All India Aakash Test Series for JEE (Main)-2020

TEST - 3 - Code-A

Test Date : 29/09/2019

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(3)	26.	(2)	51.	(3)
2.	(3)	27.	(3)	52.	(3)
3.	(4)	28.	(1)	53.	(3)
4.	(2)	29.	(1)	54.	(2)
5.	(2)	30.	(1)	55.	(4)
6.	(2)	31.	(4)	56.	(3)
7.	(4)	32.	(2)	57.	(4)
8.	(2)	33.	(1)	58.	(3)
9.	(4)	34.	(3)	59.	(1)
10.	(3)	35.	(4)	60.	(4)
11.	(2)	36.	(2)	61.	(1)
12.	(4)	37.	(1)	62.	(3)
13.	(4)	38.	(3)	63.	(3)
14.	(2)	39.	(4)	64.	(3)
15.	(1)	40.	(1)	65.	(3)
16.	(4)	41.	(4)	66.	(3)
17.	(4)	42.	(1)	67.	(3)
18.	(4)	43.	(3)	68.	(1)
19.	(3)	44.	(4)	69.	(4)
20.	(2)	45.	(4)	70.	(3)
21.	(15)	46.	(07)	71.	(03)
22.	(20)	47.	(01)	72.	(20)
23.	(92)	48.	(09)	73.	(01)
24.	(20)	49.	(03)	74.	(00)
25.	(15)	50.	(02)	75.	(00)

PART - A (PHYSICS)

1. Answer (3)

Hint : $h_0 = \sqrt{h_1 h_2} = 2$ cm

Sol. : $h_0 = \sqrt{h_1 h_2} = 2$ cm

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{40} + \frac{1}{20} = \frac{1}{f}$$

$$f = \frac{40}{3} \text{ cm}$$

2. Answer (3)

Hint : $P_{avg} = \frac{v_{rms}^2}{R}$

Sol. : $v_{rms} = \sqrt{\frac{2^2 + 1^2}{2}} = \sqrt{\frac{5}{2}}$

$$p_{avg} = \frac{5}{2 \times 20} = 0.125 \text{ W}$$

3. Answer (4)

Hint : Take the clue from $\delta - i$ graph

Sol. : Angle of deviation should decrease hence angle of emergence should also decrease on increasing angle of incidence.

4. Answer (2)

Hint : $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

Sol. : $\frac{1}{v} - \frac{1}{15} = -\frac{1}{10}$

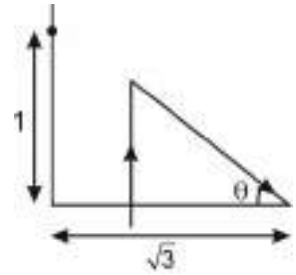
$v = -30$ cm, $m = -2$

$v_P = -20\hat{i}$ m/s

5. Answer (2)

Hint : Slope of reflected ray can be found by joining line passing through $(\sqrt{3}, 0)$ and focus (0, 1)

Sol. : $\tan\theta = \frac{1}{\sqrt{3}}$



$\theta = 30°$

$\delta = 120°$

6. Answer (2)

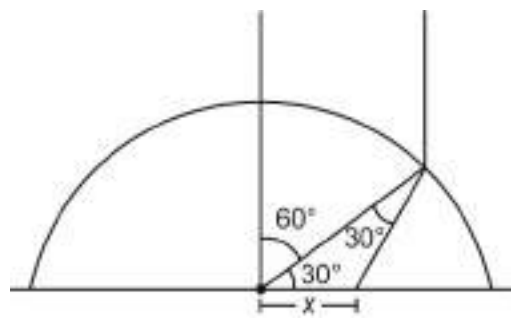
Hint : $\vec{E} \cdot \vec{B} = 0$

Sol. : $\vec{E}$ and $\vec{B}$ are always in same phase.

7. Answer (4)

Hint : Apply Snell's law and trace the path of refracted ray.

Sol. : $\frac{\sin 60°}{\sin r} = \sqrt{3}$



$r = 30°$

$$\frac{\sin 30°}{x} = \frac{\sin 120°}{R}$$

$$x = \frac{R}{\sqrt{3}}$$

8. Answer (2)

Hint : For maximum angular magnification, final image should be at 25 cm.

Sol. : $-\frac{1}{25} + \frac{1}{x} = \frac{1}{5}$

$$x = \frac{25}{6} \text{ cm}$$

9. Answer (4)

Hint : Focal length of concave lens, is

$\frac{d}{2} = 10$ cm, $d = 20$ cm

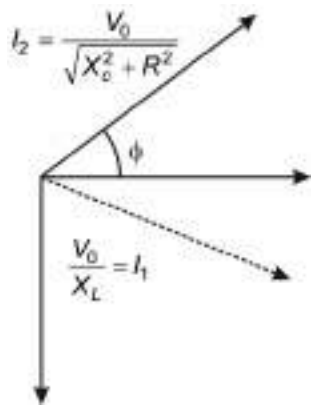
Sol. : Focal length of convex lens, is

$\frac{3d}{2} = 30$ cm

10. Answer (3)

Hint : Draw phase diagram of current in individual branch.

Sol. : $I_1 = I_2$



$\tan\phi = \frac{1}{\sqrt{3}}, \quad \phi = 30°$

Power factor = cos 30° = $\frac{\sqrt{3}}{2}$

11. Answer (2)

Hint : Ring rotates due to induced electric field.

Sol. : $E \times 2\pi R = \frac{(B \times \pi R^2)}{\Delta t}$

$\Rightarrow \quad E = \frac{BR}{2\Delta t}$

$\therefore \quad \tau\Delta t = E \times Q \times R \times \Delta t = (mR^2)$

$\Rightarrow \frac{BR}{2\Delta t} \times QR\Delta t = mR^2 \times \omega$

$\Rightarrow \omega = \frac{QB}{2m}$

12. Answer (4)

Hint : $Q = \frac{2\Delta\omega}{\omega_0}$

Sol. : $Q = \frac{\omega_0 L}{R}, \quad \omega_0^2 = \frac{1}{LC}$

$\Rightarrow \quad Q = \frac{1}{\omega_0 C} \times \frac{1}{R} = \frac{1}{\omega_0 RC}$

13. Answer (4)

Hint : Use concept of effective length.

Sol. : $v_A - v_B = \frac{1}{2}B\omega L^2$

$v_A - v_C = \frac{1}{2}B\omega L^2$

$\therefore \quad v_B - v_C = 0$

14. Answer (2)

Hint : $M \propto \sqrt{L_1 L_2}$

Sol. : Theory based

$M = \sqrt{L_1 L_2}$ when $\phi_{12} = \phi_{21}$

15. Answer (1)

Hint : $\Delta Q = \frac{\text{Change in flux}}{\text{Resistance}}$

Sol. : $\Delta\phi = (B_2 - B_1) \times A$

$= [2(2-0) + 3 \times (4-0)]a^2 = 16a^2$

$\therefore \quad \Delta Q = \frac{\Delta\phi}{R} = \frac{16a^2}{R}$

16. Answer (4)

Hint : Induced current opposes the change in flux.

Sol. : Magnetic field induced by electron in the loop is out of plane and it first increases and then decreases. So induced current is first clockwise and then anti-clockwise.

17. Answer (4)

Hint : $E = E_0\sin(\omega t - kr)$

Sol. : $k = k\cos\theta\hat{i} + k\sin\theta\hat{j}$

$\vec{r} = x\hat{i} + y\hat{j}$

$\therefore \quad E = E_0\sin(\omega t - kx\cos\theta - ky\sin\theta)$

$$= E_0\sin\left(\omega t - \frac{\sqrt{3}k}{2}x - \frac{k}{2}y\right)$$

18. Answer (4)

Hint : At resonance, $X_C = X_L$

Sol. : $Z = R$ and $X_C = X_L$

$\therefore \quad \tan\phi = \frac{(X_C - X_L)}{R} = 0$

$\Rightarrow$ Phase difference = 0

19. Answer (3)

Hint : Induced emf $= -\frac{d\phi}{dt}$

Sol. : $\phi_B = \int_{\frac{L}{2}}^{\frac{3L}{2}} \frac{\mu_0 I}{2\pi x}\cdot L dx = \frac{\mu_0 IL}{2\pi}\ln(3)$

$\Rightarrow \quad \phi_B = \frac{\mu_0(\alpha t)L}{2\pi}\ln(3)$

$\therefore \quad |\varepsilon| = \left|-\frac{d\phi}{dt}\right| = \frac{\mu_0\alpha L}{2\pi}\ln(3)$

$\therefore \quad i = \frac{|\varepsilon|}{R} = \frac{\mu_0\alpha L}{2\pi R}\ln(3)$

20. Answer (2)

Hint : RMS value is repetitive after $\frac{T}{2}$

Sol. : $V_{rms} = \sqrt{\frac{V_0^2 \times \frac{T}{4} + 0 \times \frac{T}{4}}{\frac{T}{2}}} = \frac{V_0}{\sqrt{2}}$

21. Answer (15)

Hint : Rays should fall normally on the mirror

Sol. : $\frac{1.5}{v} + \frac{1}{30} = \left(\frac{1.5-1}{R}\right)$

$v = \infty$

$R = 15$ cm

22. Answer (20)

Hint : Image formed by one will behave as object for other.

Sol. : $\frac{1}{v} + \frac{1}{60} = \frac{1}{20}$

$v = +30$ cm

Rays will be parallel to principal axis after reflection.

23. Answer (92)

Hint : Glass slab will change the actual distance

Sol. : $\Delta x = 4\left(1 - \frac{1}{4/3}\right) = 1\,\text{cm}$

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} + \frac{1}{30} = \frac{1}{20}$

$v = 60$ cm

Glass slab placed will shift the final image again by 1 cm.

Distance = 61 + 31 = 92 cm.

24. Answer (20)

Hint : $v_L + v_R = 0$ after short circuiting

Sol. : $I_0 = \frac{\varepsilon}{R} = \frac{20}{20} = 1\,\text{A}$

$\tau = \frac{L}{R} = \frac{100 \times 10^{-3}}{20} = 5$ ms

$\therefore \quad i = I_0 e^{-t/\tau} = \frac{1}{e}$ A

$\therefore \quad |v_L| = i \times R = \frac{1}{e} \times 20 = \frac{20}{e}$ volts

25. Answer (15)

Hint : $E_0 = CB_0$

Sol. : $E_0 = CB_0$

$= 3 \times 10^8 \times 50 \times 10^{-9}$

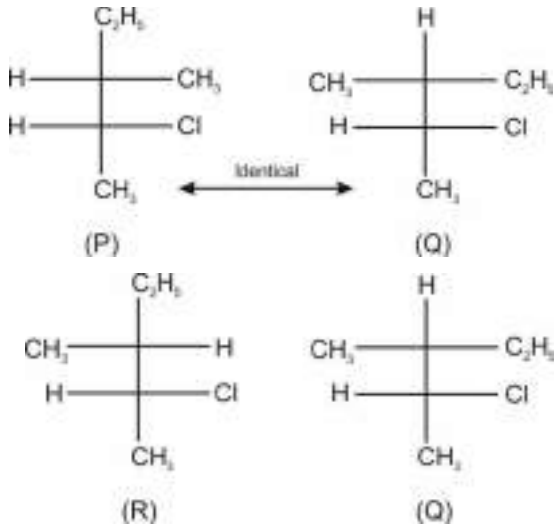
$= 15$ V/m

PART - B (CHEMISTRY)

26. Answer (2)

Hint : P and Q are identical while R is diastereomer of P

Sol. :



These are diastereomers.

