ (Here Z is impedance)

Solution : In capacitive circuit $I_C = \frac{50 \sin \omega t}{2R \angle -60°}$

$\Rightarrow \quad I_C = \frac{25}{R}\sin(\omega t + 60°)$

$\therefore \quad V_C = \frac{25}{R}\sin(\omega t + 60°) \times \sqrt{3}R\ \angle -90°$

$= 25\sqrt{3}\sin(\omega t - 30°)$

$\therefore \quad \omega t - 30° = 90°$

Now in inductive circuit $I_L = \frac{50\sin(\omega t)}{2R\ \angle 30°}$

$\therefore \quad v_{R_1} = 3\sqrt{R}\ \angle 0° \ \frac{50\sin(\omega t)}{2R\ \angle 30°}$

$= 25\sqrt{3}\sin(\omega t - 30°)$

$\therefore \quad v_{R_1} = 25\sqrt{3}$ volts

$\therefore \quad N = 3$

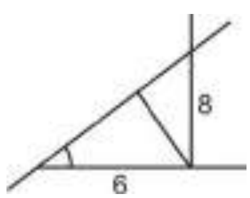
19. Answer (08)

Hint : $E_{\text{Ind}} = \left|\frac{d\phi}{dt}\right|$

Solution : $E \cdot 2\pi r = \pi r^2 \cdot \frac{dB}{dt}$

$\Rightarrow \quad \vec{E} = \frac{r}{2}\frac{dB}{dt}$

$r = 6\sin 53° = 6 \times \frac{4}{5} = \frac{24}{5}$



$\therefore \quad \int E dl = \frac{24}{5}\frac{1}{2} \times \frac{1}{3} \times 10$

$\left(\int E dl\right) = 8$

20. Answer (01)

Hint : $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \left(\frac{\mu_2 - \mu_1}{R}\right)$ use this.

Solution : Let 'x' be the distance of object from S_1 so, for first refraction let v_1 is the image distance then,

$\frac{3}{2v_1} + \frac{4}{3x} = \frac{-1}{6} \quad \Rightarrow \quad \frac{3}{2v_1} = \frac{-1}{6} - \frac{4}{3x}$

$\Rightarrow \quad v_1 = \frac{-9x}{8+x} \quad$ *i.e.* $\frac{9x}{8+x}$ from S_1

Now for 2nd refraction from S_2 object distance would be $u_2 = \frac{9x}{8+x} + 1 = \frac{8+10x}{8+x}$

For refraction from 2nd surface S_2

$\frac{1}{v_2} - \frac{3}{2u_2} = \frac{\left(1 - \frac{3}{2}\right)}{-2}$

$\Rightarrow \quad v_2 = -(x+1)$

So, $\frac{-1}{x+1} + \frac{3(8+x)}{2(8+10x)} = \frac{1}{4}$

$\Rightarrow \quad \frac{-16 - 20x + 24x + 24 + 3x^2 + 3x}{4(x+1)(4+5x)} = \frac{1}{4}$

$\Rightarrow \quad 3x^2 + 7x + 8 = 5x^2 + 9x + 4$

$\Rightarrow \quad 2x^2 + 2x - 4 = 0$

$\Rightarrow \quad x^2 + x - 2 = 0$

$\Rightarrow \quad x = 1, -2$

$\therefore \quad$ Object distance from S_1 = 1 m

PART - II (CHEMISTRY)

21. Answer (B, C)

Hint :

$$FeSO_4 + 2NaOH \longrightarrow \underset{\text{Dirty green ppt.}}{Fe(OH)_2} + Na_2SO_4$$

Solution :

$$CuSO_4 + 2NaOH \longrightarrow \underset{\text{Bluish white ppt.}}{Cu(OH)_2} + Na_2SO_4$$

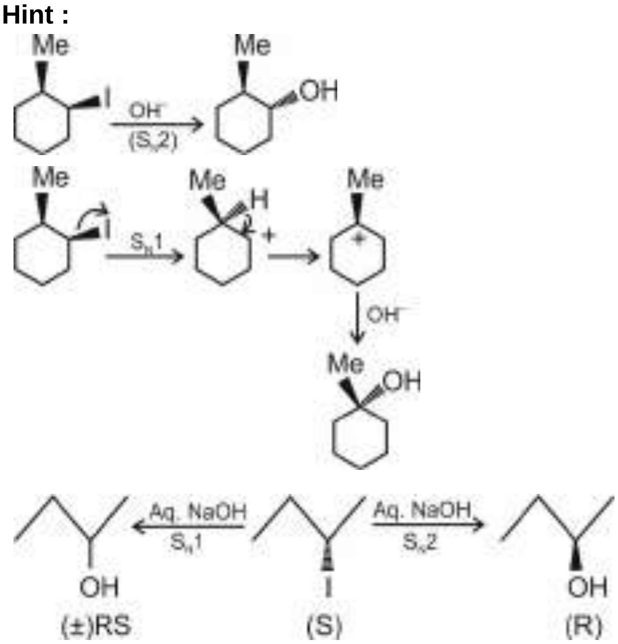
22. Answer (A, B, D)

Hint : Fact based and evident from diagram.

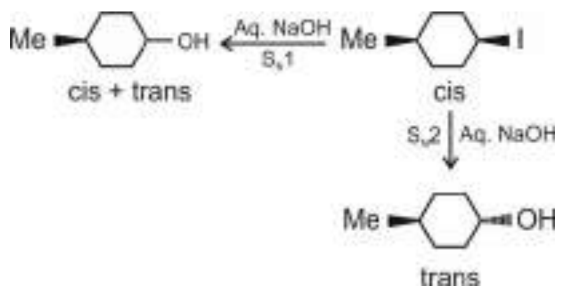
Solution : Copper (II) acetate is dimeric and hydrated as $\left[Cu(OCOCH_3)_2 \cdot H_2O\right]_2$.

23. Answer (A, B, D)

Hint :



Solution :



24. Answer (C)

Hint : Fact based.

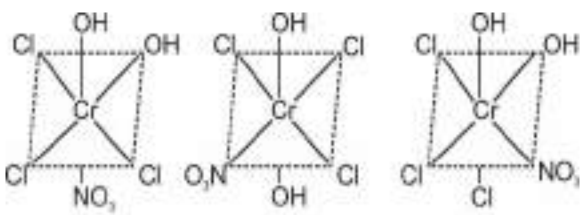
Solution : The following reactions take place in the reaction mixture

$MnCl_2 + 2NaOH + H_2O_2 \rightarrow MnO(OH)_2 + 2NaCl + H_2O$

$2CrCl_3 + 10NaOH + 3H_2O_2 \rightarrow 2Na_2CrO_4 + 6NaCl + 8H_2O$

25. Answer (B, D)

Hint : The given complex ion has three geometrical isomers and each of them is optically inactive



(I) (II) (III)

Solution : The central metal ion (Cr^{3+}) has 3 unpaired electrons. So, magnetic moment of complex is 3.87 BM

26. Answer (B)

Hint : The solution (A) contains I^- ions, as seen from the following reaction

$$\underset{(A)}{2I^-} + Hg(NO_3)_2 \longrightarrow \underset{\substack{\text{(Scarlet ppt.)} \\ (B)}}{HgI_2} + 2NO_3^-$$

Solution : $HgI_2 + 2KI \longrightarrow \underset{(C)}{K_2[HgI_4]}$

The hybridisation of $Hg^{2+}(5d^{10}6s^0)$ in (C) is sp^3. Therefore, shape of complex anion in (C) is tetrahedral.

27. Answer (C)

Hint : The reagent (X) must be $Bi(NO_3)_3$ which reacts with I^- ions to give black ppt. of BiI_3

Solution :

$$3I^- + Bi(NO_3)_3 \longrightarrow \underset{\text{Black ppt.}}{BiI_3} + 3NO_3^-$$

28. Answer (D)

Hint : The black ppt. of BiI_3 when heated in presence of water turns orange due to the formation of BiOI

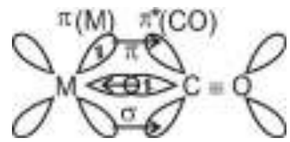
Solution :

$$BiI_3 + H_2O \longrightarrow \underset{\text{Orange ppt.}}{BiOI} + 2HI$$

29. Answer (D)

Hint : CO is π^* acceptor and σ donor.

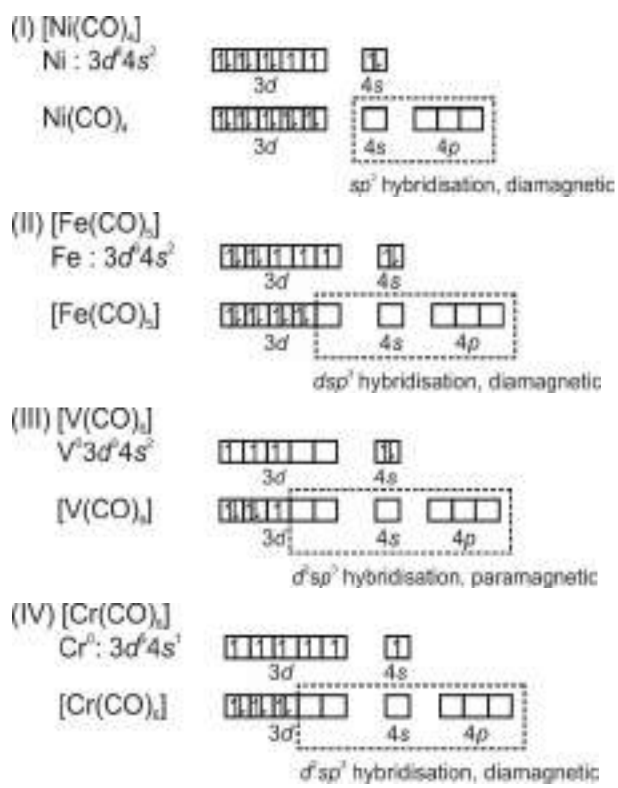
Solution : In metal carbonyl, the C-atom of CO ligand uses its filled orbital and metal uses its vacant orbital to form σ type of co-ordinate covalent bond. Simultaneously, the metal uses its filled orbital and CO uses its π-type of antibonding molecular orbital to form π -type of co-ordinate bond.