27. Answer (3)

Hint : Correct order : R > Q > P > S

Sol. : B.P. $\propto$ M.W.

If M.W. is same then

$$\text{B.P.} \propto \frac{1}{\text{Branching}}$$

28. Answer (1)

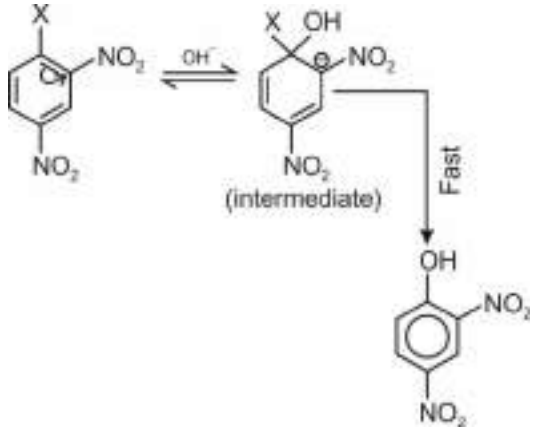
Hint : α-haloketone shows the highest S_N2 rate

Sol. : Correct order is (1) > (2) > (3) > (4)

29. Answer (1)

Hint : Order is A > B > C

Sol. :



Rate of reaction $\propto$ stability of carbanion

30. Answer (1)

Hint : A and B are diastereomers.

Sol. : Compound C has only one pair of enantiomers.

31. Answer (4)

Hint : CO and CN^- are SFL and Cl^- is WFL

Sol. : $[Ni(Cl_4)]^{2-}$ is sp^3, tetrahedral and paramagnetic

$[Ni(CN)_4]^{2-}$ is dsp^2, square planar and diamagnetic

$[Ni(CO)_4]$ is sp^3, tetrahedral and diamagnetic

32. Answer (2)

Hint : Metal is Ni

Sol. : $Ni + CO \longrightarrow [Ni(CO)_4] \xrightarrow[\Delta]{} Ni + 4CO$

33. Answer (1)

Hint : $AgNO_3$, $Pb(OAc)_2$

$$\downarrow S_2O_3^{2-}$$

$$\underset{\text{(white)}}{Ag_2S_2O_3}, \quad \underset{\text{(white)}}{PbS_2O_3}$$

$$\downarrow \Delta$$

$$\underset{\text{(Black)}}{Ag_2S} \quad \underset{\text{(Black)}}{PbS}$$

Sol. : Initially colour is white and on heating colour changes to black.

34. Answer (3)

Hint : When sulphite ion treated with Zn produces H_2S and which turns the colour of MnO_4^- to colourless due to the formation of Mn^{+2}

Sol. : $SO_3^{2-} + Zn + H^+ \longrightarrow H_2S\uparrow + Zn^{2+} + H_2O$

$MnO_4^- + H_2S \xrightarrow{H^+} Mn^{+2} + S\downarrow + H_2O$

35. Answer (4)

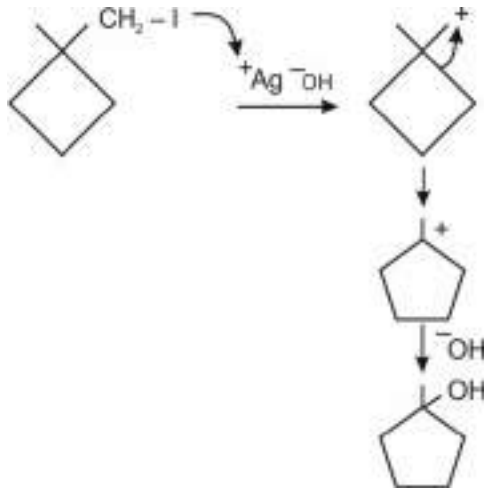
Hint : $Cd(OH)_2$, $Fe(OH)_2$ and $Mg(OH)_2$ are insoluble in excess of NaOH

Sol. : $Pb(OH)_2$ form a soluble complex $[Pb(OH)_4]^{2-}$ with excess of NaOH

36. Answer (2)

Hint : Moist Ag_2O is AgOH.

Sol. :



37. Answer (1)

Hint : NaCl or NaBr which is formed as a byproduct in the given reaction are precipitated in dry acetone.

Sol. : It facilitates the forward reaction according to Le-Chatelier principle.

38. Answer (3)

Hint : P : $CH_3-CH=CH_2$

Q : $CH_2(Br)-CH=CH_2$

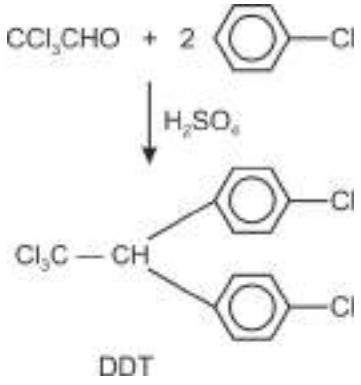
Sol. : R : $BrMg\ CH_2-CH=CH_2$

S : $D-CH_2-CH(OD)-CH_2-D$

39. Answer (4)

Hint : The product which is formed is DDT

Sol. :



40. Answer (1)

Hint : Wavelength absorbed

$$\propto \frac{1}{\text{strength of ligand}}$$

Sol. : Strength of ligand

$CN^- > NH_3 > H_2O > Cl^-$

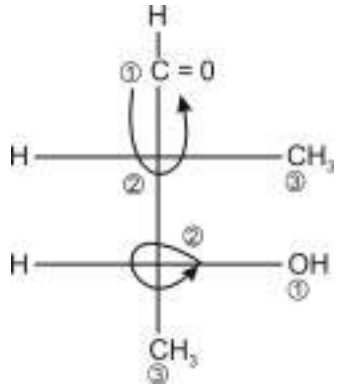
41. Answer (4)

Hint : In Wilkinson catalyst, Rh is present in +1 oxidation state.

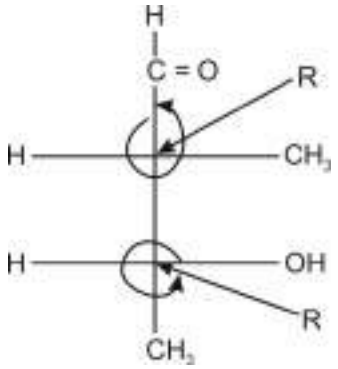
Sol. : Wilkinson catalyst : $[Rh(PPh_3)_3Cl]$

42. Answer (1)

Hint :



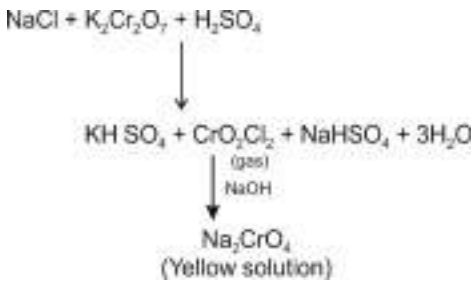
Sol. :



43. Answer (3)

Hint : Chromyl chloride test.

Sol. :



44. Answer (4)

Hint : X is Iodide

Sol. : $NaI + H_2SO_4 \longrightarrow Na_2SO_4 + SO_2 + H_2O + I_2$

I_2 + starch solution $\longrightarrow$ Blue colour.

$NaI + AgNO_3 \longrightarrow AgI + NaNO_3$

45. Answer (4)

Hint : Group 2A sulphides are insoluble in polyammonium sulphide.

Sol. : As_2S_3 is soluble in polyammonium sulphide.

46. Answer (07)

Hint : Meso compound has plane of symmetry

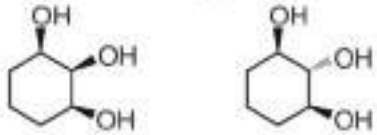
Sol. : $x_1 = 2$

$x_2 = 1$

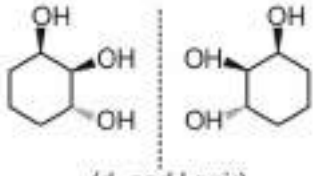
$y_1 = 0$

$y_2 = 4$

Stereoisomers of 1, 2, 3-cyclohexanetriol



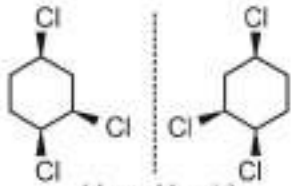
Meso-I Meso-II



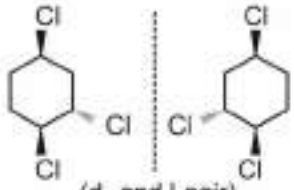
(d- and l-pair)

$\therefore x_1 = 2$ and $x_2 = 1$

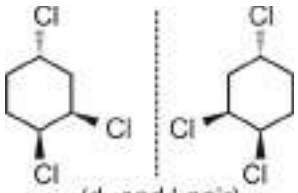
Stereoisomers of 1,2,4-Trichlorocyclohexane



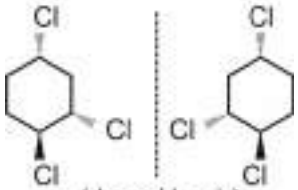
(d- and l-pair)



(d- and l-pair)



(d- and l-pair)



(d- and l-pair)

No meso isomer of it exists.

$\therefore y_1 = 0$ and $y_2 = 4$.

$x_1 + x_2 + y_1 + y_2 = 2 + 1 + 0 + 4 = 7$

47. Answer (01)

Hint : In $[M(H_2O)_4Cl(NO_2)]Cl$, M is in +1 oxidation state.

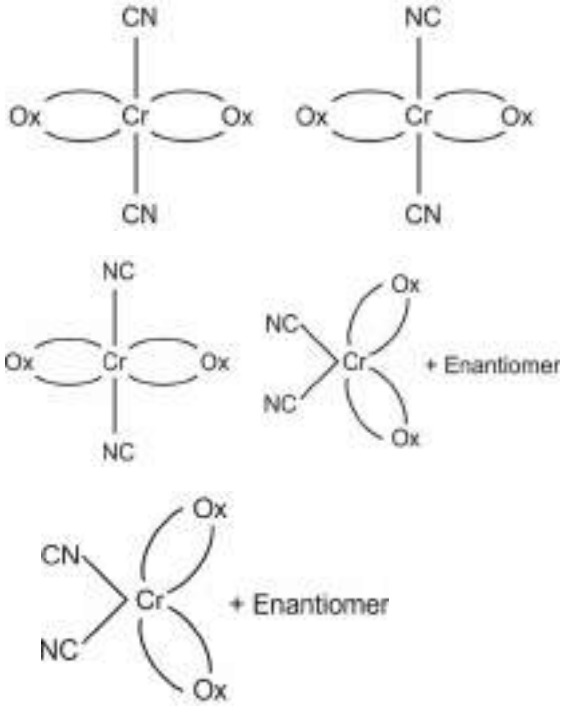
Sol. : Possible isomer is

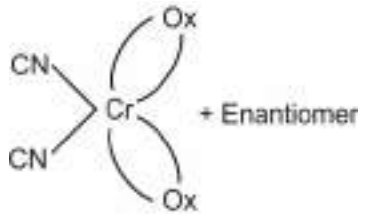
$(NO_2)^+[M(H_2O)_4\ Cl_2]^-$

48. Answer (09)

Hint : Due to presence of — CN, given complex can show linkage isomerism

Sol. :





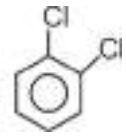
49. Answer (03)