30. Answer (A)

Hint : CO form strong field ligand with metals.

Solution :



So, (II) and (IV) are inner orbital complexes and diamagnetic.

31. Answer (B)

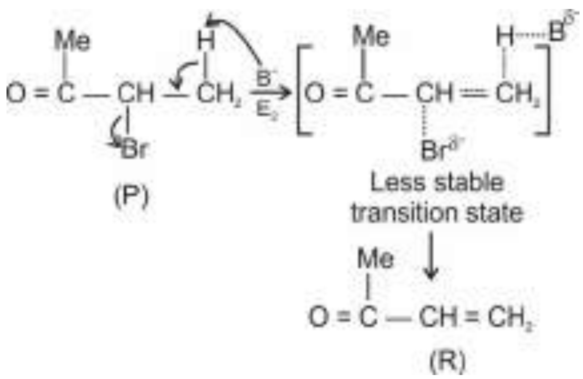
Hint : Greater the electron density on metal, lesser will be the C — O bond strength.

Solution :

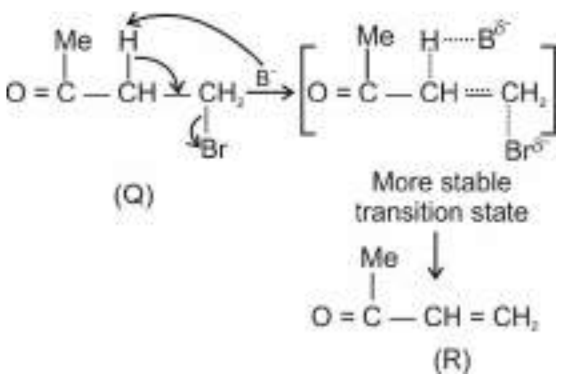
In the given metal carbonyls i.e. $[Ni(CO)_4]$, $[Fe(CO)_5]$ and $[Cr(CO)_6]$ the number of electron pairs available are 5, 4 and 3 respectively. So, the share of $d\pi$ electrons from Ni, Fe and Mn for back donating to each CO molecule in these complexes will be 5/4, 4/5 and 3/6. Hence, C — O bond length in these complexes will be in the order of (I) > (II) > (III).

32. Answer (B)

Hint :



Solution :



The H-atom attached to β C-atom with respect to Br-atom is more acidic in (Q) than in (P). Therefore, the transition state formed in elimination of HBr from (Q) is more stable than that from (P). So, (Q) reacts faster than (P) in the given elimination reaction.

33. Answer (C)

Hint : Hexachloridoplatinic acid is formed when platinum reacts with aqua regia

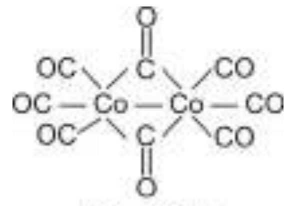
Solution : $3Pt + 16H^+ + 4NO_3^- + 18Cl^- \longrightarrow$

$$3\left[PtCl_6\right]^{2-} + 4NO + 8H_2O$$

34. Answer (A)

Hint : Fact based.

Solution :



$[Co_2(CO)_8]$

35. Answer (B)

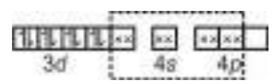
Hint : Co-ordination compound (X) must be cis isomer having permanent dipole moment since it dissolves in polar solvents.

Solution : Co-ordination compound (Y) must be trans isomer which is non-polar since it dissolves in non-polar solvents.

36. Answer A(R, T); B(P, S, T); C(P, Q, S); D(P, Q, S)

Hint : CN^- is a strong field ligand and F^- is weak field ligand.

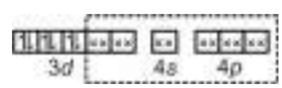
Solution :

(A) $[Ni(CN)_4]^{2-}$ 

Hybridisation : dsp^2

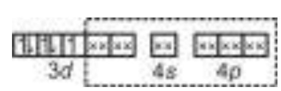
Shape : Square planar

Magnetic behaviour : Diamagnetic

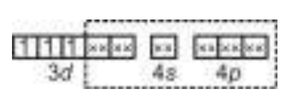
(B) $[NiF_6]^{2-}$ 

d^2sp^3 hybridisation; Octahedral; Diamagnetic

[**Note :** As oxidation state of nickel ion increases, the splitting energy increases. Thus F^- ions cause pairing of unpaired electron]

(C) $[Fe(CN)_6]^{3-}$ 

d^2sp^3 hybridisation; Octahedral; Paramagnetic

(D) $[Cr(H_2O)_6]^{3+}$ 

d^2sp^3 hybridisation; Octahedral; Paramagnetic

37. Answer A(P, R, T); B(P, R); C(S, T); D(Q, R)

Hint :

$$CoCl_2 + 7KNO_2 + 2CH_3COOH \rightarrow \underset{\text{Yellow ppt.}}{K_3\left[Co(NO_2)_6\right]}$$

$$+2KCl + 2CH_3COOK + \underset{\text{Neutral gas}}{NO} + H_2O$$

$$H_2S + 2HNO_2 \rightarrow S + \underset{\text{Neutral gas}}{2NO} + 2H_2O$$

Solution :

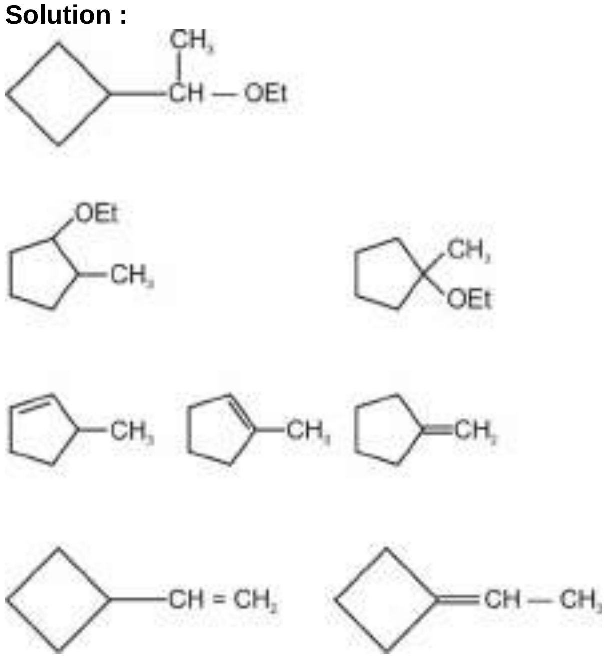
$$4FeCl_3 + 3K_4[Fe(CN)_6] \longrightarrow Fe_4[Fe(CN)_6]_3 + 12KCl$$

$$Na_2S_2O_3 + 2HCl \rightarrow 2NaCl + \underset{\substack{\text{Acidic}\\ \text{gas}}}{SO_2} + \underset{\substack{\text{Yellow}\\ \text{ppt.}}}{S} + H_2O$$

38. Answer (08)

Hint : The total number of substitution (by S_N1) and elimination (by E_1) products formed in the given reaction are 8.

Solution :



39. Answer (06)

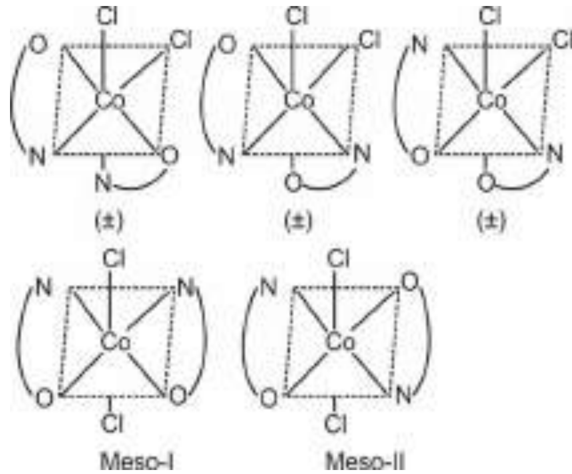
Hint : Test for G-II cations.

Solution : Only Ag^+, Pb^{2+}, Bi^{3+}, Sn^{2+}, As^{3+} and Cd^{2+} ions will get precipitated on passing H_2S gas in presence of dil HCl.

40. Answer (08)

Hint : The possible stereoisomers of the given complex ion are 8.

Solution :



PART - III (MATHEMATICS)

41. Answer (A, B, C)

Hint :

$\int \cos x\, dx = \sin x + c$ and $\int \sin x\, dx = -\cos x + c$

Solution :

$$k^2\frac{\sin 3x}{3} + \frac{k^2}{2}\sin x - \cos x - 2k\sin x\Big|_0^{\pi/2} \le 1$$

$$\Rightarrow \left(-\frac{k^2}{3} + \frac{k^2}{2} - 0 - 2k\right) - (0 + 0 - 1 - 0) \le 1$$

$\Rightarrow\ + k^2 - 12k + 6 \le 6$

$\Rightarrow\ k(k - 12) \le 0$

$k \in [0, 12]$

42. Answer (A, D)

Hint : $\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$

Solution : $\int \dfrac{x^4 dx}{1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}}$

$$= 4!x - 4!\ln\left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots + \frac{x^4}{4!}\right) + c$$