Hint : Prussian blue complex is $Fe_4[Fe(CN)_6]_3$

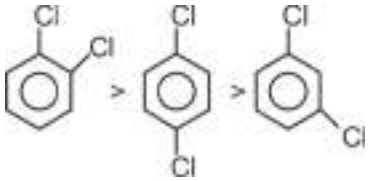
Sol. : $Fe^{3+} + [Fe(CN)_6]^{4-} \longrightarrow \overset{(III)}{Fe_4}[\overset{(II)}{Fe}(CN)_6]_3$

Prussian blue complex

50. Answer (02)

Hint : X ⟶ 

Sol. : The correct order of boiling point is



PART - C (MATHEMATICS)

51. Answer (3)

Hint : Algebraic twins

Sol. : $I_1 = \int_0^1 \frac{dx}{1+x} = \ln 2$

$I_2 = \int_0^1 \frac{dx}{1+x^2} = \frac{\pi}{4}$

$I_4 = \int_0^1 \frac{dx}{1+x^4}$

$= \frac{1}{2}\left[\int_0^1 \frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+2} - \int_0^1 \frac{\left(1-\frac{1}{x^2}\right)dx}{\left(x+\frac{1}{x}\right)^2-2}\right]$

$= \frac{1}{4\sqrt{2}}\left(\pi - \ln\left(3-2\sqrt{2}\right)\right)$

52. Answer (3)

Hint : When $x \in \left(0, \frac{\pi}{6}\right)$, $2\cos x - \sqrt{3} > 0$

Sol. : $0 < x < \frac{\pi}{6} \Rightarrow \frac{\sqrt{3}}{2} < \cos x < 1$

$\Rightarrow 2\cos x - \sqrt{3} > 0$

$\Rightarrow \cot^{-1}\left(\frac{2}{2\cos x - \sqrt{3}}\right) = \cot^{-1}\left(\frac{1}{\cos x - \frac{\sqrt{3}}{2}}\right)$

$= \tan^{-1}\left(\cos x - \frac{\sqrt{3}}{2}\right)$

$\Rightarrow I = \int_0^{\frac{\pi}{6}} \frac{\pi}{2}\,dx = \frac{\pi^2}{12}$

53. Answer (3)

Hint : Put $\tan\theta = t$ and then $t + t^3 + 2t^2 = \lambda^2$

Sol. : Put $\tan\theta = t \Rightarrow \sec^2\theta\, d\theta = dt$

$I = \int \frac{\left(\sqrt{t} + \sqrt{\frac{1}{t}}\right)(1+3t)\,dt}{\sqrt{1+t^2+2t}}$

$\int \frac{(t+1)(1+3t)}{\sqrt{t+t^3+2t^2}}dt = \int \frac{\left(3t^2+4t+1\right)dt}{\sqrt{t+t^3+2t^2}}$

Put $t + t^3 + 2t^2 = \lambda^2$

$\Rightarrow (3t^2 + 4t + 1)\, dt = 2\lambda d\lambda$

$\Rightarrow I = \int \frac{2\lambda\, d\lambda}{\lambda} = 2\lambda + C$

$= 2\sqrt{t+t^3+t^2} + C$

$= 2\sqrt{\tan\theta + \tan^3\theta + 2\tan^2\theta} + C$

54. Answer (2)

Hint : $\cot^{-1}(-x) = \pi - \cot^{-1}x$

Sol. : $I = \int_{-\pi}^{\pi} \left\{\frac{\cot^{-1}x}{\pi}\right\} dx$

$= \int_0^{\pi} \left(\left\{\frac{\cot^{-1}x}{\pi}\right\} + \left\{\frac{-\cot^{-1}x+\pi}{\pi}\right\}\right) dx$

$= \int_0^{\pi} \left(\left\{\frac{\cot^{-1}x}{\pi}\right\} + \left\{\frac{-\cot^{-1}x}{\pi}\right\}\right) dx$

$= \int_0^{\pi} \left(\frac{\cot^{-1}x}{\pi} + 1 - \frac{\cot^{-1}x}{\pi}\right) dx = \pi$

55. Answer (4)

Hint : Put $x - 2 = t$

Sol. : $I = \int_0^4 \left(x^3 - 6x^2 + 9x + \sin(x-2) - 1\right) dx$

Put $x - 2 = t \Rightarrow dx = dt$

$$I = \int_{-2}^{2} \left((t+2)^3 - 6(t+2)^2 + 9(t+2) + \sin t - 1\right) dt$$

$$= \int_{-2}^{2} \left(\left(t^2 + 4 + 4t\right)(t + 2 - 6) + 9t + 18 + \sin t - 1\right) dt$$

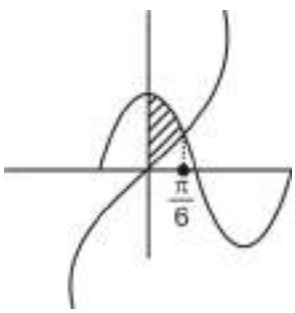
$$= \int_{-2}^{2} \left(t^3 + 4t + 4t^2 - 4t^2 - 16 - 16t + 9t + 18 + \sin t - 1\right) dt$$

$$= \int_{-2}^{2} \left(t^3 - 3t + \sin t + 1\right) dt = \int_{-2}^{2} dt = 4$$

56. Answer (3)

Hint : Draw graph

Sol. :



$$A = \int_0^{\frac{\pi}{6}} (2\cos x - 3\tan x)\, dx$$

$$= 2\sin x - 3 \ln \sec x \Big|_0^{\pi/6}$$

$$= 1 - 3 \ln\left(\frac{2}{\sqrt{3}}\right)$$

57. Answer (4)

Hint : Put $\ln y = t$ and $\frac{1}{x} = z$

Sol. : Let $\ln y = t \Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{dt}{dx}$

$$\Rightarrow (xt - 1)\frac{dt}{dx} = \frac{1}{x}$$

$$\Rightarrow \frac{1}{x^2}\frac{dx}{dt} + \frac{1}{x} = t$$

Let $\frac{1}{x} = z \Rightarrow \frac{1}{x^2}\frac{dx}{dt} = -\frac{dz}{dt}$

$$\Rightarrow \frac{dz}{dt} - z = -t$$

$$\Rightarrow e^{-t} \cdot z = -\int t\, e^{-t} dt + c$$

$\Rightarrow e^{-t} z = e^{-t}(1 + t) + c$

$\Rightarrow z = (1 + t) + ce^t$

$$\frac{1}{x} = 1 + \ln y + cy$$

Put $x = 1, y = \frac{1}{e}$

$$1 = 1 - 1 + \frac{c}{e}$$

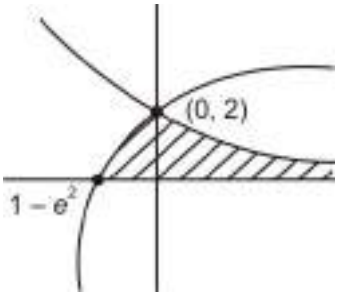
$c = e$

So $1 = x(1 + \ln y + ey)$

58. Answer (3)

Hint : Draw graph

Sol. : Required area =



$$\int_{1-e^2}^{0} \ln\left(x + e^2\right) dx + \int_0^{\infty} 2e^{-x} dx$$

$$= \int_1^{e^2} \ln t\, dt + 2\left(-e^{-x}\right)\Big|_0^{\infty}$$

$$= t \ln\left(\frac{t}{e}\right)\Big|_1^{e^2} + 2(1)$$

$= e^2 + 1 + 2$

$= e^2 + 3$

59. Answer (1)

Hint : $\frac{dx}{x} = \frac{y^2 dx - x\, d(y^2)}{x^2}$

Sol. : $xdx = y^2dx - xd(y^2)$

$$\Rightarrow \frac{dx}{x} = \frac{y^2dx - xd\left(y^2\right)}{x^2}$$

$$\Rightarrow \ln x = -\frac{y^2}{x} + c$$

Put $x = 1$, $y = 0 \Rightarrow c = 0$

$y^2 + x \ln x = 0$

60. Answer (4)

Hint : Linear differential equation

$$\text{I}\cdot\text{F}\cdot = x + \sqrt{1+x^2}$$

Sol. : $\frac{dy}{dx} + \frac{y}{\sqrt{1+x^2}} = \sqrt{1+x^2} - x$

$$\text{I}\cdot\text{F}\cdot = e^{\int \frac{dx}{\sqrt{1+x^2}}} = e^{\ln\left(x+\sqrt{1+x^2}\right)} = x + \sqrt{1+x^2}$$

Then,

$$y\cdot\left(x+\sqrt{1+x^2}\right) = \int\left(x+\sqrt{1+x^2}\right)\left(\sqrt{1+x^2} - x\right)dx + c$$

$$y = \frac{x+c}{x+\sqrt{1+x^2}}$$

At $x = 0$, $y = 1 \Rightarrow c = 1$

$$y = \frac{x+1}{x+\sqrt{1+x^2}}$$

When $x = 1$, $y = \frac{2}{\sqrt{2}+1} = 2\left(\sqrt{2}-1\right)$

61. Answer (1)

Hint : $\frac{d(xy)}{y^2x^2} = \frac{xdy - ydx}{x^2}$

Sol. : $x\,dy + y\,dx = y^2\,[x\,dy - y\,dx]$

$$\Rightarrow \frac{d\left(xy\right)}{y^2x^2} = \frac{x\,dy - y\,dx}{x^2}$$

$$\Rightarrow -\frac{1}{xy} + c = \frac{y}{x}$$

$\Rightarrow y^2 + 1 = cxy$

62. Answer (3)

Hint : Put sec$x = t$ in (2), (3) and (4)

Sol. : $I_1 = \int \sec^2 x \tan^2 x\,dx$, put $\tan x = t$

$$\Rightarrow I_1 = \frac{\tan^3 x}{3} + C$$

$I_2 = \int \sec^3 x \tan^3 x\,dx$, put $\sec x = t$

$$\Rightarrow I_2 = \frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C$$

$I_3 = \int \sec^3 x \tan^5 x\,dx$, put $\sec x = t$

$$\Rightarrow I_3 = \frac{\sec^7 x}{7} - \frac{2}{5}\sec^5 x + \frac{\sec^3 x}{3} + C$$

$I_4 = \int \sec^5 x \tan^3 x\,dx$, put $\sec x = t$

$$\Rightarrow I_4 = \frac{\sec^7 x}{7} - \frac{\sec^5 x}{5} + C$$

63. Answer (3)

Hint : $\frac{\sin^5 x + \cos^5 x}{\sin x + \cos x} = 1 - \frac{\sin 2x}{2} - \frac{\sin^2 2x}{4}$,

put $1 + \sin 2x = t$

Sol. : $I = \int \frac{(1+\sin 2x)(\cos x - \sin x)}{\sin^5 x + \cos^5 x}dx$

$$= \int \frac{\cos 2x}{\frac{\sin^5 x + \cos^5 x}{\sin x + \cos x}}dx$$

$$= \int \frac{\cos 2x\,dx}{\sin^4 x - \sin^3 x\cos x + \sin^2 x\cos^2 x - \sin x\cos^3 x + \cos^4 x}$$

$$= \int \frac{\cos 2x\,dx}{1 - \sin x\cos x - \sin^2 x\cos^2 x}$$

$$= \int \frac{\cos 2x\,dx}{1 - \frac{\sin 2x}{2} - \frac{\sin^2 2x}{4}}$$

$$= \int \frac{4\cos 2x\,dx}{4 - 2\sin 2x - \sin^2 2x} = \int \frac{4\cos 2x\,dx}{5 - (\sin 2x + 1)^2}$$