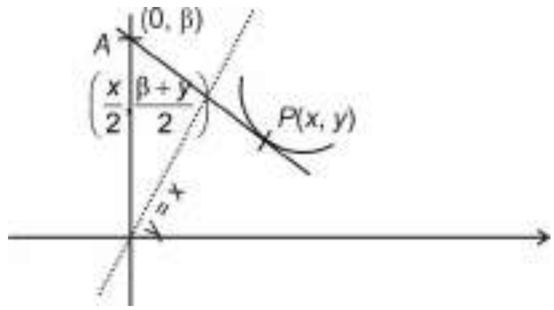
$\Rightarrow\ k = 4,\ f(1) = \frac{15}{24}$ and $f(0) = 1$

43. Answer (A, B, D)

Hint : Form differential equation using the equation of tangent at (x, y) i.e.

$$Y - y = \frac{dy}{dx}(X - x)$$

Solution :



Equation of tangent at $P(x, y)$

$$Y - y = \frac{dy}{dx}(X - x)$$

$$\beta = y - x\frac{dy}{dx}$$

Hence $$\frac{x}{2} = \frac{y - x\frac{dy}{dx} + y}{2}$$

$$\Rightarrow \quad x = 2y - x\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} - \frac{2y}{x} = -1$$

$$\Rightarrow \quad d\left(\frac{y}{x^2}\right) = -\frac{1}{x^2}dx$$

$$\Rightarrow \quad \frac{y}{x^2} = \frac{1}{x} + c$$

$\Rightarrow$ $y = x - x^2$

44. Answer (B, C)

Hint :

Minimize $\int_{x_1}^{x_2}\left((x^2+2x-3)-(kx+1)\right)dx$ where x_1 and x_2 are intersection points of given curves.

Solution : $kx + 1 = x^2 + 2x - 3$

$\Rightarrow$ $x^2 + (2 - k)x - 4 = 0$

$$\Rightarrow \quad x = \frac{k-2 \pm \sqrt{(k-2)^2 - 4(-4)}}{2}$$

$$= \frac{k-2 \pm \sqrt{k^2-4k+20}}{2} \quad \text{(say)}\ x_1, x_2 \qquad \ldots\text{(i)}$$

$$A = \left|\int_{x_1}^{x_2}\left(\left(x^2+2x-3\right)-\left(kx+1\right)\right)dx\right|$$

$$= \left|\frac{x^3}{3} + \frac{(2-k)x^2}{2} - 4x\right|_{x_1}^{x_2}$$

$$= \left|\frac{x_2^3 - x_1^3}{3} + \left(\frac{2-k}{2}\right)\left(x_2^2 - x_1^2\right) - 4\left(x_2 - x_1\right)\right| \quad \ldots\text{(ii)}$$

From (i) $x_1 + x_2 = k - 2$, $x_1x_2 = -4$ and $|x_1 - x_2|$ $= \sqrt{k^2-4k+20}$ and using these results in (ii) we get

$$= \left|\frac{(x_2-x_1)\left((x_1+x_2)^2 - x_1x_2\right)}{3} + \left(\frac{2-k}{2}\right)(x_2-x_1)(x_1+x_2) - 4(x_2-x_1)\right|$$

$$= \left|\sqrt{k^2-4k+20}\left(\frac{(k-2)^2+4}{3} + \left(\frac{2-k}{2}\right)(k-2) - 4\right)\right|$$

$$= \left|\sqrt{k^2-4k+20}\left(\frac{2(k-2)^2+8-3(k-2)^2-24}{6}\right)\right|$$

$$= \left|\sqrt{k^2-4k+20}\left(\frac{-(k-2)^2}{6} - \frac{8}{3}\right)\right|$$

$\Rightarrow$ $A_{\min}$ at $k = 2$

and $$A_{\min} = \left|\sqrt{16}\left(\frac{-8}{3}\right)\right| = \frac{32}{3} \text{ sq. units}$$

45. Answer (A, D)

Hint : Use property $\int_a^b f(a+b-x)dx = \int_a^b f(x)dx$

Solution : $$I = \int_0^{\pi/2} t(\sin 2x)\sin x\,dx \qquad \ldots\text{(i)}$$

$$I = \int_0^{\pi/2} f\left(\sin(\pi - 2x)\right)\cos x\,dx$$

$$I = \int_0^{\pi/2} t(\sin 2x)\cos x\,dx \qquad \ldots\text{(ii)}$$

Add (i) and (ii) we get

$$2I = \int_0^{\pi/2} f(\sin 2x)(\sin x + \cos x)dx$$

$$= \sqrt{2}\int_0^{\pi/2} f(\sin 2x)\sin\left(x + \frac{\pi}{4}\right)dx$$

Put $x = \frac{\pi}{4} - \theta$

$\Rightarrow$ $dx = -d\theta$

$$= \sqrt{2}\int_{+\pi/4}^{-\pi/4} f\left(\sin\left(2\left(\frac{\pi}{4} - \theta\right)\right)\right)\cos\theta(-d\theta)$$

$$= \sqrt{2}\int_{\pi/4}^{-\pi/4} f(\cos 2\theta)\cos\theta(-d\theta)$$

$$= 2\sqrt{2}\int_0^{\pi/4} f(\cos 2\theta)\cos\theta\,d\theta$$

$$I = \sqrt{2}\int_0^{\pi/4} f(\cos 2\theta)\cos\theta\,d\theta$$

46. Answer (B)
47. Answer (B)
48. Answer (C)

Hint for Q. Nos. 46 to 48

$$\frac{d}{dx}\int_{f(x)}^{g(x)} h(x) = h(g(x))\cdot g'(x) - h(f(x))\cdot f'(x)$$

Solution for Q. Nos. 46 to 48

Using Leibnitz's rule

$f(x) = x^2 - x^2 f(x)$

$\Rightarrow$ $f(x) = \frac{x^2}{1+x^2}$ ($f(x)$ is even function)

$g(x) = f(x) - 1 \Rightarrow g(x) = \frac{x^2-1-x^2}{1+x^2} = -\frac{1}{1+x^2}$

Range of $g(x) \in [-1, 0)$

$$I = \int_0^2 \left[-\frac{1}{g(x)}\right] dx = \int_0^2 1 + \left[x^2\right] dx$$

$$= x\Big|_0^2 + \int_0^1 0dx + \int_1^{\sqrt{2}} dx + \int_{\sqrt{2}}^{\sqrt{3}} 2dx + \int_{\sqrt{3}}^{2} 3dx$$

$$= 2 + 0 + \left(\sqrt{2}-1\right) + 2\left(\sqrt{3}-\sqrt{2}\right) + 3\left(2-\sqrt{3}\right)$$

$$= 7 - \sqrt{2} - \sqrt{3}$$

$\Rightarrow$ $[I] = 3$

49. Answer (A)
50. Answer (D)
51. Answer (D)

Hint for Q. Nos. 49 to 51

$\frac{dD}{dt} \propto D \Rightarrow \frac{dD}{dt} = kD$ (where k is constant of proportionality)

Solution for Q. Nos. 49 to 51 :

Let data at any time 't' is D

$$\frac{dD}{dt} \propto D$$

$$\Rightarrow \frac{dD}{dt} = kD$$

$$\Rightarrow \int_{500}^{D} \frac{dD}{D} = \int_0^t kdt$$

$$\Rightarrow \ln D\Big|_{500}^{D} = kt$$

$$\Rightarrow \frac{D}{500} = e^{kt}$$

$\Rightarrow$ $D = 500\, e^{kt}$

At $t = 2$, $D = 0.9 \times 500$

$\therefore$ $0.9 \times 500 = 500\, e^{k2}$

$\Rightarrow$ $0.9 = e^{2k}$

$\Rightarrow$ $2k = \ln 0.9$

$$\Rightarrow k = \frac{\ln(0.9)}{2}$$

$$\Rightarrow D = 500e^{\left(\frac{1}{2}\right)(\ln(0.9))t} \quad \ldots(i)$$

At $t = 4$ h, if $D = \frac{500}{2}$ in (i)

$D = 500e^{2\ln(0.9)}$

$$\Rightarrow \frac{1}{2} = e^{\frac{1}{2}\ln(0.9)t}$$

$$\Rightarrow -\ln 2 = \frac{1}{2}\ln(0.9)t$$

$$\Rightarrow t = \frac{2\ln\left(\frac{1}{2}\right)}{\ln(0.9)}$$

52. Answer (C)

Hint :

$$f(x)g'(y)dy + f'(x)g(y)dx = \int d\left(f(x)g(y)\right)$$

Solution : $xdy + \cos y \ln xdy = 6x^2dx - \sin y\, dx$

$$\Rightarrow dy + \cos y \ln xdy = 6xdx - \frac{\sin y}{x}dx$$

$$\Rightarrow dy + \cos y \ln xdy + \frac{\sin ydx}{x} = 6xdx$$

$$\Rightarrow \int dy + \int d\left(\sin y \ln x\right) = 6\int xdx$$

$\Rightarrow$ $y + \sin y \ln x = 3x^2 + c$

For $y(1) = 3 \Rightarrow 3 + 0 = 3 + c \Rightarrow c = 0$

$\Rightarrow$ $y + \sin y \ln x = 3x^2$

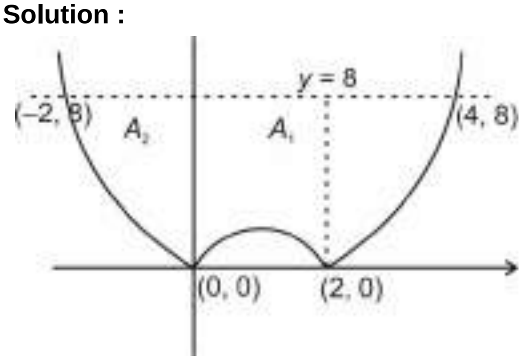
Put $x = e$ we get

$y + \sin y = 3e^2$

53. Answer (C)

Hint : Area = $\int_\alpha^\beta \left(f(x) - g(x)\right)dx$, where α and β are intersection points of given curves