Put $\sin 2x + 1 = t \Rightarrow 2\cos 2x\,dx = dt$

$$I = \int \frac{2\,dt}{5 - t^2} = -2\int \frac{dt}{t^2 - 5}$$

$$= -2\,\frac{1}{2\sqrt{5}}\ln\left|\frac{t-\sqrt{5}}{t+\sqrt{5}}\right| + C$$

$$I = \frac{1}{\sqrt{5}}\ln\left|\frac{\sin 2x + 1 + \sqrt{5}}{\sin 2x + 1 - \sqrt{5}}\right| + C$$

64. Answer (3)

Hint : $\theta \to \frac{\pi}{2} - \theta$

Sol. : $I = \int_0^{\frac{\pi}{2}} \frac{(2\sin\theta - \cos\theta)\, d\theta}{\sin\theta + \cos\theta}$ ...(i)

$\theta \to \frac{\pi}{2} - \theta$

$I = \int_0^{\frac{\pi}{2}} \frac{(2\cos\theta - \sin\theta)\, d\theta}{(\cos\theta + \sin\theta)}$ ...(ii)

From (i) + (ii)

$2I = \int_0^{\frac{\pi}{2}} d\theta = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$

65. Answer (3)

Hint : Use property

Sol. : $I = \int_0^{2\pi} \frac{\sec x\, dx}{\sec x + \cos x}$

$\Rightarrow I = 2\int_0^{\pi} \frac{\sec x\, dx}{\sec x + \cos x} = 4\int_0^{\frac{\pi}{2}} \frac{\sec x\, dx}{\sec x + \cos x}$

$= 4\int_0^{\frac{\pi}{2}} \frac{dx}{1 + \cos^2 x}$

$I = 4\int_0^{\frac{\pi}{2}} \frac{\sec^2 x\, dx}{1 + 1 + \tan^2 x}$

$= 4 \times \frac{1}{\sqrt{2}} \left[\tan^{-1} \frac{x}{\sqrt{2}}\right]_0^{\frac{\pi}{2}} = \frac{4}{\sqrt{2}} \cdot \frac{\pi}{2} = \sqrt{2}\pi$

66. Answer (3)

Hint : Use property

Sol. : $I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\tan\theta}\, d\theta}{\left(\sqrt{\tan\theta} + \sqrt{\cot\theta}\right)^3}$

$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cot\theta}\, d\theta}{\left(\sqrt{\tan\theta} + \sqrt{\cot\theta}\right)^3}$

$2I = \int_0^{\frac{\pi}{2}} \frac{d\theta}{\left(\sqrt{\tan\theta} + \sqrt{\cot\theta}\right)^2} = \int_0^{\frac{\pi}{2}} \frac{d\theta}{\tan\theta + \cot\theta + 2}$

$= \frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{\sin 2\theta}{1 + \sin 2\theta}$

$I = \frac{1}{4} \int_0^{\frac{\pi}{2}} \frac{(\sin 2\theta + 1 - 1)\, d\theta}{\sin 2\theta + 1}$

$= \frac{1}{4} \cdot \frac{\pi}{2} - \frac{1}{2} \int_0^{\frac{\pi}{4}} \frac{d\theta}{1 + \sin 2\theta}$

$I = \frac{\pi}{8} - \frac{1}{2} \int_0^{\frac{\pi}{4}} \frac{(1 - \sin 2\theta)\, d\theta}{\cos^2 2\theta}$

$= \frac{\pi}{8} - \frac{1}{2} \int_0^{\frac{\pi}{4}} \left(\sec^2 2\theta - \sec 2\theta \tan 2\theta\right) d\theta$

$I = \frac{\pi}{8} - \frac{1}{4} \left[\tan 2\theta - \sec 2\theta\right]_0^{\pi/4}$

$= \frac{\pi}{8} - \frac{1}{4} = \frac{\pi - 2}{8}$

67. Answer (3)

Hint : Put $x = \sin^2\theta$

Sol. : $I = \int_0^1 x^3 \sqrt{1 - x}\, dx$

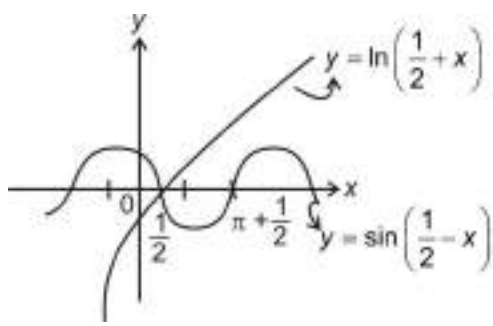
Put $x = \sin^2\theta \Rightarrow dx = 2\sin\theta\cos\theta\, d\theta$

$I = 2\int_0^{\frac{\pi}{2}} \sin^7\theta \cos^2\theta\, d\theta = 2\frac{\Gamma\left(\frac{7+1}{2}\right)\Gamma\left(\frac{2+1}{2}\right)}{\Gamma\left(\frac{7+2+2}{2}\right)} = \frac{32}{315}$

68. Answer (1)

Hint : Intersection point is $\left(\frac{1}{2}, 0\right)$

Sol. :



$A = \int_0^{\frac{1}{2}} \left(\sin\left(\frac{1}{2} - x\right) - \ln\left(\frac{1}{2} + x\right)\right) dx$

Put $\frac{1}{2} - x = t \Rightarrow dx = -dt$

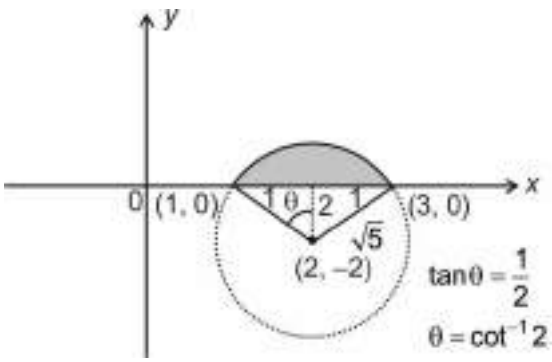
$$A = \int_0^{\frac{1}{2}} \left(\sin t - \ln(1-t)\right) dt = \frac{3}{2} - \cos\left(\frac{1}{2}\right) - \ln\sqrt{2}$$

69. Answer (4)

Hint : Area = $A_{\text{sector}} - A_{\Delta}$

Sol. : $(y + 2)^2 = 5 - (x - 2)^2$

$\Rightarrow (x - 2)^2 + (y + 2)^2 = 5$



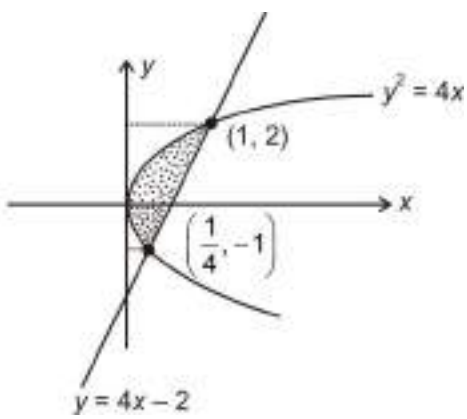
$$\Rightarrow \quad A = \frac{1}{2} \times \left(\sqrt{5}\right)^2 \times \left(2\cot^{-1}2\right) - \frac{1}{2} \times 2 \times 2$$

$= 5 \cot^{-1} 2 - 2$

70. Answer (3)

Hint : Find area on y-axis

Sol. :



$$A = \frac{1}{2}\left(1 + \frac{1}{4}\right) \times 3 - \int_{-1}^{2} \frac{y^2}{4} dy$$

$$A = \frac{15}{8} - \frac{1}{12}\left[y^3\right]_{-1}^{2} = \frac{15}{8} - \frac{3}{4} = \frac{9}{8}$$

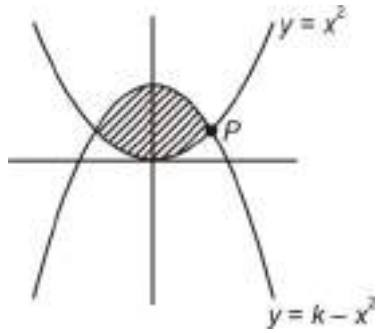
71. Answer (03)

Hint : Draw graph

Sol. : Point of intersection of two curves is

$$x = \pm\sqrt{\frac{k}{2}}$$

Area bounded by these two curves,



$$2\sqrt{6} = 2\int_0^{\sqrt{\frac{k}{2}}} \left(k - 2x^2\right) dx$$

$$\Rightarrow \sqrt{6} = k\sqrt{\frac{k}{2}} - \frac{2}{3}\left(\sqrt{\frac{k}{2}}\right)^3$$

$$\Rightarrow \sqrt{6} = \frac{k^{\frac{3}{2}}}{\sqrt{2}} - \frac{2k^{\frac{3}{2}}}{6\sqrt{2}}$$

$$k^{\frac{3}{2}}\left(\frac{4}{6\sqrt{2}}\right) = \sqrt{6} \Rightarrow k = 3$$

72. Answer (20)

Hint : $\frac{dP}{dt} \propto P \Rightarrow \frac{dP}{P} = kt$ (where k is constant)

Sol. : $\frac{dP}{dt} \propto P$

$$\Rightarrow \frac{dP}{dt} = kP$$

$$\Rightarrow \frac{dP}{P} = kdt$$

$$\Rightarrow \ln P = kt + c$$

When $t = 0$, $P = 40 \Rightarrow C = \ln 40$

When $t = 10$, $P = 20 \Rightarrow k = -\frac{\ln 2}{10}$

$$\ln P = -\frac{\ln 2}{10} t + \ln 40$$

If $P = 5$

Then $\ln 5 = -\frac{\ln 2}{10}t + \ln 40$

$\Rightarrow \frac{\ln 2}{10}t = \ln\left(\frac{40}{5}\right)$

$\Rightarrow t$ = 30 minutes

So 20 minutes are still required

73. Answer (01)

Hint : $\int e^{ax} \sin bx\, dx = \frac{e^{ax}}{a^2+b^2}$

$(a \sin bx - b \cos bx) + C$

Sol. :

$\int e^{2x} \sin x \cos 3x\, dx = \frac{1}{2}\int e^{2x}\left[\sin 4x - \sin 2x\right] dx$

$= \frac{1}{2}\left[\int e^{2x} \sin 4x\, dx - \int e^{2x} \sin 2x\, dx\right]$

$= \frac{e^{2x}}{40}\left[2\sin 4x - 4\cos 4x - 5\sin 2x + 5\cos 2x\right] + C$

$\Rightarrow f(x) = 2\sin 4x - 4\cos 4x - 5\sin 2x + 5\cos 2x$

$f\left(\frac{\pi}{8}\right) = 2$

$f\left(\frac{\pi}{4}\right) = -1$

74. Answer (00)

Hint : $\{\sin x\} = \sin x - [\sin x]$

Sol. :

$I = 2\left(\int_0^{\pi} \cos x\, e^{\sin x} dx + \int_{\pi}^{2\pi} \cos x\, e^{(\sin x + 1)} dx\right)$

$I = 2\left(\left[e^{\sin x}\right]_0^{\pi} + \left[e^{\sin x + 1}\right]_{\pi}^{2\pi}\right)$

= 2 (0 + 0) = 0

75. Answer (00)

Hint : Odd function

Sol. : $f(x) = \sin(\tan x) + \tan(\sin x)$ is odd function

$\Rightarrow \int_{-\pi}^{\pi} f(x) dx = 0$

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All India Aakash Test Series for JEE (Main)-2020

TEST - 3 - Code-B

Test Date : 29/09/2019

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(2)	26.	(4)	51.	(3)
2.	(3)	27.	(4)	52.	(4)
3.	(4)	28.	(3)	53.	(1)
4.	(4)	29.	(1)	54.	(3)
5.	(4)	30.	(4)	55.	(3)
6.	(1)	31.	(1)	56.	(3)
7.	(2)	32.	(4)	57.	(3)
8.	(4)	33.	(3)	58.	(3)
9.	(4)	34.	(1)	59.	(3)
10.	(2)	35.	(2)	60.	(1)
11.	(3)	36.	(4)	61.	(4)
12.	(4)	37.	(3)	62.	(1)
13.	(2)	38.	(1)	63.	(3)
14.	(4)	39.	(2)	64.	(4)
15.	(2)	40.	(4)	65.	(3)
16.	(2)	41.	(1)	66.	(4)
17.	(2)	42.	(1)	67.	(2)
18.	(4)	43.	(1)	68.	(3)
19.	(3)	44.	(3)	69.	(3)
20.	(3)	45.	(2)	70.	(3)
21.	(15)	46.	(02)	71.	(00)
22.	(20)	47.	(03)	72.	(00)
23.	(92)	48.	(09)	73.	(01)
24.	(20)	49.	(01)	74.	(20)
25.	(15)	50.	(07)	75.	(03)

PART - A (PHYSICS)

1. Answer (2)

Hint : RMS value is repetitive after $\frac{T}{2}$

Sol. : $V_{rms} = \sqrt{\frac{V_0^2 \times \frac{T}{4} + 0 \times \frac{T}{4}}{\frac{T}{2}}} = \frac{V_0}{\sqrt{2}}$

2. Answer (3)

Hint : Induced emf $= -\frac{d\phi}{dt}$

Sol. : $\phi_B = \int_{\frac{L}{2}}^{\frac{3L}{2}} \frac{\mu_0 I}{2\pi x} \cdot L dx = \frac{\mu_0 IL}{2\pi} \ln(3)$

$\Rightarrow \quad \phi_B = \frac{\mu_0 (\alpha t) L}{2\pi} \ln(3)$

$\therefore \quad |\varepsilon| = \left| -\frac{d\phi}{dt} \right| = \frac{\mu_0 \alpha L}{2\pi} \ln(3)$

$\therefore \quad i = \frac{|\varepsilon|}{R} = \frac{\mu_0 \alpha L}{2\pi R} \ln(3)$

3. Answer (4)

Hint : At resonance, $X_C = X_L$

Sol. : $Z = R$ and $X_C = X_L$

$\therefore \quad \tan\phi = \frac{(X_C - X_L)}{R} = 0$

$\Rightarrow$ Phase difference = 0

4. Answer (4)

Hint : $E = E_0 \sin(\omega t - kr)$

Sol. : $k = k\cos\theta \hat{i} + k\sin\theta \hat{j}$

$\vec{r} = x\hat{i} + y\hat{j}$

$\therefore \quad E = E_0 \sin(\omega t - kx\cos\theta - ky\sin\theta)$

$= E_0 \sin\left(\omega t - \frac{\sqrt{3}k}{2}x - \frac{k}{2}y\right)$

5. Answer (4)

Hint : Induced current opposes the change in flux.

Sol. : Magnetic field induced by electron in the loop is out of plane and it first increases and then decreases. So induced current is first clockwise and then anti-clockwise.

6. Answer (1)

Hint : $\Delta Q = \frac{\text{Change in flux}}{\text{Resistance}}$

Sol. : $\Delta\phi = (B_2 - B_1) \times A$

$= [2(2-0) + 3 \times (4-0)]a^2 = 16a^2$

$\therefore \quad \Delta Q = \frac{\Delta\phi}{R} = \frac{16a^2}{R}$

7. Answer (2)

Hint : $M \propto \sqrt{L_1 L_2}$

Sol. : Theory based

$M = \sqrt{L_1 L_2}$ when $\phi_{12} = \phi_{21}$

8. Answer (4)

Hint : Use concept of effective length.

Sol. : $v_A - v_B = \frac{1}{2} B\omega L^2$

$v_A - v_C = \frac{1}{2} B\omega L^2$

$\therefore \quad v_B - v_C = 0$

9. Answer (4)

Hint : $Q = \frac{2\Delta\omega}{\omega_0}$

Sol. : $Q = \frac{\omega_0 L}{R}, \quad \omega_0^2 = \frac{1}{LC}$

$\Rightarrow \quad Q = \frac{1}{\omega_0 C} \times \frac{1}{R} = \frac{1}{\omega_0 RC}$

10. Answer (2)

Hint : Ring rotates due to induced electric field.

Sol. : $E \times 2\pi R = \frac{(B \times \pi R^2)}{\Delta t}$

$\Rightarrow \quad E = \frac{BR}{2\Delta t}$

$\therefore \quad \tau \Delta t = E \times Q \times R \times \Delta t = (mR^2)$

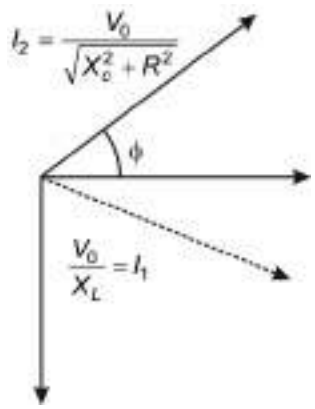
$\Rightarrow \frac{BR}{2\Delta t} \times QR\Delta t = mR^2 \times \omega$

$\Rightarrow \omega = \frac{QB}{2m}$

11. Answer (3)

Hint : Draw phase diagram of current in individual branch.

Sol. : $I_1 = I_2$



$\tan\phi = \frac{1}{\sqrt{3}}, \quad \phi = 30°$

Power factor = cos 30° = $\frac{\sqrt{3}}{2}$

12. Answer (4)

Hint : Focal length of concave lens, is

$\frac{d}{2} = 10$ cm, d = 20 cm

Sol. : Focal length of convex lens, is

$\frac{3d}{2} = 30$ cm

13. Answer (2)

Hint : For maximum angular magnification, final image should be at 25 cm.

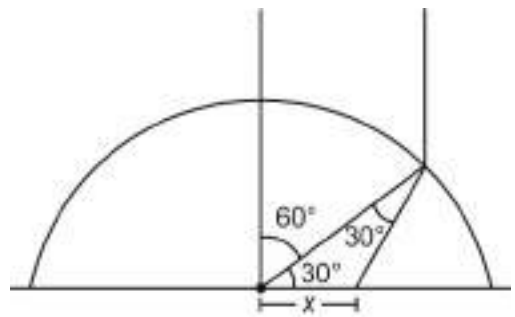
Sol. : $-\frac{1}{25} + \frac{1}{x} = \frac{1}{5}$

$x = \frac{25}{6}$ cm

14. Answer (4)

Hint : Apply Snell's law and trace the path of refracted ray.

Sol. : $\frac{\sin 60°}{\sin r} = \sqrt{3}$



$r = 30°$

$\frac{\sin 30°}{x} = \frac{\sin 120°}{R}$

$x = \frac{R}{\sqrt{3}}$

15. Answer (2)

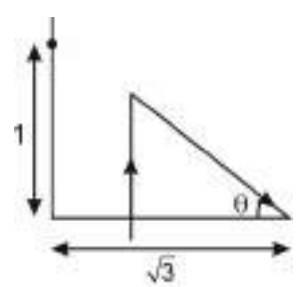
Hint : $\vec{E} \cdot \vec{B} = 0$

Sol. : $\vec{E}$ and $\vec{B}$ are always in same phase.

16. Answer (2)

Hint : Slope of reflected ray can be found by joining line passing through $(\sqrt{3}, 0)$ and focus (0, 1)

Sol. : $\tan\theta = \frac{1}{\sqrt{3}}$



$\theta = 30°$

$\delta = 120°$

17. Answer (2)

Hint : $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

Sol. : $\frac{1}{v} - \frac{1}{15} = -\frac{1}{10}$

$v = -30$ cm, $m = -2$

$v_P = -20\hat{i}$ m/s

18. Answer (4)

Hint : Take the clue from $\delta - i$ graph

Sol. : Angle of deviation should decrease hence angle of emergence should also decrease on increasing angle of incidence.

19. Answer (3)

Hint : $P_{avg} = \frac{v_{rms}^2}{R}$

Sol. : $v_{rms} = \sqrt{\frac{2^2 + 1^2}{2}} = \sqrt{\frac{5}{2}}$

$p_{avg} = \frac{5}{2 \times 20} = 0.125$ W

20. Answer (3)

Hint : $h_0 = \sqrt{h_1 h_2} = 2$ cm

Sol. : $h_0 = \sqrt{h_1 h_2} = 2$ cm

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{40} + \frac{1}{20} = \frac{1}{f}$

$f = \frac{40}{3}$ cm

21. Answer (15)

Hint : $E_0 = CB_0$

Sol. : $E_0 = CB_0$

$= 3 \times 10^8 \times 50 \times 10^{-9}$

$= 15$ V/m

22. Answer (20)

Hint : $v_L + v_R = 0$ after short circuiting

Sol. : $I_0 = \frac{\varepsilon}{R} = \frac{20}{20} = 1$ A

$\tau = \frac{L}{R} = \frac{100 \times 10^{-3}}{20} = 5$ ms

$\therefore \quad i = I_0 e^{-t/\tau} = \frac{1}{e}$ A

$\therefore \quad |v_L| = i \times R = \frac{1}{e} \times 20 = \frac{20}{e}$ volts

23. Answer (92)

Hint : Glass slab will change the actual distance

Sol. : $\Delta x = 4\left(1 - \frac{1}{4/3}\right) = 1$ cm

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} + \frac{1}{30} = \frac{1}{20}$

$v = 60$ cm

Glass slab placed will shift the final image again by 1 cm.

Distance = 61 + 31 = 92 cm.

24. Answer (20)

Hint : Image formed by one will behave as object for other.

Sol. : $\frac{1}{v} + \frac{1}{60} = \frac{1}{20}$

$v = +30$ cm

Rays will be parallel to principal axis after reflection.

25. Answer (15)

Hint : Rays should fall normally on the mirror

Sol. : $\frac{1.5}{v} + \frac{1}{30} = \left(\frac{1.5 - 1}{R}\right)$

$v = \infty$

$R = 15$ cm

PART - B (CHEMISTRY)

26. Answer (4)

Hint : Group 2A sulphides are insoluble in polyammonium sulphide.

Sol. : As_2S_3 is soluble in polyammonium sulphide.

27. Answer (4)

Hint : X is Iodide

Sol. : $NaI + H_2SO_4 \longrightarrow Na_2SO_4 + SO_2 + H_2O + I_2$

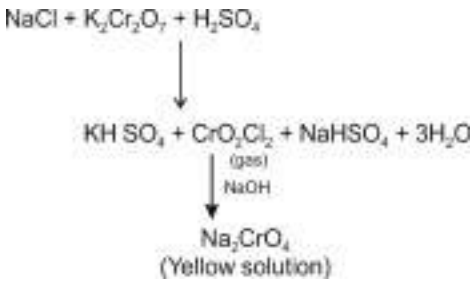
I_2 + starch solution $\longrightarrow$ Blue colour.

$NaI + AgNO_3 \longrightarrow AgI + NaNO_3$

28. Answer (3)

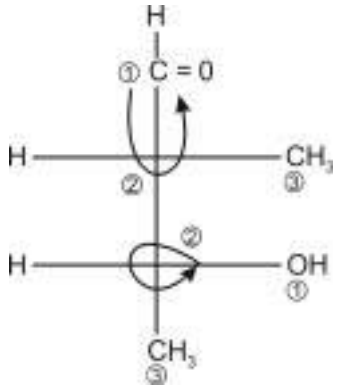
Hint : Chromyl chloride test.