Solution :



$$A_1 = \int_0^2 8-\left(2x-x^2\right)dx + \int_2^4 8-\left(x^2-2x\right)dx$$

$$= 8x - x^2 + \frac{x^3}{3}\Big|_0^2 + 8x - \frac{x^3}{3} + x^2\Big|_2^4$$

$$= \left(16-4+\frac{8}{3}\right)+\left(32-\frac{64}{3}+16-16+\frac{8}{3}-4\right)$$

$$= \frac{44}{3}+\left(28-\frac{56}{3}\right)=\frac{44}{3}+\frac{28}{3}=\frac{72}{3}=24 \text{ sq.units}$$

$$A_2 = \int_{-2}^0 8-\left(x^2-2x\right)dx = 8x-\frac{x^3}{3}+x^2\Big|_{-2}^0$$

$$= 0-\left(-16+\frac{8}{3}+4\right)=12-\frac{8}{3}=\frac{28}{3} \text{ sq. units}$$

$$k = \frac{24}{28}\cdot 3 \quad \Rightarrow \quad [k] = 2$$

54. Answer (C)

Hint : $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{kn} f\left(\frac{r}{n}\right)\cdot\frac{1}{n} = \int_0^k f(x)dx$

Solution : Let $L = \lim\limits_{n\to\infty}\sum\limits_{r=1}^{4n}\frac{1}{r+\sqrt{rn}}$

$$= \lim_{n\to\infty}\sum_{r=1}^{4n}\frac{1}{n}\left(\frac{1}{\left(\frac{r}{n}\right)+\sqrt{\frac{r}{n}}}\right)$$

$$= \int_0^4 \frac{dx}{\left(x+\sqrt{x}\right)}$$

Put $x = t^2$

$\Rightarrow$ $dx = 2t\,dt$

$\Rightarrow$ $L = \int_0^2 \frac{2t\,dt}{t(1+t)} = 2\ln(1+t)\Big|_0^2 = 2\ln 3 = \ln 9$

55. Answer (B)

Hint : Put $\sin 2x = 2\sin x\cos x$ and make partial fractions

Solution : $\int \frac{2\sin x(1-\cos x)dx}{2\sin x\cos x(1+\cos x)}$

$\Rightarrow$ $\int \frac{1-\cos x\,dx}{(1+\cos x)\cos x}$

$\Rightarrow$ $\int\left(\frac{1}{\cos x}-\frac{2}{1+\cos x}\right)dx$

$\Rightarrow$ $\int \sec x dx - 2\int \frac{dx}{2\cos^2\frac{x}{2}}$

$\Rightarrow$ $\int \sec x dx - \int \sec^2\frac{x}{2}dx$

$\Rightarrow$ $\ln|\sec x+\tan x| - 2\tan\frac{x}{2}+c$

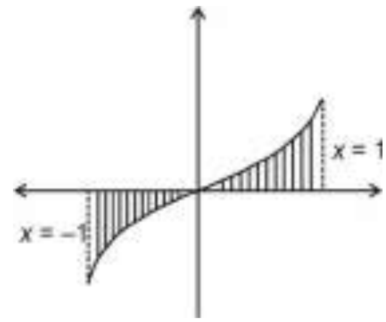
56. Answer A(Q, R, S, T); B(Q, R, S, T); C(T); D(P, Q, R, S)

Hint : Area between two curve $f(x)$ and $g(x)$

$= \int_\alpha^\beta \left(f(x)-g(x)\right)dx$, where α, β are the point of intersection.

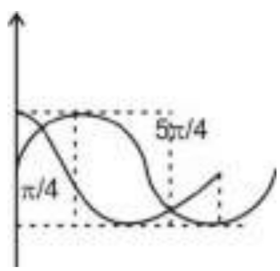
Solution :

(A) $y = |x|x$



$$A = \int_{-1}^0 -x^2dx + \int_0^1 x^2dx = \frac{1}{3}+\frac{1}{3}=\frac{2}{3} \text{ sq. units}$$

(B)