Sol. :

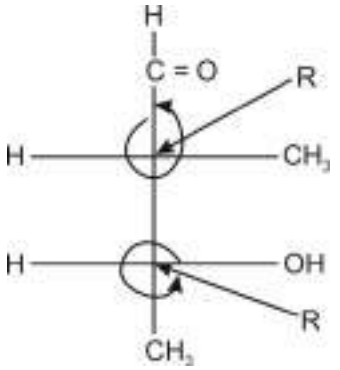


29. Answer (1)

Hint :



Sol. :



30. Answer (4)

Hint : In Wilkinson catalyst, Rh is present in +1 oxidation state.

Sol. : Wilkinson catalyst : $[Rh(PPh_3)_3Cl]$

31. Answer (1)

Hint : Wavelength absorbed

$$\propto \frac{1}{\text{strength of ligand}}$$

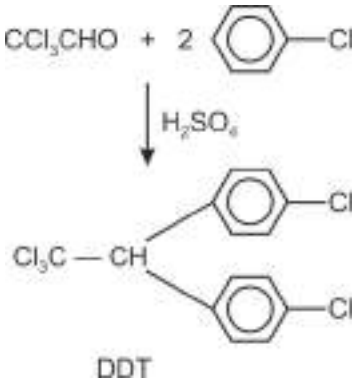
Sol. : Strength of ligand

$CN^- > NH_3 > H_2O > Cl^-$

32. Answer (4)

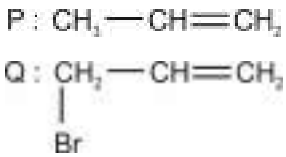
Hint : The product which is formed is DDT

Sol. :

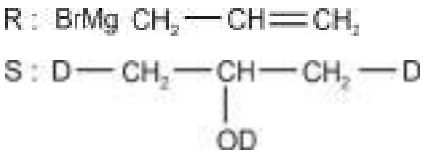


33. Answer (3)

Hint :



Sol. :



34. Answer (1)

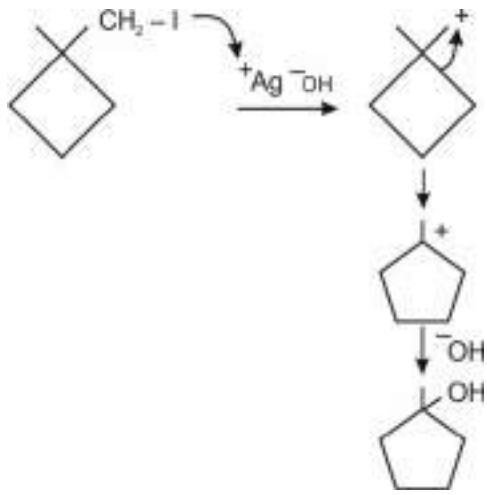
Hint : NaCl or NaBr which is formed as a byproduct in the given reaction are precipitated in dry acetone.

Sol. : It facilitates the forward reaction according to Le-Chatelier principle.

35. Answer (2)

Hint : Moist Ag_2O is AgOH.

Sol. :



36. Answer (4)

Hint : $Cd(OH)_2$, $Fe(OH)_2$ and $Mg(OH)_2$ are insoluble in excess of NaOH

Sol. : $Pb(OH)_2$ form a soluble complex $[Pb(OH)_4]^{2-}$ with excess of NaOH

37. Answer (3)

Hint : When sulphite ion treated with Zn produces H_2S and which turns the colour of MnO_4^- to colourless due to the formation of Mn^{+2}

Sol. : $SO_3^{2-} + Zn + H^+ \longrightarrow H_2S\uparrow + Zn^{2+} + H_2O$

$MnO_4^- + H_2S \xrightarrow{H^+} Mn^{+2} + S\downarrow + H_2O$

38. Answer (1)

Hint : $AgNO_3$, $Pb(OAc)_2$

$\downarrow S_2O_3^{2-}$

$\underset{\text{(white)}}{Ag_2S_2O_3}$, $\underset{\text{(white)}}{PbS_2O_3}$

$\downarrow \Delta$

$\underset{\text{(Black)}}{Ag_2S}$ $\underset{\text{(Black)}}{PbS}$

Sol. : Initially colour is white and on heating colour changes to black.

39. Answer (2)

Hint : Metal is Ni

Sol. : $Ni + CO \longrightarrow [Ni(CO)_4] \xrightarrow[\Delta]{} Ni + 4CO$

40. Answer (4)

Hint : CO and CN^- are SFL and Cl^- is WFL

Sol. : $[Ni(Cl_4)]^{2-}$ is sp^3, tetrahedral and paramagnetic

$[Ni(CN)_4]^{2-}$ is dsp^2, square planar and diamagnetic

$[Ni(CO)_4]$ is sp^3, tetrahedral and diamagnetic

41. Answer (1)

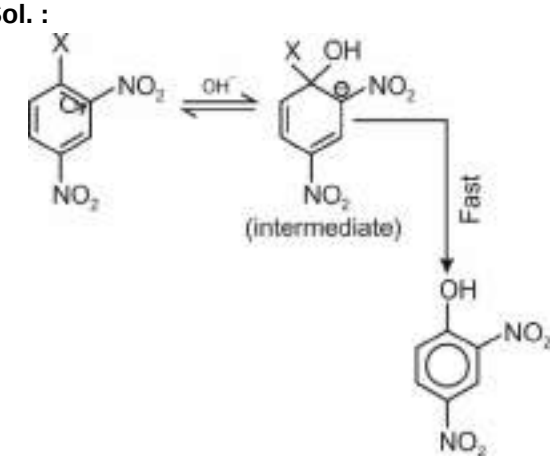
Hint : A and B are diastereomers.

Sol. : Compound C has only one pair of enantiomers.

42. Answer (1)

Hint : Order is A > B > C

Sol. :



Rate of reaction $\propto$ stability of carbanion

43. Answer (1)

Hint : α-haloketone shows the highest S_N2 rate

Sol. : Correct order is (1) > (2) > (3) > (4)

44. Answer (3)

Hint : Correct order : R > Q > P > S

Sol. : B.P. $\propto$ M.W.

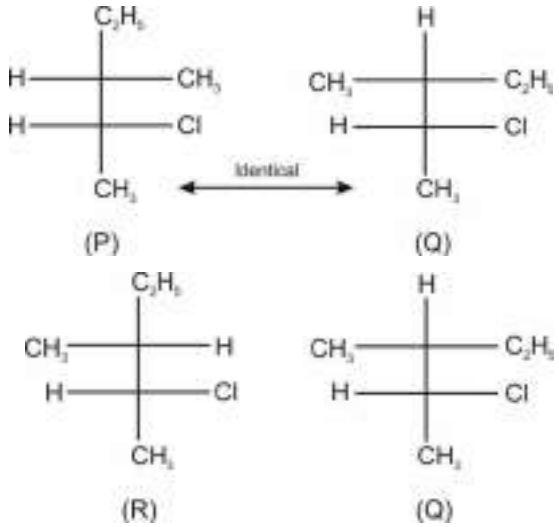
If M.W. is same then

$$\text{B.P.} \propto \frac{1}{\text{Branching}}$$

45. Answer (2)

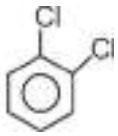
Hint : P and Q are identical while R is diastereomer of P

Sol. :

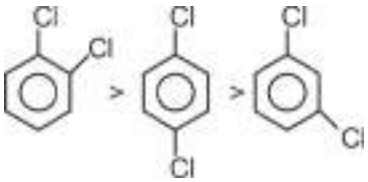


These are diastereomers.

46. Answer (02)

Hint : X ⟶ 

Sol. : The correct order of boiling point is



47. Answer (03)

Hint : Prussian blue complex is $Fe_4[Fe(CN)_6]_3$

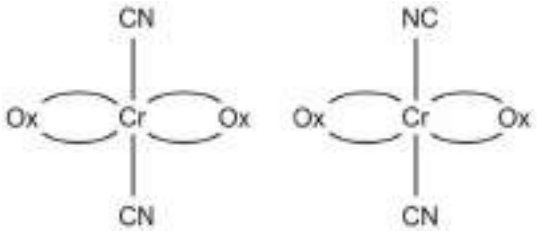
Sol. : $Fe^{3+} + [Fe(CN)_6]^{4-} \longrightarrow \overset{(III)}{Fe_4}[\overset{(II)}{Fe}(CN)_6]_3$

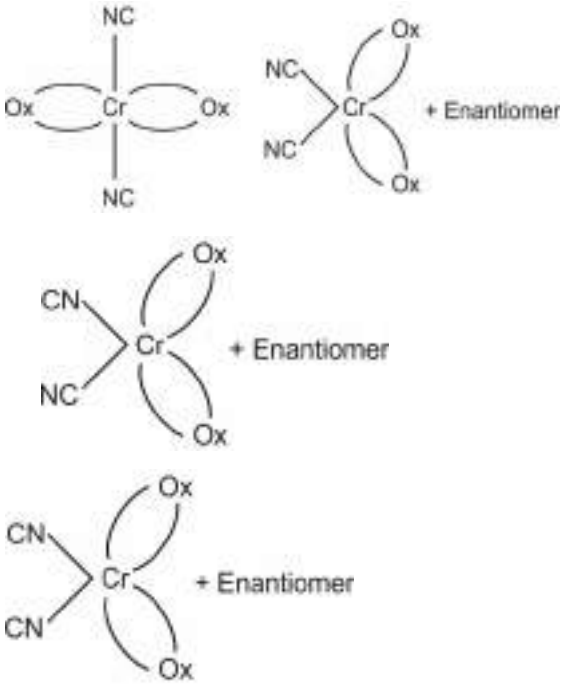
Prussian blue complex

48. Answer (09)

Hint : Due to presence of — CN, given complex can show linkage isomerism

Sol. :





49. Answer (01)

Hint : In $[M(H_2O)_4Cl(NO_2)]Cl$, M is in +1 oxidation state.

Sol. : Possible isomer is

$(NO_2)^+[M(H_2O)_4\ Cl_2]^-$

50. Answer (07)

Hint : Meso compound has plane of symmetry

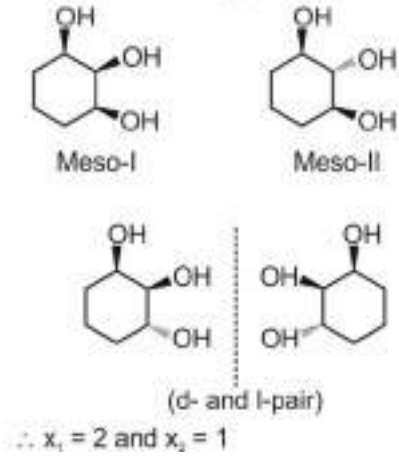
Sol. : $x_1 = 2$

$x_2 = 1$

$y_1 = 0$

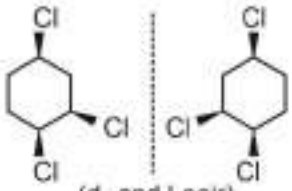
$y_2 = 4$

Stereoisomers of 1, 2, 3-cyclohexanetriol

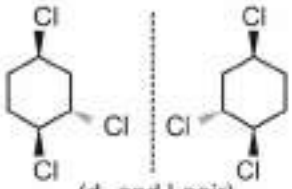


$\therefore x_1 = 2$ and $x_2 = 1$

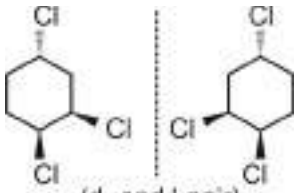
Stereoisomers of 1,2,4-Trichlorocyclohexane



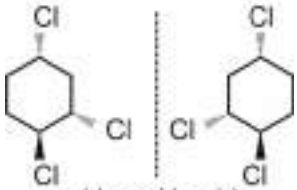
(d- and l-pair)



(d- and l-pair)



(d- and l-pair)



(d- and l-pair)

No meso isomer of it exists.

$\therefore$ $y_1 = 0$ and $y_2 = 4$.

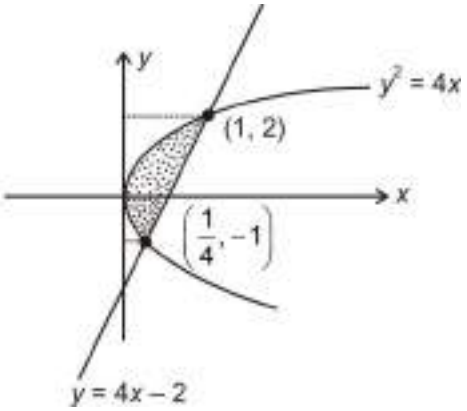
$x_1 + x_2 + y_1 + y_2 = 2 + 1 + 0 + 4 = 7$

PART - C (MATHEMATICS)

51. Answer (3)

Hint : Find area on *y*-axis

Sol. :



$$A = \frac{1}{2}\left(1+\frac{1}{4}\right)\times 3 - \int_{-1}^{2} \frac{y^2}{4}\,dy$$

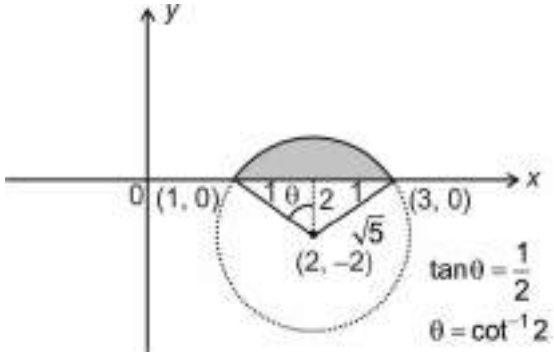
$$A = \frac{15}{8} - \frac{1}{12}\left[y^3\right]_{-1}^{2} = \frac{15}{8} - \frac{3}{4} = \frac{9}{8}$$

52. Answer (4)

Hint : Area = $A_{\text{sector}} - A_{\Delta}$

Sol. : $(y + 2)^2 = 5 - (x - 2)^2$

$\Rightarrow (x - 2)^2 + (y + 2)^2 = 5$



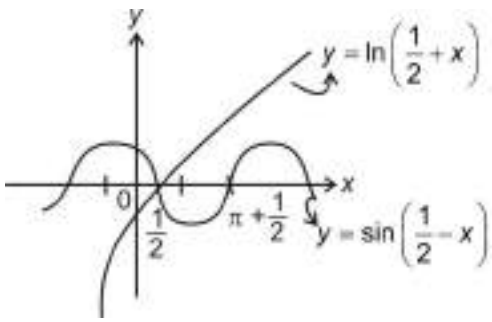
$$\Rightarrow \quad A = \frac{1}{2}\times\left(\sqrt{5}\right)^2\times\left(2\cot^{-1}2\right) - \frac{1}{2}\times 2\times 2$$

$= 5\cot^{-1} 2 - 2$

53. Answer (1)

Hint : Intersection point is $\left(\frac{1}{2}, 0\right)$

Sol. :



$$A = \int_0^{\frac{1}{2}} \left(\sin\left(\frac{1}{2} - x\right) - \ln\left(\frac{1}{2} + x\right)\right) dx$$

Put $\frac{1}{2} - x = t \Rightarrow dx = -dt$

$$A = \int_0^{\frac{1}{2}} \left(\sin t - \ln(1-t)\right) dt = \frac{3}{2} - \cos\left(\frac{1}{2}\right) - \ln\sqrt{2}$$

54. Answer (3)

Hint : Put $x = \sin^2\theta$

Sol. : $I = \int_0^1 x^3 \sqrt{1-x}\, dx$

Put $x = \sin^2\theta \Rightarrow dx = 2\sin\theta\cos\theta\, d\theta$

$$I = 2\int_0^{\frac{\pi}{2}} \sin^7\theta \cos^2\theta\, d\theta = 2\frac{\Gamma\left(\frac{7+1}{2}\right)\Gamma\left(\frac{2+1}{2}\right)}{\Gamma\left(\frac{7+2+2}{2}\right)} = \frac{32}{315}$$

55. Answer (3)

Hint : Use property

Sol. : $I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\tan\theta}\, d\theta}{\left(\sqrt{\tan\theta} + \sqrt{\cot\theta}\right)^3}$

$$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cot\theta}\, d\theta}{\left(\sqrt{\tan\theta} + \sqrt{\cot\theta}\right)^3}$$

$$2I = \int_0^{\frac{\pi}{2}} \frac{d\theta}{\left(\sqrt{\tan\theta} + \sqrt{\cot\theta}\right)^2} = \int_0^{\frac{\pi}{2}} \frac{d\theta}{\tan\theta + \cot\theta + 2}$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{\sin 2\theta}{1+\sin 2\theta}$$

$$I = \frac{1}{4}\int_0^{\frac{\pi}{2}} \frac{(\sin 2\theta + 1 - 1)\, d\theta}{\sin 2\theta + 1}$$

$$= \frac{1}{4}\cdot\frac{\pi}{2} - \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{d\theta}{1+\sin 2\theta}$$

$$I = \frac{\pi}{8} - \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{(1-\sin 2\theta)\, d\theta}{\cos^2 2\theta}$$

$$= \frac{\pi}{8} - \frac{1}{2}\int_0^{\frac{\pi}{4}} \left(\sec^2 2\theta - \sec 2\theta \tan 2\theta\right) d\theta$$

$$I = \frac{\pi}{8} - \frac{1}{4}\left[\tan 2\theta - \sec 2\theta\right]_0^{\pi/4}$$

$$= \frac{\pi}{8} - \frac{1}{4} = \frac{\pi - 2}{8}$$

56. Answer (3)

Hint : Use property

Sol. : $I = \int_0^{2\pi} \frac{\sec x\, dx}{\sec x + \cos x}$

$$\Rightarrow I = 2\int_0^{\pi} \frac{\sec x\, dx}{\sec x + \cos x} = 4\int_0^{\frac{\pi}{2}} \frac{\sec x\, dx}{\sec x + \cos x}$$

$$= 4\int_0^{\frac{\pi}{2}} \frac{dx}{1+\cos^2 x}$$

$$I = 4\int_0^{\frac{\pi}{2}} \frac{\sec^2 x\, dx}{1+1+\tan^2 x}$$

$$= 4\times\frac{1}{\sqrt{2}}\left[\tan^{-1}\frac{x}{\sqrt{2}}\right]_0^{\frac{\pi}{2}} = \frac{4}{\sqrt{2}}\cdot\frac{\pi}{2} = \sqrt{2}\pi$$

57. Answer (3)

Hint : $\theta \to \frac{\pi}{2} - \theta$

Sol. : $I = \int_0^{\frac{\pi}{2}} \frac{(2\sin\theta - \cos\theta)\, d\theta}{\sin\theta + \cos\theta}$ ...(i)

$\theta \to \frac{\pi}{2} - \theta$

$I = \int_0^{\frac{\pi}{2}} \frac{(2\cos\theta - \sin\theta)\, d\theta}{(\cos\theta + \sin\theta)}$ ...(ii)

From (i) + (ii)

$$2I = \int_0^{\frac{\pi}{2}} d\theta = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$

58. Answer (3)

Hint : $\frac{\sin^5 x + \cos^5 x}{\sin x + \cos x} = 1 - \frac{\sin 2x}{2} - \frac{\sin^2 2x}{4}$,

put $1 + \sin 2x = t$

Sol. : $I = \int \frac{(1+\sin 2x)(\cos x - \sin x)}{\sin^5 x + \cos^5 x} dx$

$$= \int \frac{\cos 2x}{\frac{\sin^5 x + \cos^5 x}{\sin x + \cos x}} dx$$

$$= \int \frac{\cos 2x\, dx}{\sin^4 x - \sin^3 x \cos x + \sin^2 x \cos^2 x - \sin x \cos^3 x + \cos^4 x}$$

$$= \int \frac{\cos 2x\, dx}{1 - \sin x \cos x - \sin^2 x \cos^2 x}$$

$$= \int \frac{\cos 2x\, dx}{1 - \frac{\sin 2x}{2} - \frac{\sin^2 2x}{4}}$$

$$= \int \frac{4\cos 2x\, dx}{4 - 2\sin 2x - \sin^2 2x} = \int \frac{4\cos 2x\, dx}{5 - (\sin 2x + 1)^2}$$

Put $\sin 2x + 1 = t \Rightarrow 2\cos 2x\, dx = dt$

$$I = \int \frac{2\, dt}{5 - t^2} = -2\int \frac{dt}{t^2 - 5}$$

$$= -2\frac{1}{2\sqrt{5}} \ln\left|\frac{t - \sqrt{5}}{t + \sqrt{5}}\right| + C$$

$$I = \frac{1}{\sqrt{5}} \ln\left|\frac{\sin 2x + 1 + \sqrt{5}}{\sin 2x + 1 - \sqrt{5}}\right| + C$$

59. Answer (3)

Hint : Put $\sec x = t$ in (2), (3) and (4)

Sol. : $I_1 = \int \sec^2 x \tan^2 x\, dx$, put $\tan x = t$

$$\Rightarrow I_1 = \frac{\tan^3 x}{3} + C$$

$I_2 = \int \sec^3 x \tan^3 x\, dx$, put $\sec x = t$

$$\Rightarrow I_2 = \frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C$$

$I_3 = \int \sec^3 x \tan^5 x\, dx$, put $\sec x = t$

$$\Rightarrow I_3 = \frac{\sec^7 x}{7} - \frac{2}{5}\sec^5 x + \frac{\sec^3 x}{3} + C$$

$I_4 = \int \sec^5 x \tan^3 x\, dx$, put $\sec x = t$

$$\Rightarrow I_4 = \frac{\sec^7 x}{7} - \frac{\sec^5 x}{5} + C$$

60. Answer (1)

Hint : $\frac{d(xy)}{y^2x^2} = \frac{xdy - ydx}{x^2}$

Sol. : $x\, dy + y\, dx = y^2 [x\, dy - y\, dx]$

$$\Rightarrow \frac{d(xy)}{y^2x^2} = \frac{x\, dy - y\, dx}{x^2}$$

$$\Rightarrow -\frac{1}{xy} + c = \frac{y}{x}$$

$\Rightarrow y^2 + 1 = cxy$

61. Answer (4)

Hint : Linear differential equation

$$\text{I}\cdot\text{F}\cdot = x + \sqrt{1+x^2}$$

Sol. : $\frac{dy}{dx} + \frac{y}{\sqrt{1+x^2}} = \sqrt{1+x^2} - x$

$$\text{I}\cdot\text{F}\cdot = e^{\int \frac{dx}{\sqrt{1+x^2}}} = e^{\ln\left(x+\sqrt{1+x^2}\right)} = x + \sqrt{1+x^2}$$