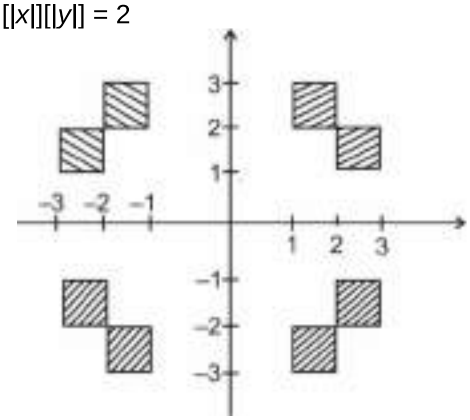


$$A = \int_{\pi/4}^{5\pi/4}\left(\sin x-\cos x\right)dx = \left(-\cos x-\sin x\right)\Big|_{\pi/4}^{5\pi/4}$$

$$= \left(+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)-\left(-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\right)$$

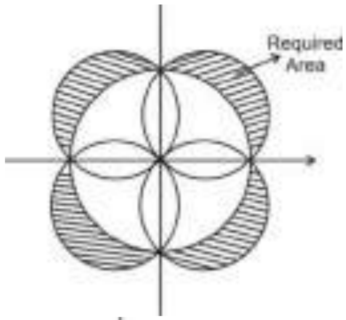
$$= 2\sqrt{2} \text{ sq. units}$$

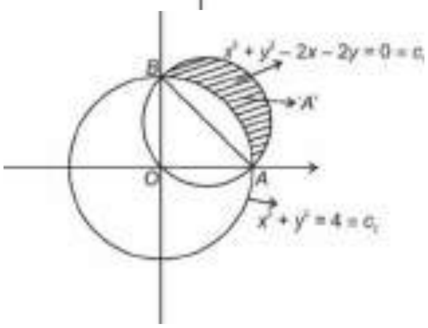
(C) $[|x|][|y|] = 2$



A = 8 sq. units

(D)





Required Area = 4A = 4(Area of semicircle of C_1 – (Area of quarter circle of C_2) + Area of ΔAOB)

$$= 4\left(\frac{\pi\left(\sqrt{2}\right)^2}{2}-\frac{\pi(2)^2}{4}+\frac{1}{2}\cdot 2\cdot 2\right)$$

$$= 4(\pi - \pi + 2) = 8 \text{ sq. units}$$

57. Answer A(P, Q, R); B(P, Q, R); C(S, T); D(P, Q, R, S, T)

Hint : Apply estimation of integrals

Solution : $I_1=\int_0^1 e^{x^2}dx \Rightarrow 1<I<e-1$

$$I_2=\int_0^{\pi/2}\frac{\sin x}{x}dx \Rightarrow \frac{2}{\pi}<I<\frac{\pi}{2}$$

$$I_3=\left|\int_0^{\pi/2}(\sin x)dx\right|=\frac{\pi}{2}\ln 2$$

$$=\frac{\pi}{2}\times(2.303)\times(0.301)\cong 0.9$$

$$I_4=0<\int_0^1\frac{x^7}{\left(1+x^8\right)^{1/7}}dx<x^7$$

$$\Rightarrow \quad 0<I_4<\frac{1}{8}$$

58. Answer (00)

Hint : Differentiate expression of area using Leibnitz's rule

Solution : Given

$$\alpha^3+8=\int_2^{\alpha}\left(x^2+2-f(x)\right)dx$$

Differentiate both sides we get

$3\alpha^2 = \alpha^2 + 2 - f(\alpha)$

$\Rightarrow \ f(\alpha) = -2\alpha^2 + 2$

$\Rightarrow \ f(x) = -2x^2 + 2$

$\Rightarrow \ x_0 = 0$

59. Answer (07)

Hint : Put $y = vx$ and solve as homogeneous differential equation.

Solution : Put $y = vx$

$$u+x\frac{du}{x}=\frac{v}{1+v^2}$$

$$\Rightarrow \quad x\frac{du}{dx}=\frac{v-v-v^3}{1+v^2}$$

$$\Rightarrow \quad \int\frac{(1+v)^2}{v^3}dv=-\int\frac{dx}{x}$$

$$\Rightarrow \quad -\frac{1}{2v^2}+\ln u=-\ln x+c$$

$$\Rightarrow \quad -\frac{x^2}{2y^2}+\ln\left(\frac{y}{x}\right)+\ln x=c\Rightarrow-\frac{1}{2}$$

And $\dfrac{-a^2}{2e^2}+1=-\dfrac{1}{2}\Rightarrow\dfrac{a^2}{3}=e^2$

60. Answer (02)

Hint : If $f(x) = t$

$\Rightarrow \ f'(x)dx = dt$

Solution : $\displaystyle\int\frac{e^{2x}}{\left(e^{2x}+1\right)^2}dx$

Put $e^{2x} + 1 = t$

$2.e^{2x}\,dx = dt$

$$\Rightarrow \quad \frac{1}{2}\int\frac{dt}{t^2}=-\frac{1}{2}\frac{1}{t}+c=-\frac{1}{2\left(e^{2x}+1\right)}+c\Rightarrow k=2$$

□ □ □