Then,

$$y\cdot\left(x + \sqrt{1+x^2}\right) = \int\left(x + \sqrt{1+x^2}\right)\left(\sqrt{1+x^2} - x\right)dx + c$$

$$y = \frac{x + c}{x + \sqrt{1+x^2}}$$

At $x = 0$, $y = 1 \Rightarrow c = 1$

$$y = \frac{x+1}{x + \sqrt{1+x^2}}$$

When $x = 1$, $y = \frac{2}{\sqrt{2}+1} = 2\left(\sqrt{2} - 1\right)$

62. Answer (1)

Hint : $\frac{dx}{x} = \frac{y^2dx - x\, d(y^2)}{x^2}$

Sol. : $xdx = y^2dx - xd(y^2)$

$$\Rightarrow \frac{dx}{x} = \frac{y^2dx - xd\left(y^2\right)}{x^2}$$

$$\Rightarrow \ln x = -\frac{y^2}{x} + c$$

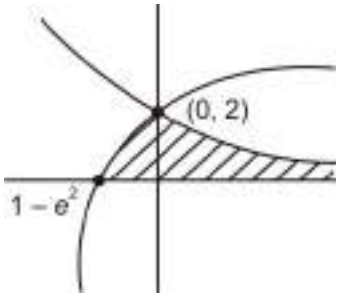
Put $x = 1$, $y = 0 \Rightarrow c = 0$

$y^2 + x \ln x = 0$

63. Answer (3)

Hint : Draw graph

Sol. : Required area =



$$\int_{1-e^2}^{0} \ln\left(x+e^2\right)dx + \int_{0}^{\infty} 2e^{-x}dx$$

$$= \int_{1}^{e^2} \ln t\, dt + 2\left(-e^{-x}\right)\Big|_0^{\infty}$$

$$= t\ln\left(\frac{t}{e}\right)\Bigg|_1^{e^2} + 2(1)$$

$= e^2 + 1 + 2$

$= e^2 + 3$

64. Answer (4)

Hint : Put $\ln y = t$ and $\frac{1}{x} = z$

Sol. : Let $\ln y = t \Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{dt}{dx}$

$$\Rightarrow (xt-1)\frac{dt}{dx} = \frac{1}{x}$$

$$\Rightarrow \frac{1}{x^2}\frac{dx}{dt} + \frac{1}{x} = t$$

Let $\frac{1}{x} = z \Rightarrow \frac{1}{x^2}\frac{dx}{dt} = -\frac{dz}{dt}$

$$\Rightarrow \frac{dz}{dt} - z = -t$$

$$\Rightarrow e^{-t}\cdot z = -\int t\, e^{-t}dt + c$$

$\Rightarrow e^{-t}z = e^{-t}(1 + t) + c$

$\Rightarrow z = (1 + t) + ce^t$

$$\frac{1}{x} = 1 + \ln y + cy$$

Put $x = 1, y = \frac{1}{e}$

$$1 = 1 - 1 + \frac{c}{e}$$

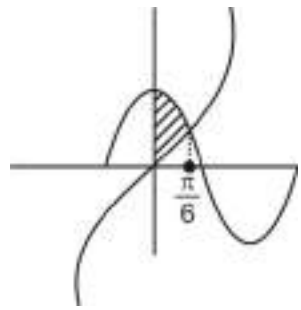
$c = e$

So $1 = x(1 + \ln y + ey)$

65. Answer (3)

Hint : Draw graph

Sol. :



$$A = \int_0^{\frac{\pi}{6}} (2\cos x - 3\tan x)\,dx$$

$$= 2\sin x - 3\ln\sec x\Big|_0^{\pi/6}$$

$$= 1 - 3\ln\left(\frac{2}{\sqrt{3}}\right)$$

66. Answer (4)

Hint : Put $x - 2 = t$

Sol. : $I = \int_0^4 \left(x^3 - 6x^2 + 9x + \sin(x-2) - 1\right)dx$

Put $x - 2 = t \Rightarrow dx = dt$

$$I = \int_{-2}^{2} \left((t+2)^3 - 6(t+2)^2 + 9(t+2) + \sin t - 1\right)dt$$

$$= \int_{-2}^{2} \left(\left(t^2 + 4 + 4t\right)(t + 2 - 6) + 9t + 18 + \sin t - 1\right)dt$$

$$= \int_{-2}^{2} \left(t^3 + 4t + 4t^2 - 4t^2 - 16 - 16t + 9t + 18 + \sin t - 1\right)dt$$

$$= \int_{-2}^{2} \left(t^3 - 3t + \sin t + 1\right)dt = \int_{-2}^{2} dt = 4$$

67. Answer (2)

Hint : $\cot^{-1}(-x) = \pi - \cot^{-1}x$

Sol. : $I = \int_{-\pi}^{\pi} \left\{\frac{\cot^{-1}x}{\pi}\right\} dx$

$= \int_{0}^{\pi} \left(\left\{\frac{\cot^{-1}x}{\pi}\right\} + \left\{\frac{-\cot^{-1}x+\pi}{\pi}\right\}\right) dx$

$= \int_{0}^{\pi} \left(\left\{\frac{\cot^{-1}x}{\pi}\right\} + \left\{\frac{-\cot^{-1}x}{\pi}\right\}\right) dx$

$= \int_{0}^{\pi} \left(\frac{\cot^{-1}x}{\pi} + 1 - \frac{\cot^{-1}x}{\pi}\right) dx = \pi$

68. Answer (3)

Hint : Put $\tan\theta = t$ and then $t + t^3 + 2t^2 = \lambda^2$

Sol. : Put $\tan\theta = t \Rightarrow \sec^2\theta\, d\theta = dt$

$$I = \int \frac{\left(\sqrt{t} + \sqrt{\frac{1}{t}}\right)(1+3t)\, dt}{\sqrt{1+t^2+2t}}$$

$$\int \frac{(t+1)(1+3t)}{\sqrt{t+t^3+2t^2}} dt = \int \frac{(3t^2+4t+1)\, dt}{\sqrt{t+t^3+2t^2}}$$

Put $t + t^3 + 2t^2 = \lambda^2$

$\Rightarrow (3t^2 + 4t + 1)\, dt = 2\lambda d\lambda$

$\Rightarrow I = \int \frac{2\lambda\, d\lambda}{\lambda} = 2\lambda + C$

$= 2\sqrt{t+t^3+t^2} + C$

$= 2\sqrt{\tan\theta + \tan^3\theta + 2\tan^2\theta} + C$

69. Answer (3)

Hint : When $x \in \left(0, \frac{\pi}{6}\right)$, $2\cos x - \sqrt{3} > 0$

Sol. : $0 < x < \frac{\pi}{6} \Rightarrow \frac{\sqrt{3}}{2} < \cos x < 1$

$\Rightarrow 2\cos x - \sqrt{3} > 0$

$\Rightarrow \cot^{-1}\left(\frac{2}{2\cos x - \sqrt{3}}\right) = \cot^{-1}\left(\frac{1}{\cos x - \frac{\sqrt{3}}{2}}\right)$

$= \tan^{-1}\left(\cos x - \frac{\sqrt{3}}{2}\right)$

$\Rightarrow I = \int_{0}^{\frac{\pi}{6}} \frac{\pi}{2} dx = \frac{\pi^2}{12}$

70. Answer (3)

Hint : Algebraic twins

Sol. : $I_1 = \int_{0}^{1} \frac{dx}{1+x} = \ln 2$

$I_2 = \int_{0}^{1} \frac{dx}{1+x^2} = \frac{\pi}{4}$

$I_4 = \int_{0}^{1} \frac{dx}{1+x^4}$

$= \frac{1}{2}\left[\int_{0}^{1} \frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+2} - \int_{0}^{1} \frac{\left(1-\frac{1}{x^2}\right)dx}{\left(x+\frac{1}{x}\right)^2-2}\right]$

$= \frac{1}{4\sqrt{2}}\left(\pi - \ln\left(3-2\sqrt{2}\right)\right)$

71. Answer (00)

Hint : Odd function

Sol. : $f(x) = \sin(\tan x) + \tan(\sin x)$ is odd function

$\Rightarrow \int_{-\pi}^{\pi} f(x)dx = 0$

72. Answer (00)

Hint : $\{\sin x\} = \sin x - [\sin x]$

Sol. :

$I = 2\left(\int_{0}^{\pi} \cos x\, e^{\sin x} dx + \int_{\pi}^{2\pi} \cos x\, e^{(\sin x+1)} dx\right)$

$I = 2\left(\left[e^{\sin x}\right]_0^{\pi} + \left[e^{\sin x+1}\right]_{\pi}^{2\pi}\right)$

$= 2\,(0 + 0) = 0$

73. Answer (01)

Hint : $\int e^{ax} \sin bx\, dx = \frac{e^{ax}}{a^2+b^2}$

$(a \sin bx - b \cos bx) + C$

Sol. :

$$\int e^{2x}\sin x\cos 3x\,dx = \frac{1}{2}\int e^{2x}\left[\sin 4x - \sin 2x\right]dx$$

$$= \frac{1}{2}\left[\int e^{2x}\sin 4x\,dx - \int e^{2x}\sin 2x\,dx\right]$$

$$= \frac{e^{2x}}{40}\left[2\sin 4x - 4\cos 4x - 5\sin 2x + 5\cos 2x\right] + C$$

$\Rightarrow f(x) = 2\sin 4x - 4\cos 4x - 5\sin 2x + 5\cos 2x$

$$f\left(\frac{\pi}{8}\right) = 2$$

$$f\left(\frac{\pi}{4}\right) = -1$$

74. Answer (20)

Hint : $\frac{dP}{dt} \propto P \Rightarrow \frac{dP}{P} = kt$ (where k is constant)

Sol. : $\frac{dP}{dt} \propto P$

$$\Rightarrow \frac{dP}{dt} = kP$$

$$\Rightarrow \frac{dP}{P} = kdt$$

$$\Rightarrow \ln P = kt + c$$

When $t = 0$, $P = 40 \Rightarrow C = \ln 40$

When $t = 10$, $P = 20 \Rightarrow k = -\frac{\ln 2}{10}$

$$\ln P = -\frac{\ln 2}{10}t + \ln 40$$

If $P = 5$

Then $\ln 5 = -\frac{\ln 2}{10}t + \ln 40$

$$\Rightarrow \frac{\ln 2}{10}t = \ln\left(\frac{40}{5}\right)$$

$\Rightarrow t = 30$ minutes

So 20 minutes are still required

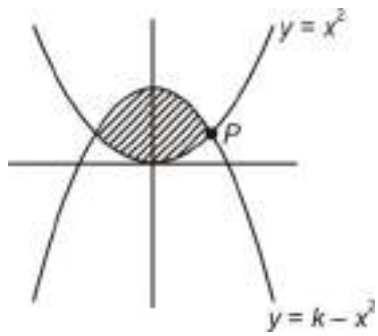
75. Answer (03)

Hint : Draw graph

Sol. : Point of intersection of two curves is

$$x = \pm\sqrt{\frac{k}{2}}$$

Area bounded by these two curves,



$$2\sqrt{6} = 2\int_0^{\sqrt{\frac{k}{2}}}\left(k - 2x^2\right)dx$$

$$\Rightarrow \sqrt{6} = k\sqrt{\frac{k}{2}} - \frac{2}{3}\left(\sqrt{\frac{k}{2}}\right)^3$$

$$\Rightarrow \sqrt{6} = \frac{k^{\frac{3}{2}}}{\sqrt{2}} - \frac{2k^{\frac{3}{2}}}{6\sqrt{2}}$$

$$k^{\frac{3}{2}}\left(\frac{4}{6\sqrt{2}}\right) = \sqrt{6} \Rightarrow k = 3$$

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